

Mechanical Measurement Systems
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Lecture-34
Force and Torque Measurement

Hello I welcome you all in this course of Mechanical Measurement Systems, today we will discuss on force and torque measurement. Now force is expressed in I mean as per the Newton's second law it is F is proportional to Acceleration and F is equal to ma.

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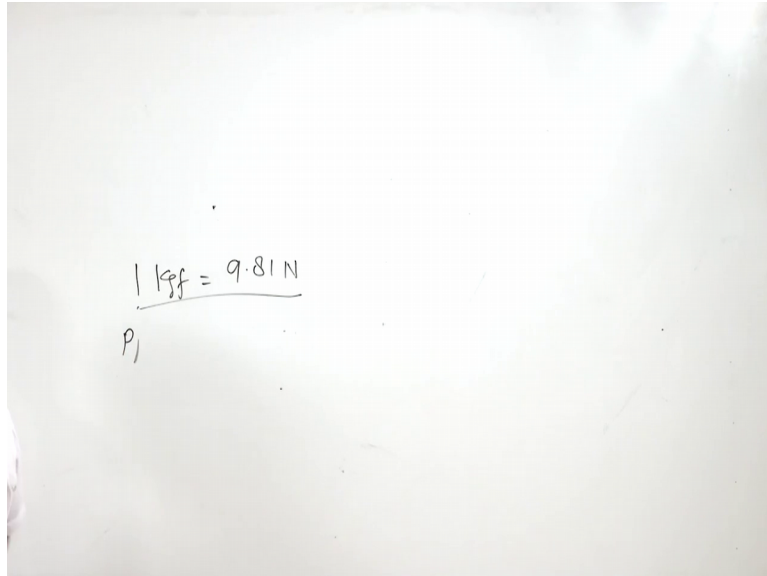
$$F \propto a$$
$$F = ma \rightarrow F \cdot x = W$$
$$\textcircled{T} = T \times \omega = P$$
$$T \cdot 2\pi n = P$$
$$T = \frac{P}{2\pi n}$$

So, in today's lecture we will be discussing force and torque. So, this is force and torque is suppose there is a shaft right and twisting movement is exerted on the shaft, when the twisting movement is exerted on the shaft that will cause the torque right. If we multiply force with displacement F into x it is work, energy and if we multiply torque with omega that is also work energy, torque omega can also be expressed by $2\pi n$ right and this is again work.

So, torque is equal to output divided or this is this is the output not work it will give the output in power, it is work per second it is going to be power. So, torque is going to be the power of the shaft power transmitted by the shaft divided by rotational speed of the shaft. But how to measure the force and the work we are going to discuss here force also yes force is also expressed in terms of kilo gram force it is an n case unit. So, 1 kilo gram

force is equal to 9.81 Newton's, but in SI unit in SI unit force is expressed in terms of Pascal.

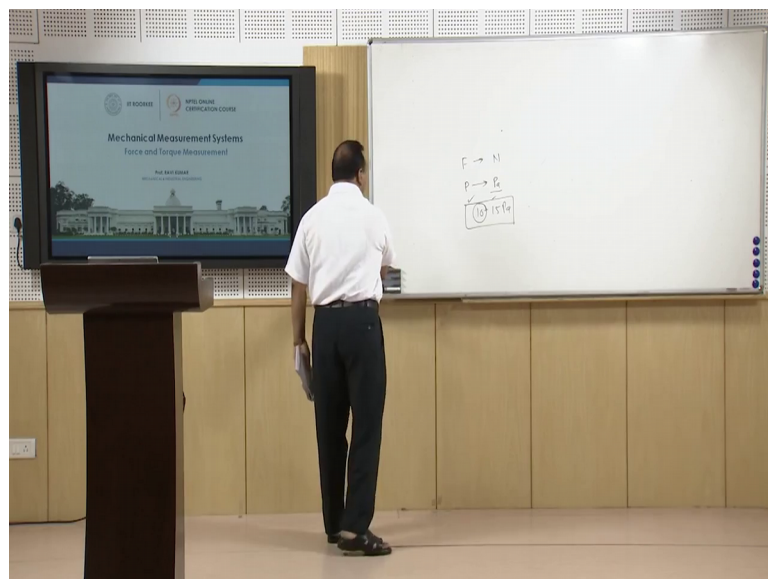
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$$1 \text{ kgf} = 9.81 \text{ N}$$

P

Sorry Newton force is expressed in terms of in the unit of Newton's and pressure any amount of pressure is expressed in terms of Pascal and kilo Pascal even very low pressure is expressed in terms of Pascal's.

