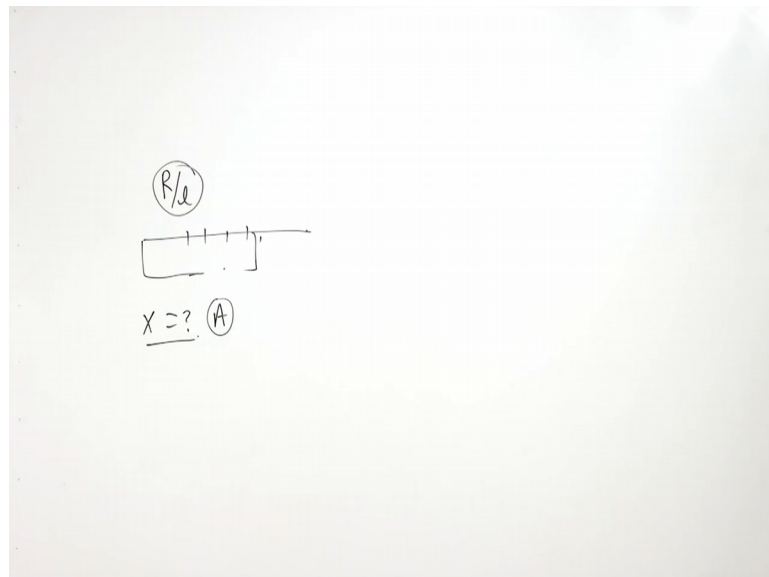


Mechanical Measurement Systems
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Lecture - 31
Strain Gauges

Hello. I welcome you all in the course Mechanical Measurement Systems. Today we will discuss the strain gauges the topics to be covered is only strain gauges today. Say for displacement measurements, we have discussed about the potentiometer.

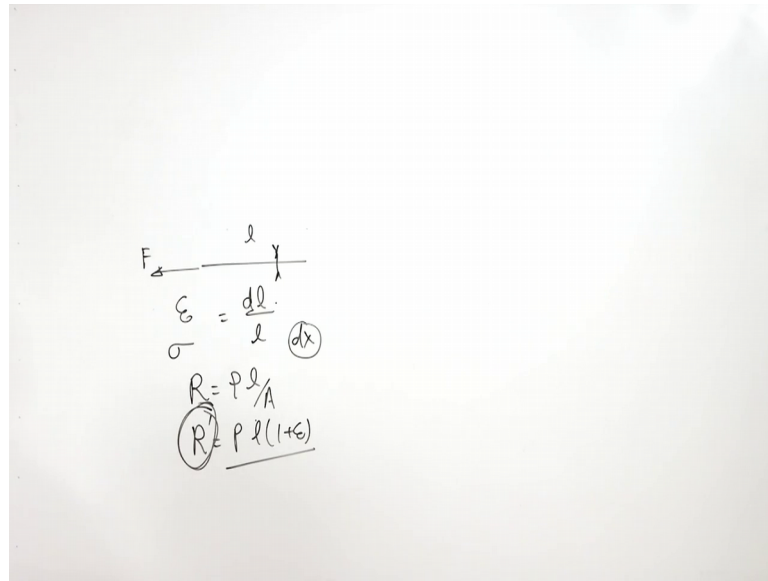
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Where there is a wire and effective length of the wire is changed in order to measure the displacement or any phenomena which is converted into the displacement. But in this process physical property or physical properties of wire are not changed, physical properties means in the entire process the resistance per meter length; length of the wire is not changed or any other physical property. For example, length of the wire is not changed it is fixed, effective length changed or cross section area of the wire is not changed. Now there is another device which is used for measuring the displacement and many other physical phenomenas, but the working principle is the strain developed in a wire.

Now, this strain developed is directly related with the resistance of the wire for example, a wire of a thin wire of length let us say particular length l .

