Mechanical Measurement Systems
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Lecture-35
Displacement and Acceleration Measurement

Hello I welcome you all in this course on mechanical measurement systems, today we will discuss the displacement and acceleration measurement; regarding displacement measurement we will discuss first we will discuss on LVDTs, then capacitive transducers; how capacitive transducers can be used for the measurement purpose and for acceleration measurement there accelerometer.

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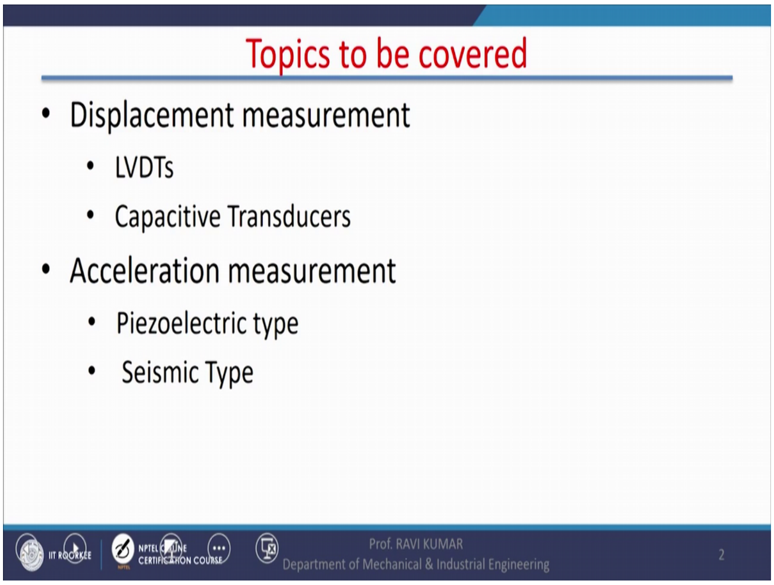

There are accelerometer which are piezoelectric type and seismic type, so both type accelerometers will be discussing here to begin with LVDT it is a very popular displacement measuring device this is linear.

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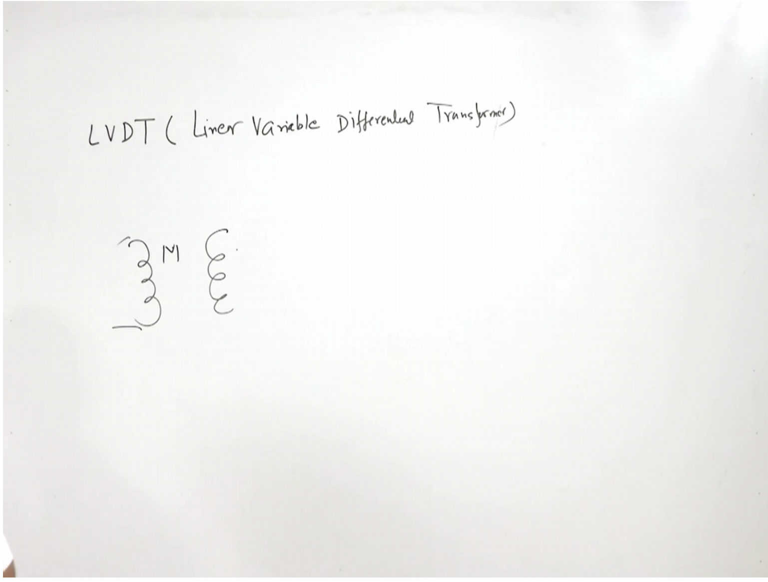

So, LVDT is linear variable differential transformer and as like other transformers this also works on the principle of mutual inductions, mutual inductors means there is a flux change in primary coil there is going to be the induced EMF in secondary coil and which depends upon the mutual inductance between the 2 coils, but in LVDT there is 1 primary coil and there are 2 secondary coils.

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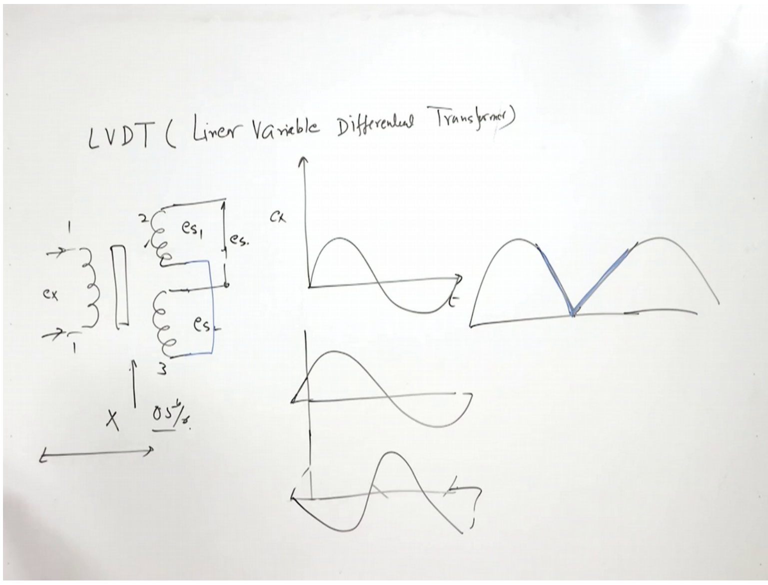

There is 1 primary coil and there are 2 secondary coils, the reason I will explain to you later. Now in this between these 2 coils we simply put an iron bar or an iron rod, so the flux interaction between these 2 coils can be increased. So, the moment this core it is called core right, so when the core moves in either direction right, there is a flux interaction between this coil 1 2 and 2 and 1 and 3 and due to this induced EMF produced right.