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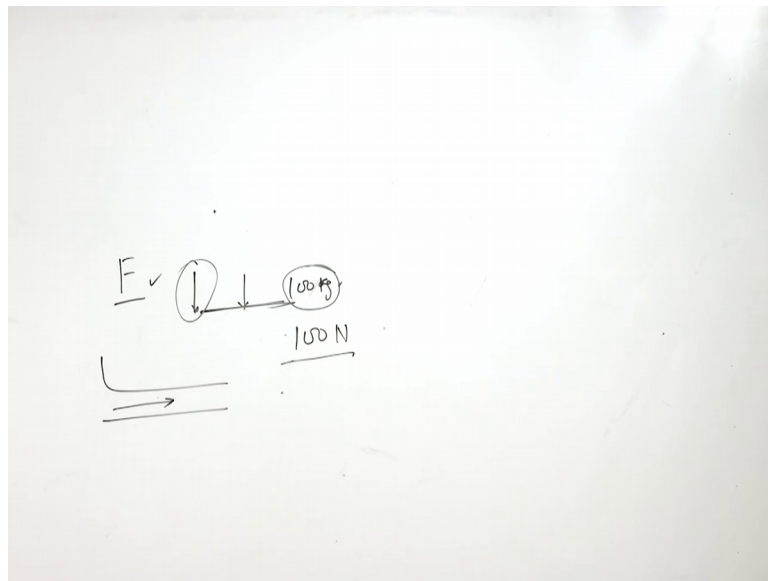


For example, some of suppose for example, this room if I pressurize this room, so pressurization may go up to 10 to 15 Pascal. If you look at the 10 Pascal it is equivalent

to 1 millimeter head of the water right. So, this is of the this is or this is this is the order of pressurization of the buildings large this buildings are also pressurized, if the building is pressurized then dust and all will not enter the building. So, the system will remain I mean specially when the building is occupied by the electrolysis system there is a lab occupied by the electronic system it will remain neat and clean.

So, now we are discussing about the force we are discussing about the force.

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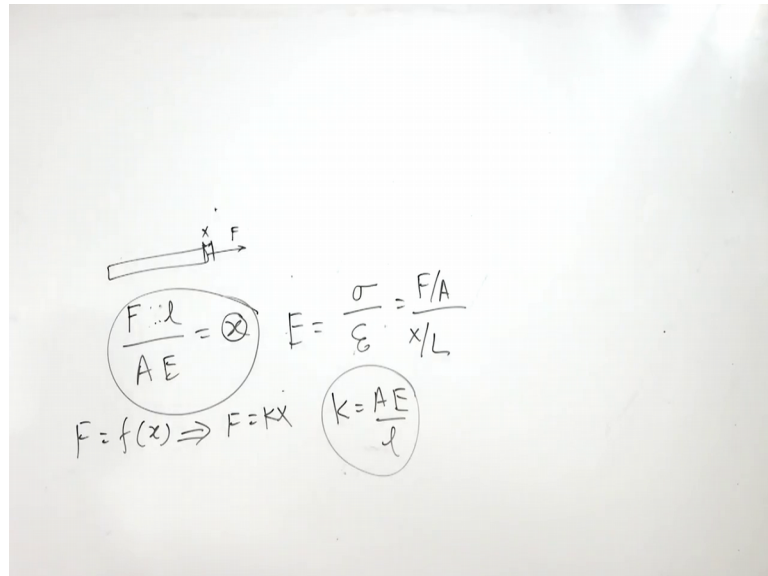


How to calculate the force? There are many ways of calculating the force and there are many reliable ways of calculating the force, first of all we can use balance; balance can also be used for measuring the force because the force against the known gravitational force and if it is balanced if it is balanced, then we will say that the force is equivalent to at gravit suppose on this side we have having 100 kg, this is the known gravitational force 100 kg force.

So, 100 Newton's if this balance is this side also this side also we are applying 100 Newton's or the balance if the balance is or the arm of the balance is in the central position or is no deflection then this unknown force is also 100 Newton. So, that is very primitive and very relative way of reliable way of measuring the force. Another is transmission of force through a fluid as a pressure in the form of pressure, the force is converted into the pressure and this fluid pressure is exerted on primary cell let it be any primary sensing element right. So, that is that is another way of measuring the force,

third way is suppose there is an elastic member elastic member suppose there is a bar. I want to make I want to use this bar for measurement of force.

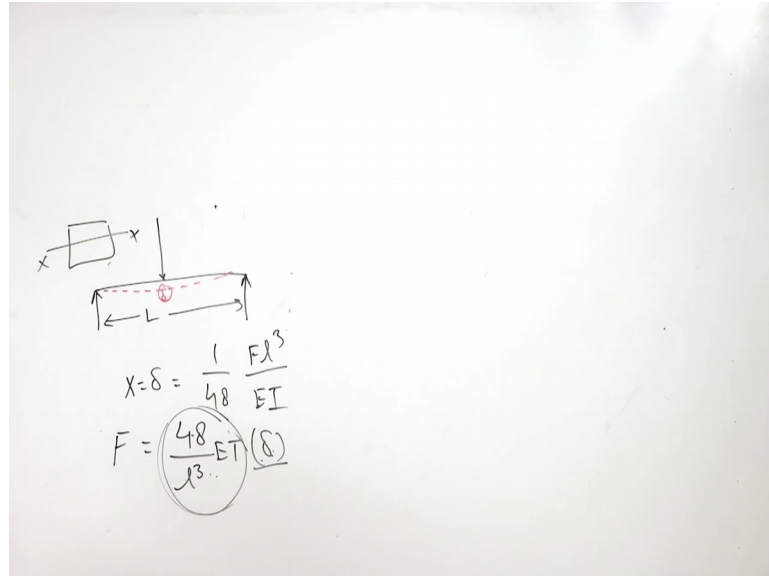
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So, suppose the force is applied here right, when the force is applied here then force multiplied by the length divided by A into E is equal to x , force into length $F l$ by $A E$ because, is change in the length of the bar it is simply case of modulus of elasticity stress upon strain. Stress is force by area strain is x by L and from here we got the value of x like this and this x can be measured here and we can find.