If you are able to measure this induced EMF and if we can relate this with the displacement in this core, then it this transformer can very well work for a as a displacement measuring device, the property of LVDT is it can measure wide displacement with 0.5 percent linearity that is why it is very popular. Now regarding the working principle if we start with suppose there is an excitation voltage, so it has to have ac excitation voltage with an RMS value ranging between 3 to 15 volts, this order of excitation is given to the primary coil and which produces induces this secondary coil secondary coil ES1.

Let us say this is es1 and this is ES2 right, so it is something like this excitation voltage excitation voltage and this is time. So, excitation voltage is like this it is an ac alternating current, there is a core if core is in the middle then it is balanced right and in the both the coils will get the wave form as like this in both the both the coils, but this is phase length right.

Now if I reverse the polarity I mean if I connect this 1 2 to 4 if I connect this 2 to 4 and take out this and measure voltage here right; that is why we take 2 coils now it will be clear to you when the core is in the middle in the neutral position there is going to be no output because, the output in the second coil has reversed right.

So, it will be something like this reverse of this, so the output will be cancelling out in es1 and ES2 and we are getting no deflection in the LVDT, but the moment we move in the either direction the deflection will start and the variation of this output ES is linear with the displacement in the core and finally we are going to get if we super impose these 2 wave forms and this is the linear part this is the linear part where we will be operating LVDT will be operating right because, here the variation of voltage with distance is linear.

Now let us do some analytical work on this LVDTs. So, first of all we will apply the Kirchhoff's law for voltage loop, so for here so for primary it is going to be current in primary sorry current in primary coil right, multiplied by the resistance of primary coil plus me self inductance of primary coil and primary coil divided by dt minus excitation is equal to 0 right.

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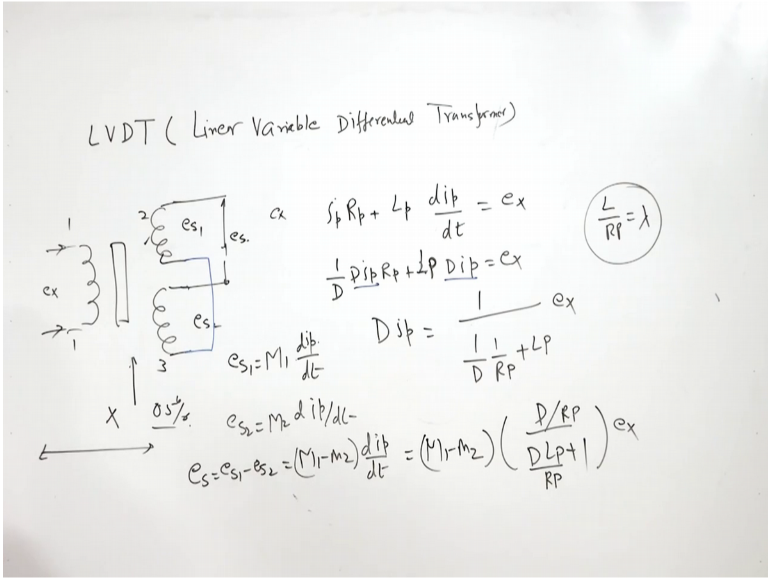

Now here excitation voltage we can take to the right hand side, now in the secondary side the secondary side es1 is equal to M1 Dip by dt, es2 is equal to M2 d ip by dt. So, difference of these 2 this is differential that is why it is called differential, now difference of es1 and es2 is coming here. So, it is going to be ES is equal to es1 minus es2 is going to be M1 minus M2 Dip by dt.

Now, let us go back to this equation now ip Rp can always be written 1 by D Dip Rp plus Lp Dip is equal to ex and from here we can get Dip as 1 by 1 by D 1 by Rp plus Lp ex; this Dip Dip from here Dip from here can be taken out and once we take the Dip and Dip out this is Lp right. So, we are going to get this expression, if you want to further simply this then we will get now this Dip is we can put here, if you are putting Dip here then output we are getting as, so output is M1 minus M2 d by DLp plus Rp ex.

Now from here if we divide this by Rp, so we will be getting this by Rp this is also Rp and here it will be 1. Now L by Rp can be taken as lambda time constant and we can all comfortably used first order equation solution for the first order equation and that is going to be equal to eo by this is the excitation voltage ex, D is equal to M1 minus M2 D sorry by lambda D plus 1.

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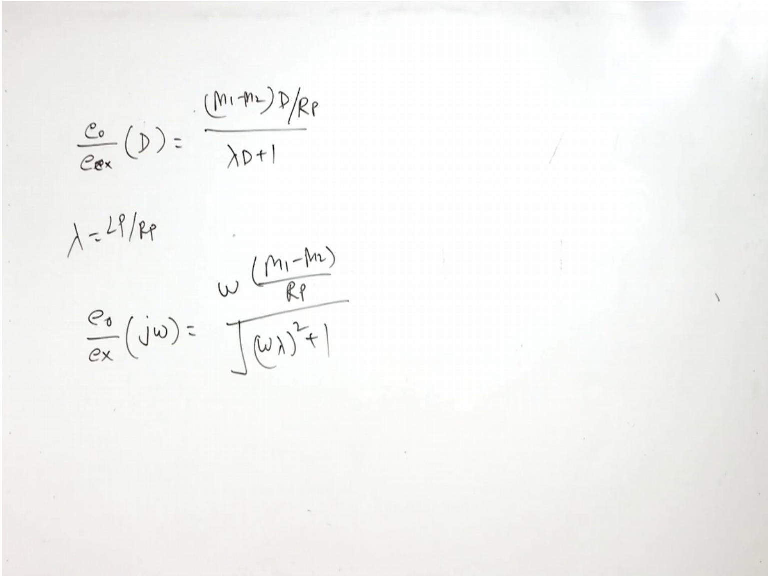