So, we can have a force as a function of x or we can say if we say F is equal to kx , then here the k is $A E$ by l a constant because, a cross section area is a constant modulus of elasticity is a constant and length is also a constant. Now like this we can also make use of a simply supported beam simply supported beam can also be used for force measurement.

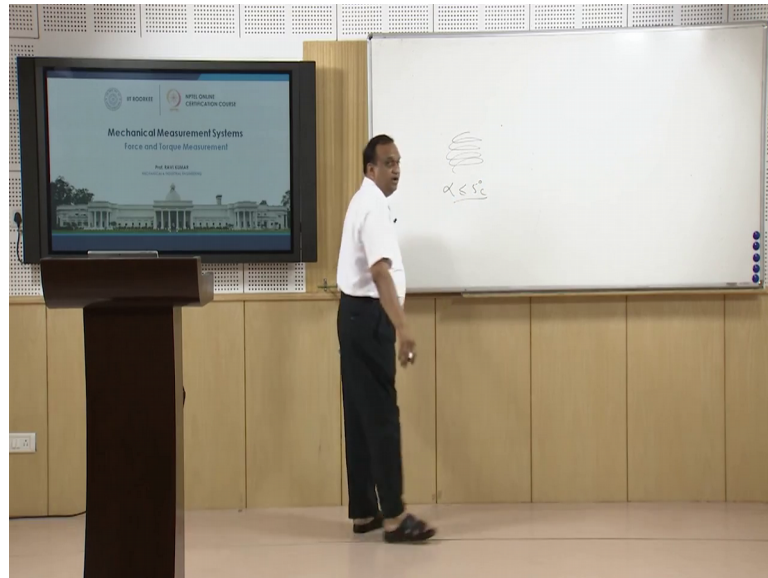
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Because there is a simply supported beam of length l , in the middle of the beam the force F is applied. And if you are able to measure the deflection; if we are able to measure the deflection of the beam δ and there are many devices to measure this deflection, then we can write the formula the deflection or x is equal to $\frac{1}{48} \frac{F l^3}{E I}$ $E I$ is the moment of inertia of the cross section of cross section of this bar along the axis x axis.

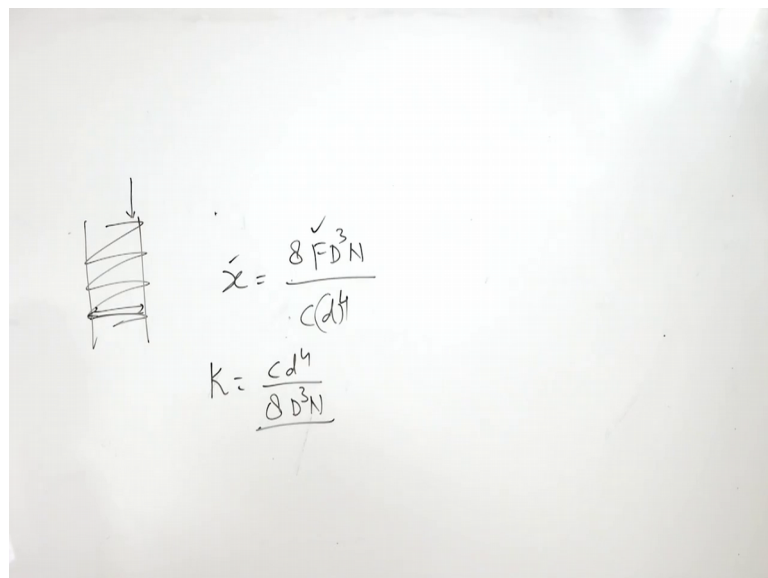
So, here F is equal to 48 by l^3 $E I$ multiplied by δ or x and this is also a constant here. So, the moment we calculate the δ will multiply δ or x by this constant k and we will get the value of F that is another way; this is for the basics of very basic things for the measurement of the force. We can make use of springs also for measurement of force F is equal to kx is a known formula.

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So, there is a closed spring, closed spring is dead spring for which helix angle is less than or equal to 5 degree, when it is more than 5 degree it is an open spring right. So, let us take 1 spring right it can be an open spring also right.

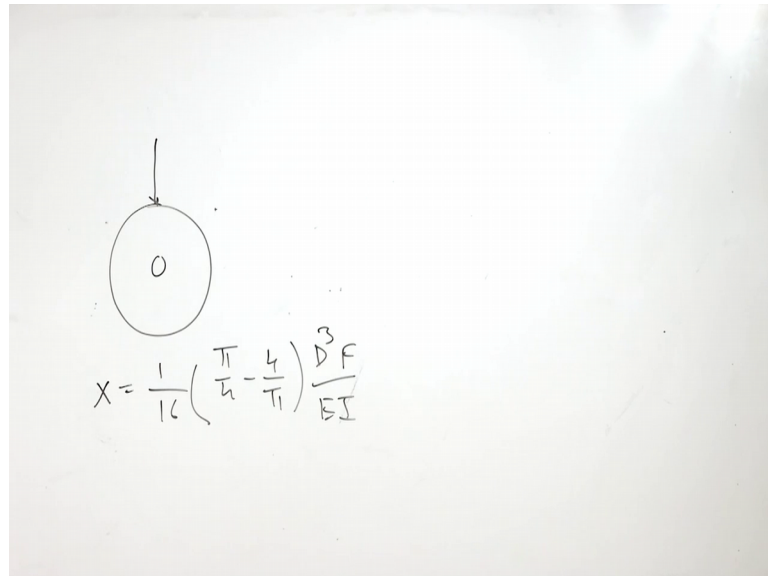
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So, let us take an spring in the force is applied, when force is applied on the spring the deflection of the spring is equal to 8 FD cube N divided by Cd to the power 4, C is the modulus of rigidity of the wire N is the number of turns D is the diameter spring diameter coil diameter and D is the diameter of the wire right. from here also we get the function displacement as the function of F. So, k here will be Cd raise to the power 4 divided by 8 D cube N. So, spring can also be used as a force measurement a thin ring or