Where already lambda I have explained lambda is equal to Lp by Rp. Now LVDT can also be used suppose the frequency of input is 20 hertz Lp LVDT can go for a high frequency input also because, in potentiometer there is a limitation it cannot go of a more than 50 hertz; but LVDT we can go for higher input and in that case eo by excitation j omega or I omega this is D by Rp ok, then it is equal to omega M1 minus M2 divided by Rp divided by omega lambda whole square plus 1 right and rest of the analysis we can use we can do as we did in the case of first order systems.

Now LVDTs have variety of applications right and the best part of the LVDT is it can accommodate very high order of displacement, if you compare with the potentiometer and linearity is another benefit of using LVDTs and linearity is another benefit of using LVDT. So, after LVDTs we will go for the capacitive transducers, now in capacitive transducers it works on the principle that if the 2 plates which are charged with positive and negative charge right.

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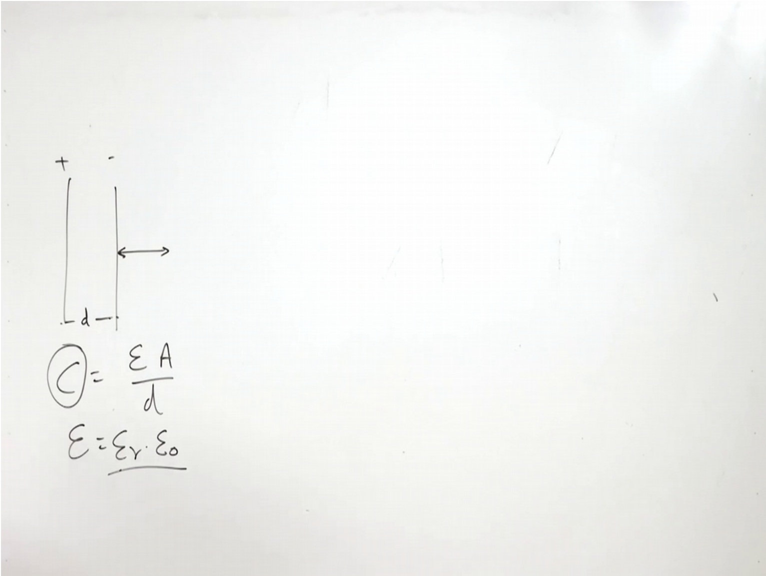

Then capacitance is equal to epsilon A by d epsilon is permittivity and if permittivity is equal to relative permittivity multiplied by absolute permittivity, d is the distance between 2 these 2 capacitance A is the cross section area.

So, the moment this plate is moved either in either direction this direction or this direction there is going to be change in C right and if you are able to measure the change in C or we can relate the C with this movement we can find the displacement. Now instead of using 1 if we have to use bank of capacitance or let us say 2 capacitance.

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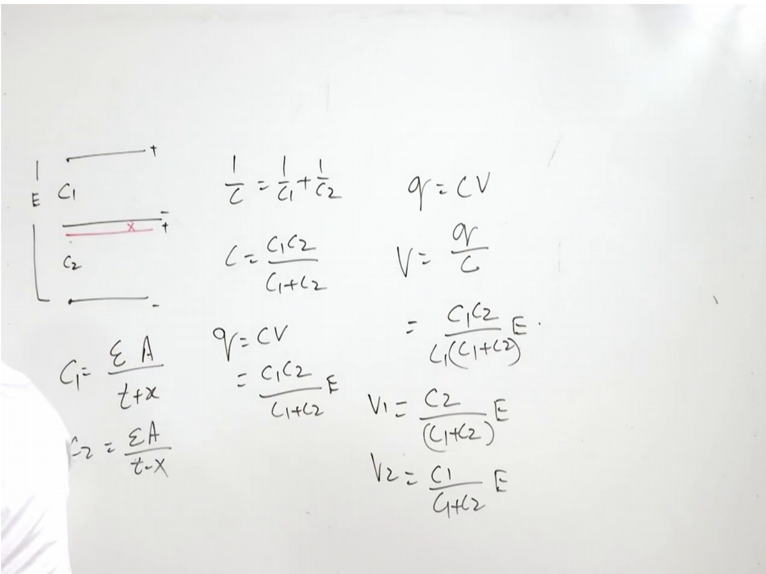

If 2 capacitance are used and they are put in series, so plus minus plus minus right and they are put certain voltage let the voltage E is applied between these 2 right and this capacitor has capacity C1 and this C2 right. So, total capacity of the capacitors is going to be because they are in series, 1 by C1 plus 1 by C2 or C is equal to C1 C2 by C1 plus C2 and charge on the plate is going to be q is equal to CV is equal to C1 C2, C1 plus C2 and E.

Now here there is a displacement initially there is a displacement in the middle plate and displacement in the middle plate is let us say it is t sorry X (Refer Time: 13:53) displacement sorry this is plus minus. Now displacement of middle plate is X this the middle plate displacement is X, now there is a displacement in the this plate of x and voltage is q is equal to CV right. If you want to have voltage V then we can take q by c right and then this q is nothing but C1 C2 by C1 plus C2 into E and this voltage q is divided by C2 it is divided by C1.

So, voltage is going to be C2 by C1 plus C2 multiplied by E right and similarly we can find the voltage for the second capacitor this is V1, similarly we can find the V2 as C1, C1 plus C2 into E right when we are talking if you are using this formula C is equal to epsilon naught d by sorry epsilon not A, A is constant epsilon sorry epsilon is same for every capacitor A is same only it is going to be C1 is d sorry t plus x and C2 is equal to epsilon A t minus x. Now putting these values here, here we will get the final expression as and will take the difference of V1 and V2.

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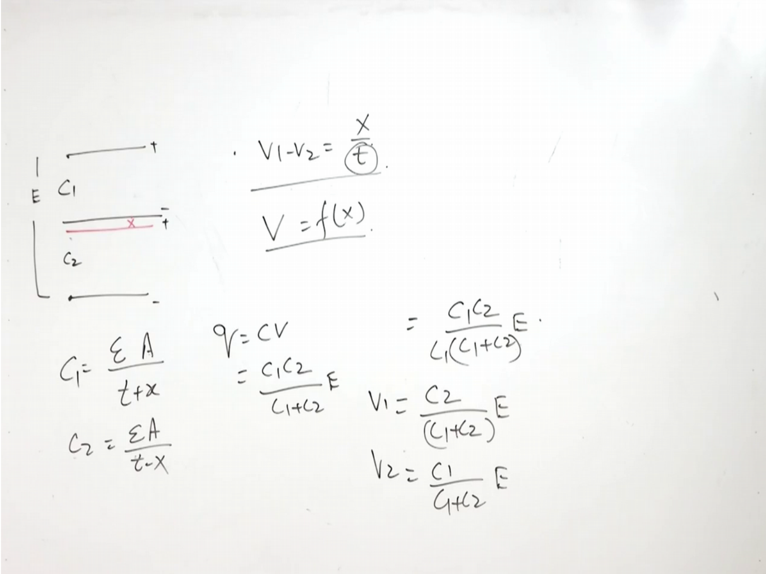

Then final expression for V1 minus V2 is going to be X by t. So, simply we can take V is a function of x because t is constant. So, that is how we can use these capacitive transducers for distance measurement and the property is the main property of these transducers is, they can sense very small displacement, they can sense the displacement of the order of let us say 2.5 microns 2.5 micrometer because you know 0.0025 millimeter.

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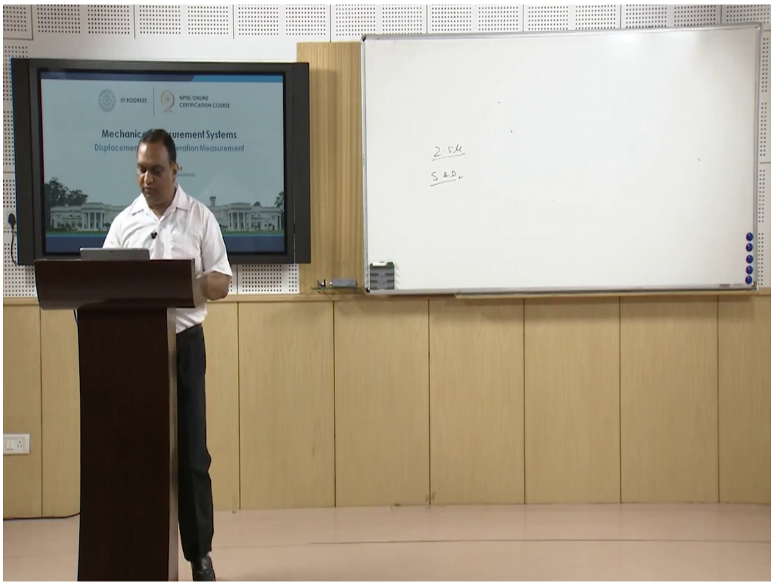

They are easy to fabricate they are not very costly and output linearity is also good in these transducers also, they do not have wide range about the output linearity is good for these trans transducers and they also may make measure static and dynamic input as well.

So, they are good for both for this static and dynamic input as well. So, after this velocity this displacement measurement, we will go for the acceleration measurement, acceleration measurement techniques.

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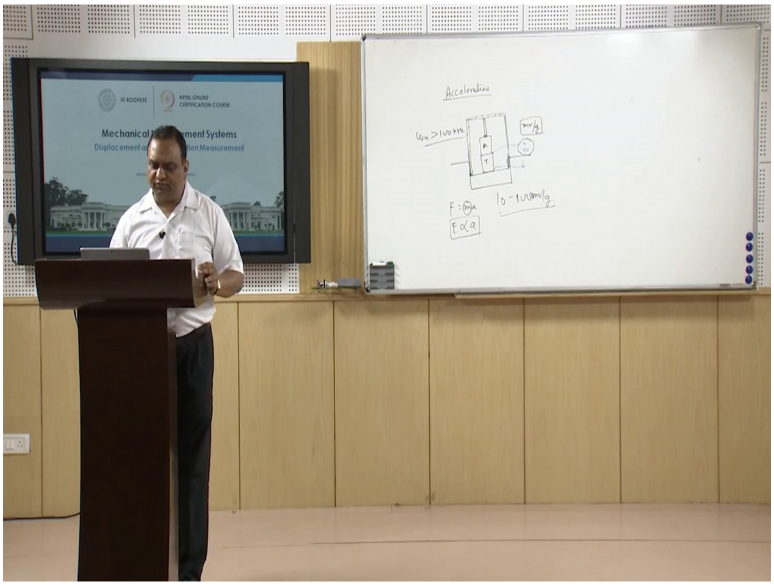