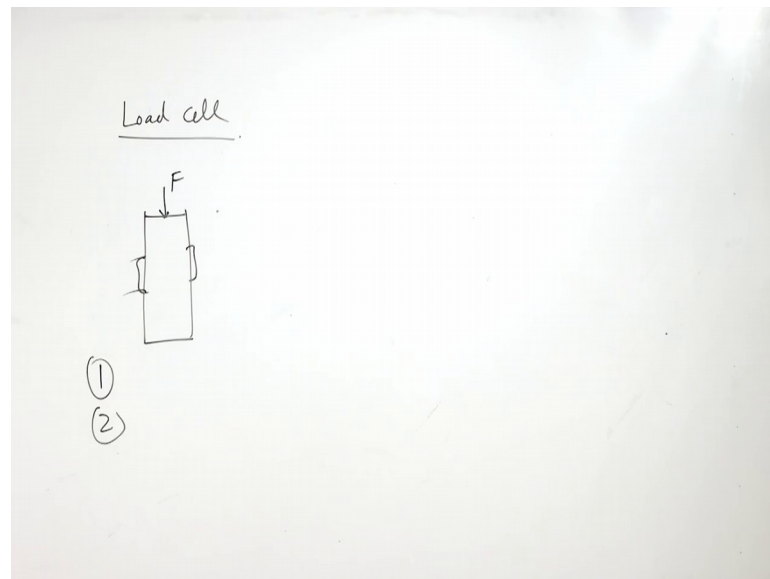
proving ring can also be used for force measurement. We have already discussed this type of rings in our earlier when we were discussing about a transducers.

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If the force is applied here and this change of the diameter will take place and it is the curved beam, so displacement X is going to be equal to 1 by 16 pi by 4 minus 4 by pi D cube F divided by EI. So, from here also we can get the value of k and then we can make this is of the formula kx for finding out the force. So, this is the these are the this is the description of force measurement using elastic members there are load cells also.

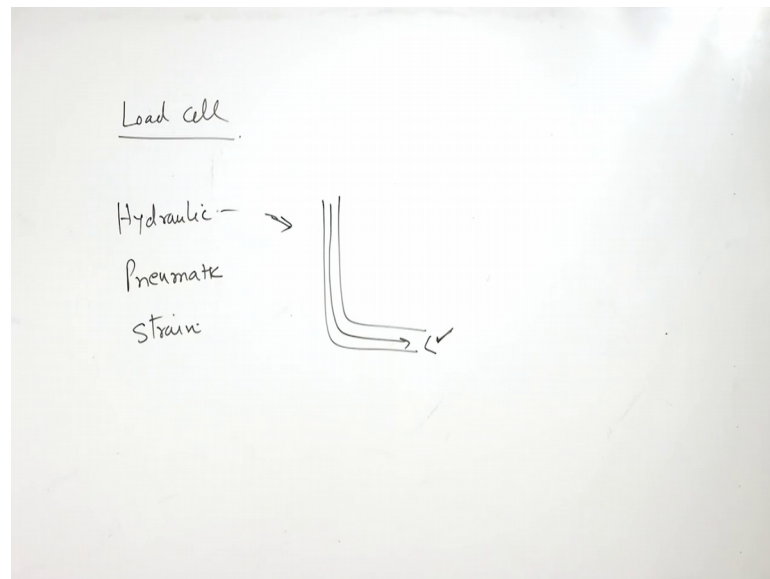
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Load cell and load cell are very popular for the force measurement and they have variety of applications. So, the load cell is used to describe a variety of force transducers, which utilize the deflection on a strain of elastic member or increase the pressure in closed fluid. So, the force transducers either they use deflection or strain is measured that is 1 way, in a load cell either deflection or strain is measured or second thing is when the force is exerted the pressure is also exerted.

So, rise in pressure so there are 2 ways of force measurement, but this load cells either increase in pressure due to application of force. So, first 1 is increase in pressure due to application of force that is 1 way, another way is strain developed in the column; suppose there is the vertical column and force is applied. So, whatever strains is developed in this column, if you are able to measure that that will also help us in finding out the force applied on the load. So, they are hydraulic there are 3 types of load cells.