Now, acceleration measurement we can go for the piezoelectric type of accelerometer where piezoelectric sensor is used for acceleration measurement. So, in a in a base in the base this is a sub straight, where the this acceleration has to be measure and there is housing right and in this housing piezoelectric transducers is fixed right and above the piezoelectric transducer there is mass M mass M and this is transducer and this transducer and we have connections to this transducer, so we can always have output of this transducer. Now we know that force F is equal ma mass is constant here mass is not varying.

So, force is proportional to acceleration, it means force exerted on this transducers transducer is proportional to acceleration. Now acceleration is always expressed in terms of g acceleration due to gravitation. So, output will be expressed in terms of let us say mille volts per g because, acceleration is always this is this is standard acceleration. So, when the entire system is moving up when it is moving up, the force on this transducer will increase and when it is coming down, then this force on this transducer will reduce right, because acc acceleration due to gravity will change when it is moving either moving up or it is either moving down with certain acceleration. Accordingly we will get the output from the transducer and that is how we can make or we can we can find output of transducer is a function of acceleration in the body and this is how the acceleration is made is measured in with the help of piezoelectric transducers.

And first of all they are not very expensive why we use piezoelectric transducers because, they are not very expensive, they have output impedance is high ok; frequency response we can go up to 10 to 15 hertz for frequency response, sensitivity is approximately 10 to 100 mille volts per g right. This is the sensitivity of the this is the range of sensitivity of this transducers, but these transducers are temperature sensitive, I mean if there is a change in temperature the output will be affected.

So, temperature has always interfering effect in the case of these type of accelerometer and these accelerators are also subjected to hysteresis error, so there are 2 drawbacks of these type of transducers. Now in these transducers we use the piezoelectric crystal which has natural frequency having 100 kilo hertz. So, natural frequency of the piezoelectric crystal has to be higher than 100 kilo hertz right and the benefit of these type of accelerometer is they are very light in weight because, primary sensing element has I mean very I mean very 5 grams or 10 grams. So, they are very light in weight that goes in the favor of these type of transducers. Now another type of acceleration measuring transducer is seismic type.

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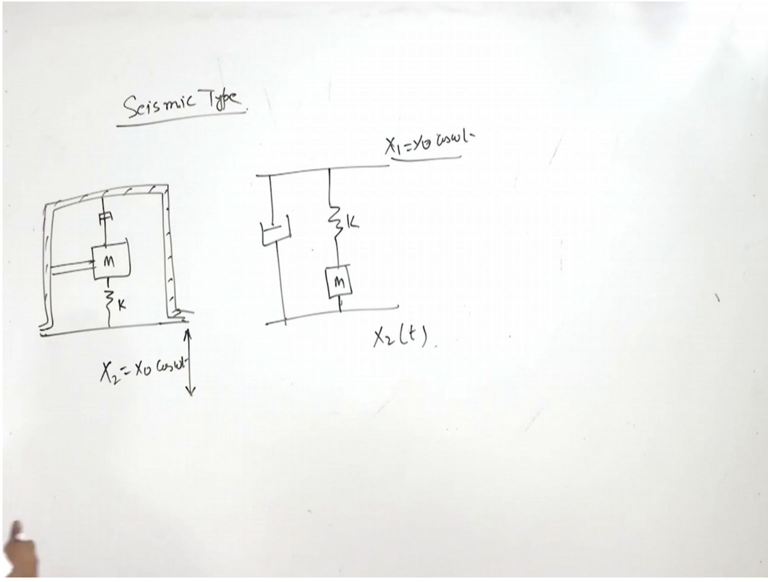

Now, seismic type of acceleration measuring devices. So, is sesic seismic type of system again there is a substrate and on the substrate this transducer is fixed for this for measurement of acceleration, it has a seismic mass M it has seismic mass M balanced on a spring and a dash board. So, there is a mass hanging in this accelerometer on a spring with some constant K and a dashboard right and for the measurement of displacement there is a arrangement for the measurement of displacement X and now if this substrate is under vibrations.

So, vibrations are let us say X1 is equal to Xo cos omega t. So, this will also vibrate and this vibration will have certain amplitude right. So, we can transform this arrangement like this, there is a spring k fixed to mass M mass M there is a dashboard and this is X2 t, this is let us say this is X2 and this is X1 is equal to Xo cos omega t. Now we can do this vibrations analysis. So, will start with the basic equation the basic equation is I will rub it off.