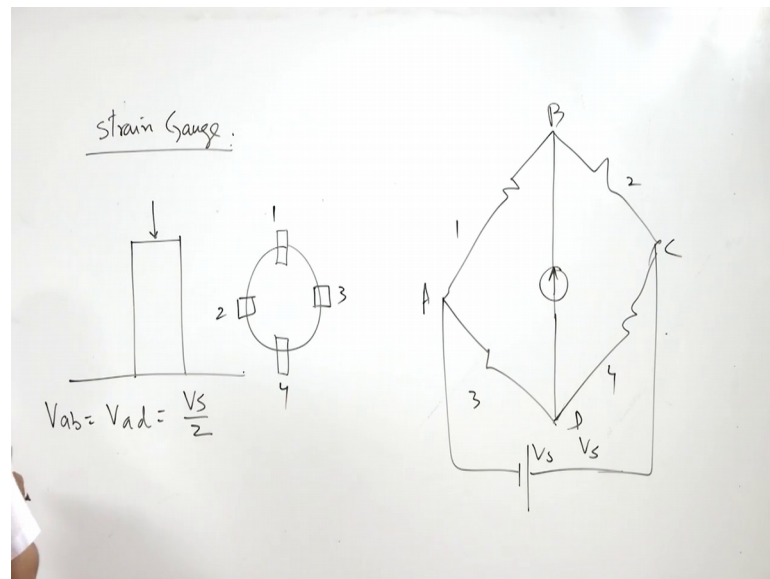
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One is hydraulic, second is pneumatic and third is strain type. Now in hydraulic load cells as I mentioned earlier, the force the I mean the primary sensing element force pulse is converted into the pressure and the pressure pulse is transmitted through the hydraulic pressure to the primary to the sensing element and the measurement of force is done and same is case with the pneumatic type of load cells right.

So, here we get the pneumatic pressure waves which is subsequently cells and the measurement is done, but the most popular is the strain gauge type of load cells. Now strain gauge of type of load cells strain gauges, definitely the concept of strains is de strain developed is used here.

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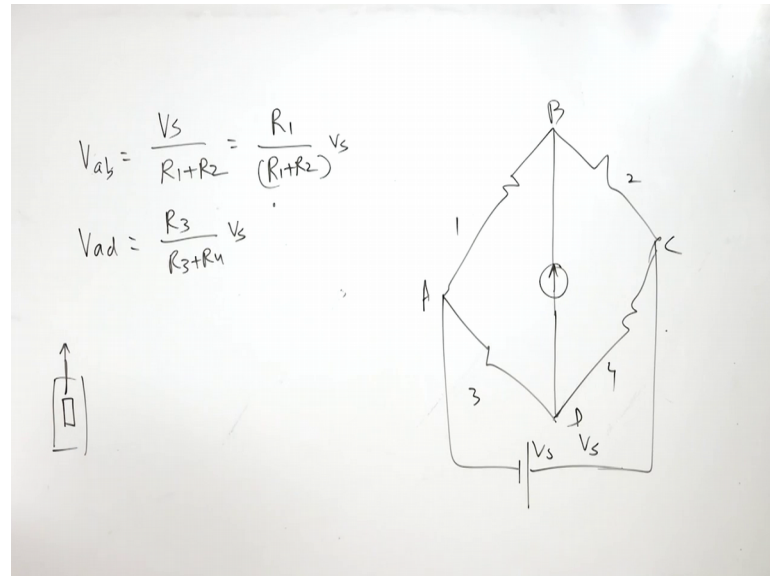


So, there is a vertical cylinder in the strain gauges type of load cells and 4 strain gauges are fixed, this is also known as poisons arrangements this is also known as poisons arrangement and this force when it is exerted on the cell the strain is developed. Now in strain gauges when they are stressed or compressed there is a change in the output signal right, now 2 4 strain gauges are fixed and they represent each arc of a wheat stone bridge and the arm is 1 2 3 and 4 right and some input is provided this is let us say is V_S and these arms are balanced, so there is no deflection in galvanometer.

Now fixing of strain gauge, how to fix strain gauge on the tube wall or the walls of this structure. So, on opposite to opposite wall they are fixed in this direction along the axis on 2 opposite walls, 2 opposite surfaces not wall opposite surfaces and on 2 opposite surfaces it is transverse arrangement it is a transverse arrangement right. So, there 4 strain gauges now 1 2 3 4 right, so 1 is 1 and 4 are along the axis and 2 and 3 are perpendicular to the axis. Now we will measure because this is ABCD when this bridge is balanced, when this bridge is balanced in that case potential difference between this and this are equal because there is no potential difference between this and this because there is no flow of current.

So, V_{ab} is equal to V_{ad} is equal to V_S by 2, this is V_S and this again this is true for this also. So, potential difference between this and this is equal to half of the V_S now I will remove this.

(Refer Slide Time: 16:24)



Now, V_{ab} is equal to current, current between A and C is V_s by R_1 plus R_2 , this is the current flowing from C to A and now we know the V_s by 2 is 0 is equal to V_{ab} right. So, and it is equal to R_1 by R_1 plus R_2 V_s , R_1 is suppose once we have the current V is equal to IR . So, this is the potential difference between V_{ab} and V_{ac} also we can add we can also calculate V_{ad} V_{ad} is equal to R_3 divided by R_3 plus R_4 V_s right.

Now the vertical column is put under force when it is put under force, the strain gauge is which are along the axis they will be compressed and strain gauge is which are on the diameter perpendicular to the angle, because the diameter will increase they come under tension or vice versa. If the load cell there is a tensile force on load cell, then the strain gauge for which are along the axis the resistance will increase let us say resistance will increase and which are perpendicular to the axis the resistance will decrease right. So, in this case where the load applied on the strain gauge, because all the resistance have the same R , this is R this is R this is R and this is R right.

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$$\underline{dV_0} = \frac{R + \nu dR}{2R - dR(1-\nu)} V_s - \frac{R - dR}{2R - dR(1+\nu)} V_s$$

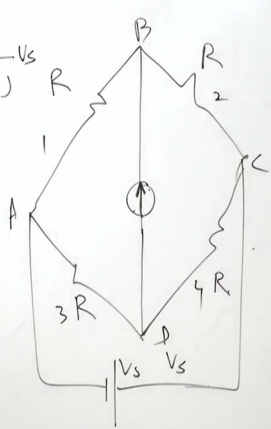
$$\underline{V_{ab}} = \frac{R - dR}{(R - dR) + (R + \nu dR)} V_s$$

$$\underline{V_{ad}} = \frac{R + \nu dR}{(R - dR) + (R + \nu dR)} V_s$$

So, when all the same then we can say this is the 1 by 2 $V_s R$ is equal to R_1 plus R_2 right. Now the strain gauge is compressed resistance has reduced. So, it has become R minus V_{ab} is equal to R minus dR divided by R minus dR plus R plus νdR right multiplied by V_s [FL]. Now the potential difference between ab is R minus dR divided by R minus dR plus R plus νdR multiplied by V_s right and potential difference between ad is R plus νdR R minus dR plus R plus νdR multiplied by V_s because, now the bridge is not the equilibrium is disturbed.

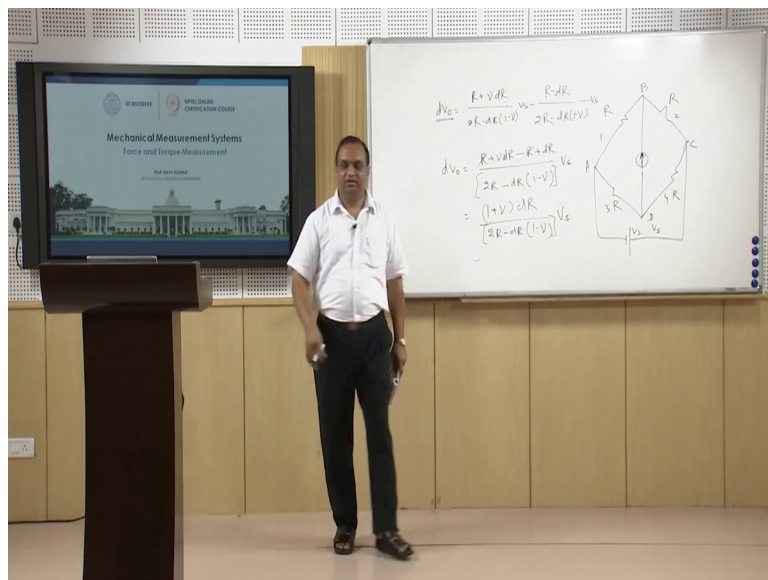
So, we will have a deflection here and the change in the volume sorry change in the voltage DV_0 output between D and D is going to be difference of these 2. So, difference of these 2 will be taken here as like this and so DV_0 is equal to R plus μdR minus R plus dR divided by $2 R$ minus dR 1 minus μ .

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$$\begin{aligned} dV_0 &= \frac{R + \nu dR}{2R - dR(1-\nu)} V_s - \frac{R - dR}{2R - dR(1+\nu)} V_s \\ dV_0 &= \frac{R + \nu dR - R + dR}{2R - dR(1-\nu)} V_s \\ &= \frac{(1+\nu) dR}{2R - dR(1-\nu)} V_s \end{aligned}$$


So, this R and R will be cancelled out multiplied by V_s right. So, we will be getting this is 1 plus mu dR divided by 2 R minus dR 1 minus mu multiplied by V_s right.

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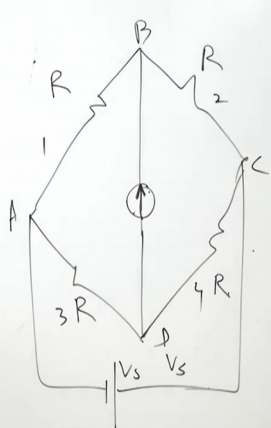


Now, this can further be written as ΔV_0 or dV_0 is equal to $\frac{2(1+\nu)}{2 - \nu} \frac{dR}{R} V_s$ now this is dR .

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$$dV_o = 2(1+\nu) \left(\frac{dR}{R} \frac{V_s}{4} \right)$$

$$dV_o = \frac{R + \nu dR - R + dR}{2R - dR(1-\nu)} V_s$$

$$= \frac{(1+\nu) dR}{2R - dR(1-\nu)} V_s$$


So, DR upon R and then we get V_s by 4 because, numerator and denominator they are multiplied by 4. So, because if we take this dR multiplied by 1 minus ν . So, this we have a neglected here and then we have got this expression, so if we neglect this then this is going to be the final expression.

So, this expression shows that what is going to be the change in the output voltage when there is a dR change in the resistance of strain gauges. Now for further understanding of this, let us solve 1 example a strain gauge a strain gauge load cell consist of a solid steel cylinder which has 4 identical strain gauges this is strain gauges, strain gauges mounted upon it in the poisons arrangement; poisons arrangement in the same arrangement 2 along the x axis 2 perpendicular to the axis for each gauge the nobler resistance are is 100 ome and gauge factor is 2. So, gauge factor F or g is to resistance is 100.