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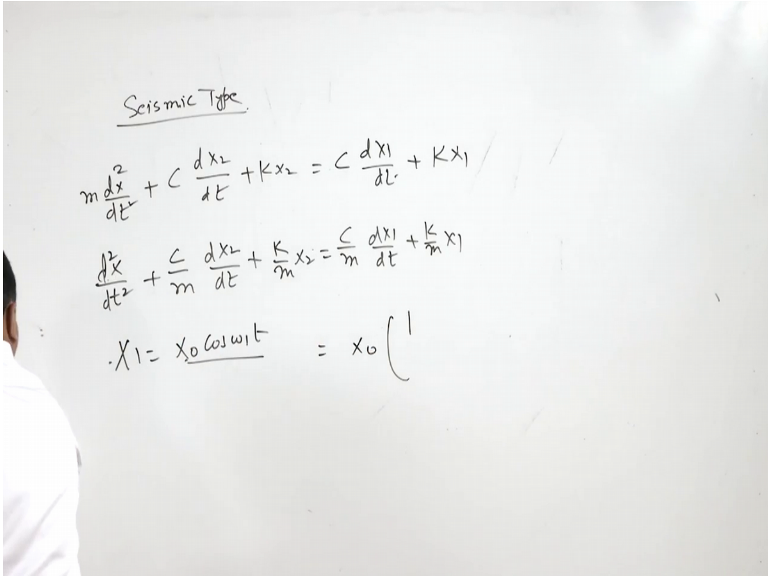

So, the basic equation is md square x by dt square plus c dx 2 by dt plus kx 2, this is generalized equation for the vibration having a spring and is equal to here C dx 1 by dt plus kx 1 right, so this is input this is input and this is output. Now here we can further modify this equation as d square x dt square this is dt square plus C by m dx 2 by dt plus k by m x 2 is equal to C by m dx 1 by dt plus k by m x 1 right. Now after this if we want to have solution for this then X1 can be put as X1 is equal to Xo cos omega 1 t now the moment we put this X1 is equal to Xo cos omega t, now this equation becomes Xo now this will be copied here the same.

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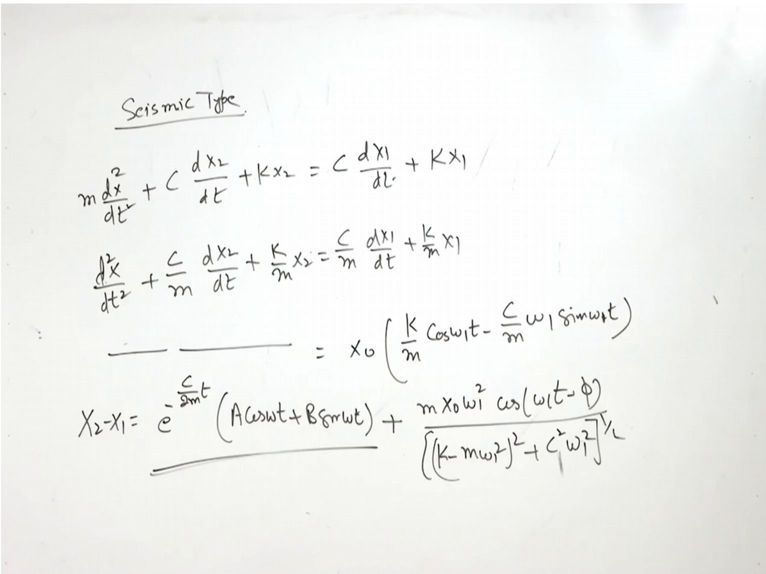

It will remain same is equal to xo k by m cos omega 1 t minus C by m omega 1 sin omega 1 t ok, because x is equal to xo cos omega t omega 1 t.

So, we are just differentiating it first once differentiating it once and then k by m also vibrating right. Now solution for this now if you want to have solution for this that is the X2 minus X 1 are going to be e this to power c by 2 mt A cos omega t plus B sin omega t plus m Xo omega 1 square cos omega 1 t minus phi divided by k minus m omega 1 square whole square plus C square omega 1 square raise to power 1 by 2 and this is out of this solution out of this solution it is we will deal it as a second order equation, out of this solution this is a transient phenomena.

So, it will die out after sometime so if this is vibrating I mean the sub straight is vibrating this is going to be the response final response, because this is the transient phenomena with time it will die out right.

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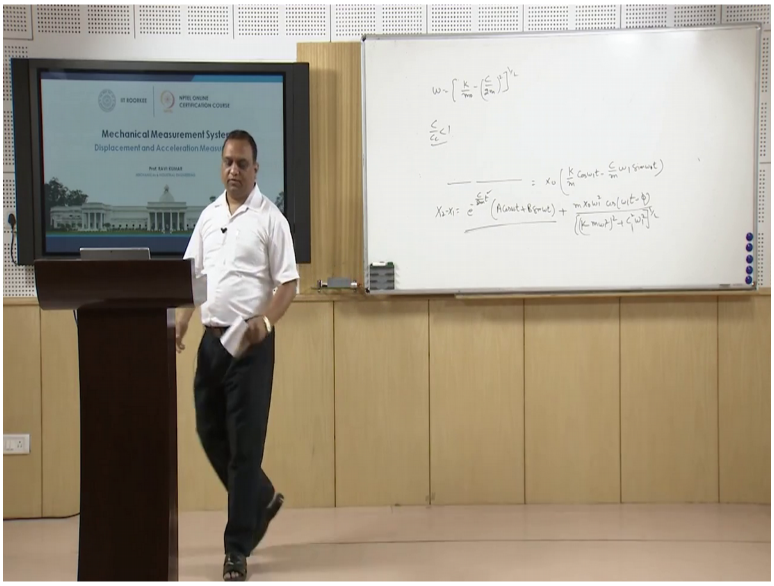

So, we can find from here omega is equal to k by above minus c by 2 m whole square 1 by 2 when for C by C is less than this C by C is less than 1 for 100 m type of system. Now after this if we if we want to have steady state harmonics they are going to be X 2 minus X1 o is equal to X0 omega 1 by omega n whole square divided by 1 minus omega 1 by omega n square plus 2C by CC omega 1 by omega n square n total rest power 1 by n.