(Refer Slide Time: 23:11)

$R = 100 \Omega$
 $F/g = 2.0$
 $V_s = 6 \text{ VDC}$
 50 mm,
 $E = 200 \text{ GPa}$
 $\nu = 0.3$

$\sigma = \frac{\text{Load}}{\text{Area}} = \frac{1 \times 10^3}{\frac{\pi}{4} (0.05)^2}$
 $= 0.5095 \text{ MPa}$
 $0.5095 \times 10^6 \text{ Pa}$

$\epsilon = \frac{0.5095 \times 10^6}{200 \times 10^9} = 2.5475 \times 10^{-6}$

$\frac{dR}{R} = F \epsilon = 2 \times 2.5475 \times 10^{-6}$
 $= 5.095 \times 10^{-6}$

And gauge factor F or g is 2.0 and the gauges are connected elastically to 4 arms of a wheat stone bridge I explained earlier which is and energies with the supply of voltage, so it is 6 volts.

So, VS is 6 volt DC right make calculations for the sensitivity of the load cell the cylinder is 50 mm in diameter, this is very common dimension of a load cell 2 inches that is 50 mm approximately and modulus of elasticity for steel is 200 gega Pascal and this poison ratio is 0.3; now for this we have to find sensitivity of the load cell. So, let us take 1 kilo Newton of force which is working on the load cell, so sigma is load divided by area. So, 1 kilo Newton means 1 into 10 to the power 3 divided by area pi by 4, 0.05 square pi by 4 D square and this will give the stress as you know 0.5095 mega Pascal or 0.5095 into 10 to the power 6 Pascal.

Strain is equal to stress by E, so it is 0.5095 into 10 to the power 6 divided by 200 into 2 to the power 9 and that is going to be equal to 2.5475 into 10 to the power minus 6, that is the order of a strain which is developed by the application of 1 kilo Newton of force. Now the DR by R change in the resistance is equal to this is F into epsilon or the F by G into epsilon and that is equal to 2 into 2.5474 into 10 to the power minus 6 and is equal to 5.095 into 10 to the power minus 6. This is change in the resistance DR over R now we can calculate the value of Vo unbalanced voltage this Vo is equal to 2 into 1 plus mu [noise] DR over R VS by 4 dVo ok.

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$$\begin{aligned}
 R &= 100 \Omega \\
 F/G &= 2.0 \\
 V_s &= 6 \text{ V DC} \\
 50 \text{ mm} \\
 E &= 200 \text{ GPa} \\
 \nu &= 0.3
 \end{aligned}$$

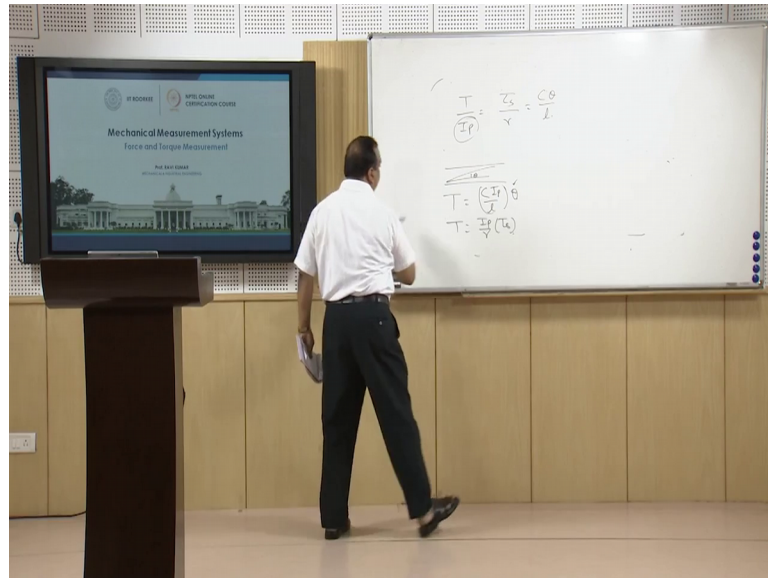
$$\begin{aligned}
 dV_o &= 2(1+\nu) \left(\frac{dR}{R} \frac{V_s}{4} \right) \\
 &= 2(1+0.3) \left(\frac{5.095 \times 10^{-6}}{2} \times \frac{6}{4} \right) \\
 &= 19.87 \times 10^{-6} = \boxed{19.87 \mu\text{V}}
 \end{aligned}$$

$$\begin{aligned}
 \epsilon &= \frac{0.5095 \times 10^{-6}}{200 \times 10^9} = 2.5475 \times 10^{-6} \\
 \frac{dR}{R} &= F \epsilon = 2 \times 2.5475 \times 10^{-6} \\
 &= 5.095 \times 10^{-6}
 \end{aligned}$$

And that is going to be equal to 2 into 1.0 plus 0.3 5.095 into 10 to the power minus 6 into 6 by 4 and that is equal to divided by this is DR by R. So, it is equal to 19.87 into 10 to the power minus 6 or 19.87 micro volt. So, sensitivity of the gauge is this output voltage right, that is we are getting here 19.87 into 10 to the power minus 6 per kilo Newton. So, per new kilo Newton application of the force this is the output of the voltmeter which is connected with a opposite arms.

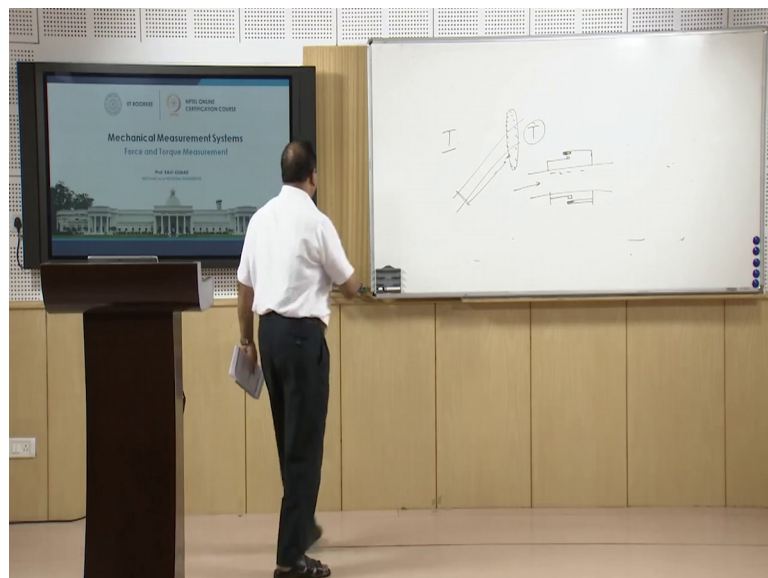
So, this is the sensitivity of the strain gauge, now after this we will take up the torque measurement; for the torque measurement the governing equation is T is equal to Ip this is the governing equation is equal to strain stress upon a is equal to C theta over l, l is the length of the shaft theta is the angle of twist, R is the radius of shaft this is polar moment of inertia right and from here we can always calculate the value of torque if we have the value of C Ip by l as a function.

(Refer Slide Time: 28:05)



So, torque as a function of theta this is the multiplying factor or torque is also I_p upon R multiplied by shearing stress. So, both ways we can measure the torque from the shaft, now measurement of theta is important if you are able to measure the theta the torque can be easily measured. So, there are certain techniques for torque measurement first of all is mechanical torque meter, mechanical torque meter is mechanically fixed to shaft.

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So, on the shaft there is a marked disc, if I fix a rod parallel to the shaft on the surface of the shaft and when the shaft is twisted right, the indicator will indicate the reading on the marked disc and from where we can calculate the value of theta; length of the distance

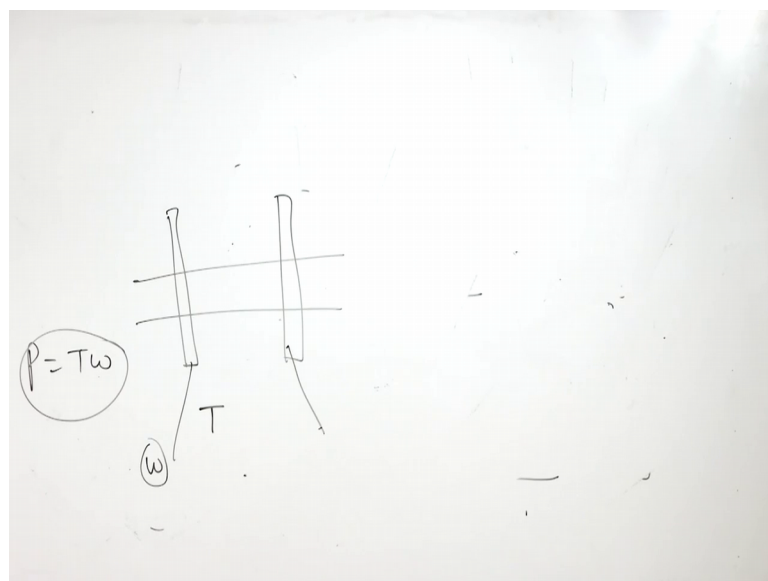
between the indicator or the length of this pointer is known to us value of C of the shaft is known to us modulus of sensitivity, we can find value of torque exerted on the shaft.

So, that is the mechanical method of torque measurement there are optical methods, also in optical methods on the shaft I mean there are 2 fixers. Now optical methods you can reserve the lateral thrust also on the shaft and 1 optical measure this is the transmitter and this is the fo optically sensitive receiver and this is 1 and this is optically sensible sensitive receiver. So, when the torsion takes place we will we know the length, there is going to be the lateral displacement between these 2 right.

Now this lateral displacement can always be correlated with torque because, if the measurement most of the signals are converted into displacement. So, 1 is fixed here another is fixed here when there is a lateral displacement in alignment of this arm and this arm, this is the displacement and this can always be correlated with T ; if there is a lateral thrust then they will move over each other in this direction, then we can find the thrust or lateral thrust also.

So, that is another technique which is optical technique for a torsion measurement, for electrical torsion measurement the 2 disc and the 2 transducers effect the 2 magnetic disc or slotted disc fixed on the shaft there are 2 slotted disc.

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Fixed on the shaft right and we get signals from both the disc, the signals are received from both the disc and when the shaft rotates we get pulses from this disc. So, from a single disc pulse will give the omega and pulse difference between time difference between the pulse is between 2 disc will can be always be related with torque and if take multiple of these 2 signals then we can find the power also, because power is $t \omega$. So, this is how the torsional meter is used electrical torso meter works and that is all for today.

Thank you very much.