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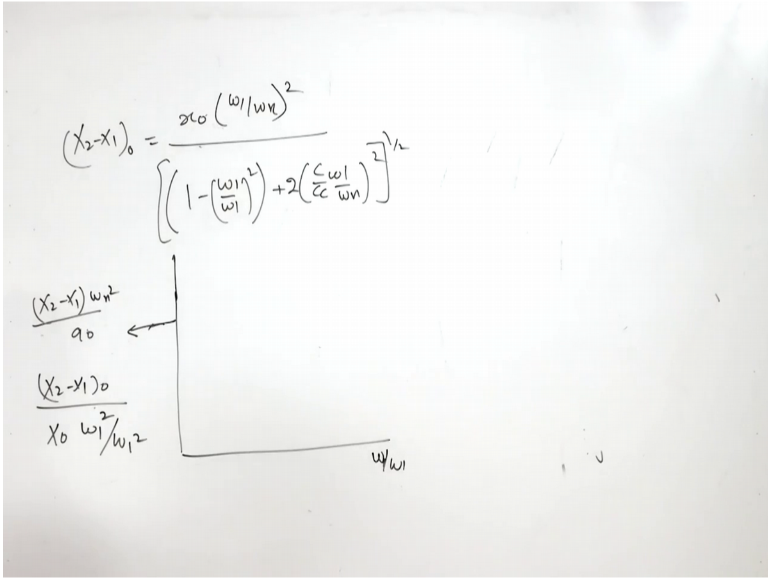

Now, if you draw a curve for this, now if you draw a curve for on axis we can take omega 1 by omega n on ordinate on ordinate we will take x 2 minus x 1 omega n square divided by ao. Now how we are taking ao is the amplitude of input where ao is the amplitude, where a is the amplitude x 2 minus x 1 omega how we are getting this? We are getting this when we just take this x 2 minus x 1 o divided by x o omega 1 square divided by omega n square. Now xo omega 1 square is ao omega 1 square by omega minus square right. So, this x o omega 1 square is ao. So, between these 2 x 2 minus x 1 omega square by ao and omega 1 by omega n. So, this omega 1 by omega n right.

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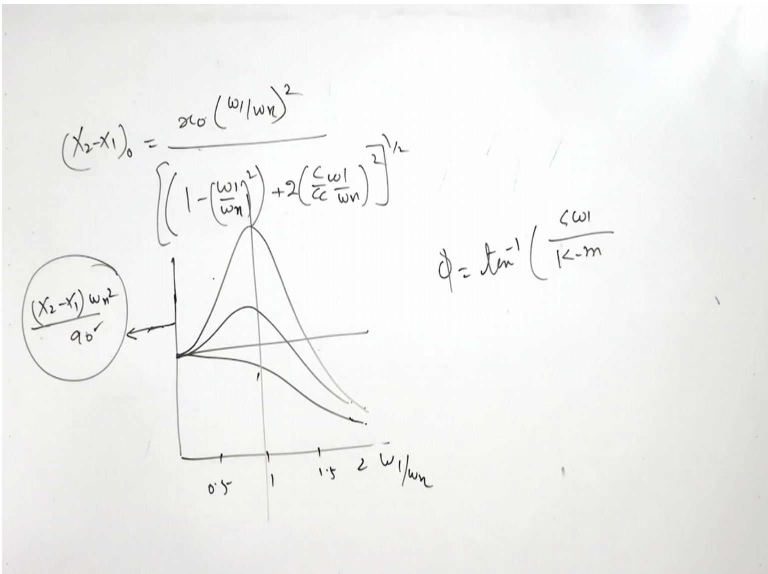

So, this is omega 1 by omega n and if we draw a curve for this for different value of c by cc. So, when c by cc is 1 the curve is like this, and omega by omega n I will give you the values this is 0.5, 1, 1.5 and 2. So, this is c by cc 1. When we take c by cc point let us say 0.4, it is going to be like this and definitely if it is 0.1 the curve is going to be like this right. Now from here we have to find the relation between displacement and the acceleration of the body. Now for this purpose regarding yes there is a going to phase angle also and the phase angle is going to be in this case phi is equal to ten inverse c w 1 divided by k minus m this is not for this 1.

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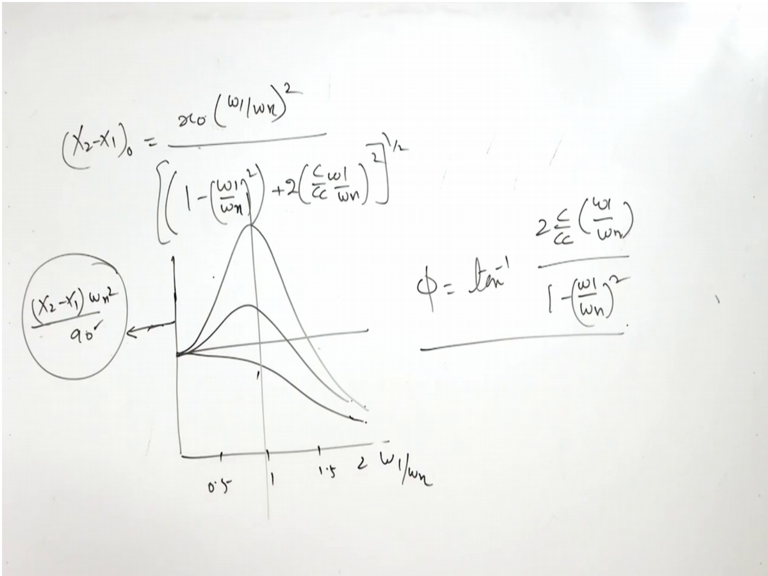

Now phase angle phase angle here is going to be phi is equal to 10 inverse 2 c by c c omega 1 by omega n divided by 1 minus omega 1 by omega n whole square because it is a cyclic input.

So, for cyclic input this is going to be the phase angle right. I will take one numerical then it will be clear how the acceleration is measured with the help of these type of instruments, then this seismic instrument to be used for the measurement of linear acceleration, it has omega and natural frequency 100 hertz radius radiance per second. So, there is a small seismic instrument used for measuring the acceleration and omega n is equal to 100 cycles per second.

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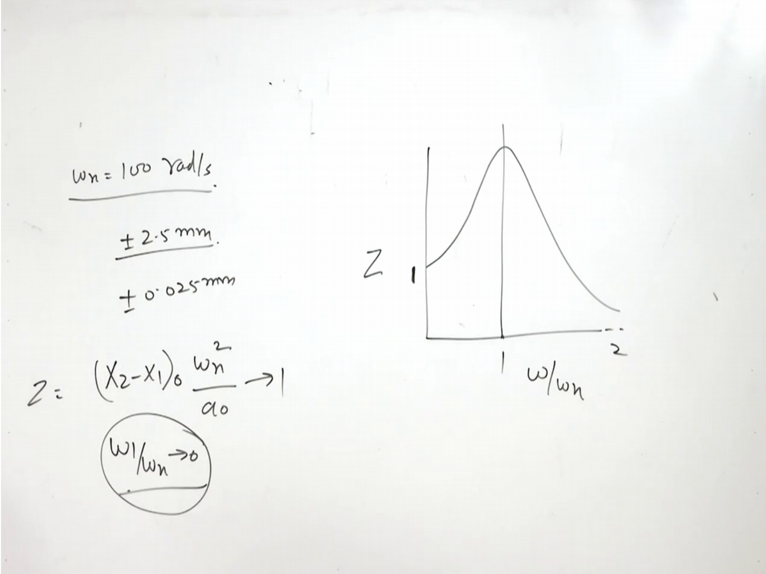

Sorry radiance per second not cycles per second radiance per second fine at a displacement sensing transducer, which detects a maximum of plus minus 2 mm. So, maximum detection of displacement transducer is plus minus 2.5 mm ok. The certainty in displacement measurement is uncertainty; uncertainity in the displacement measurement is plus minus 0.025 mm this is the uncertainty in the measurement.

Calculate the maximum acceleration that being measured with this instrument and the uncertainty in measurement assuming omega n is known exactly. So, we know the natural frequency, it is exact 100 radiance per second there is no error in this. Now for this is the maximum measurement which can be measured with a with a LVDT.

Now, let us look back the expression for acceleration parameter that is X 2 minus X 1 o omega n square divided by ao right. Where ao is maximum w 1 by w n is standing to 0. So, if you look at this graph where we took this let us say this is z parameter. So, z parameter and omega by omega n omega by omega by and omega n minus 1 then z parameter was the highest and then it came down this is 2 let say, omega by omega n 2 and this is value 1. So, when omega by omega 1 by omega n standing to 0, and this is standing to 1 or this is equal to 1.

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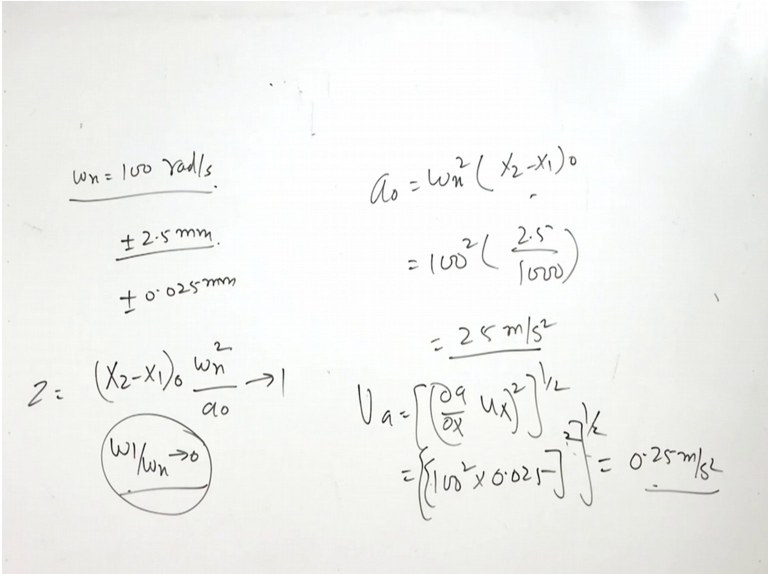

So, ao can be written as omega n square x 2 minus x 1 o. So, it is 100 square divided by 2.5 divided by 1000 and we are going to get 2.5 meter per second square. So, uncertainty the measurement of a is equal to del a by del x, uncertainty in the measurement of x whole square rest to the power 1 by 2. So, del a by del x is omega n square is 100 square, and uncertainty in the measurement of the x is 0.025 and then we take sorry whole square and then it is half.

So, out of this if we solve this we will be getting like 0.25 meter per second square as the uncertainty in the measurement acceleration that is all for today.

Thank you very much.