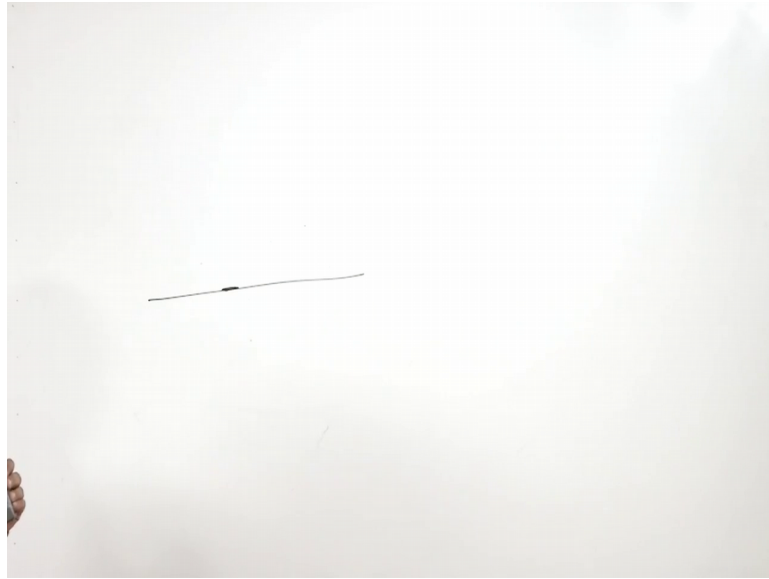
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If it is subjected to some force, the length the wire will get elongated it is a metallic wire it will get elongated and it will develop a strain, and at the same time it will develop stress also. Under this strain the length of the wire will change due to because strain is being developed strain is being Δl upon l . So, length of the wire will be changed. Since the length of the wire is increased volume is constant cross section area will reduce it will reduce. So, if we take resistance of the wire is ρl by A , l is the length of the wire a is the cross section area then the resistance will change to r dash with ρl 1 plus ϵ something like this divided by change in the area right and this r dash can also can always be related with the change in the length of the wire right.

So, in strain gauge this is the basic working principle, this is the working principle of strain gauges that when strain gauges put under stress right strains are developed and in the resistance of the strain gauges and corresponding change in the dimension or the length of the strain gauges measured. For example, there is a bar of one meter length.

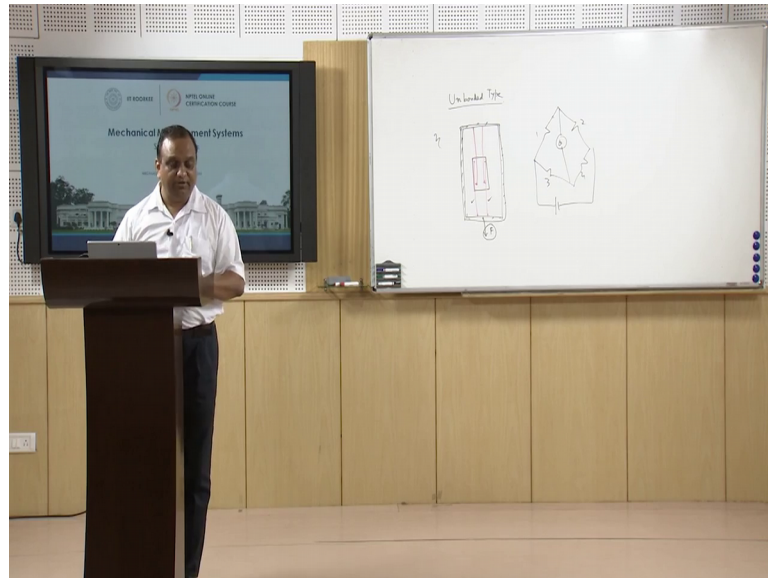
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I fix a strain gauge here, strain developed in a strain gauge will be equal to the strain developed in the bar. So, once am able to maintain eh able to calculate the strain gauge and developed in the strain gauge, on any sub straight for example, there is luster. This suppose here in if I put a strain gauge and I put this under tensional compression. In that case same strain level of a strain is going to because strain is a dimensionless unit. So, strain in the strain gauge is equal to the strain in the luster.

Now once I calculate the strain in the luster calculate the change in the length of the luster. And definitely when strain is developed in the wire the resistance will change and we will be taking the signals from the change in the resistance. Now there are two types of strain gauges bounded type and unbounded type.

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So, will start with the unbounded type of strain gauge unbounded type. In unbounded type of strain gauge there is a fixed frame and there is a moving frame. The two frames one frame is fixed another is moving and on this moving frame strain gauges are fixed or these wires they connect moving frame with the fixed frame, there are four wires and these wires are pre stressed wires all the wires are pre stressed wires. Suppose force F is exerted in this direction and displacement takes place ok.

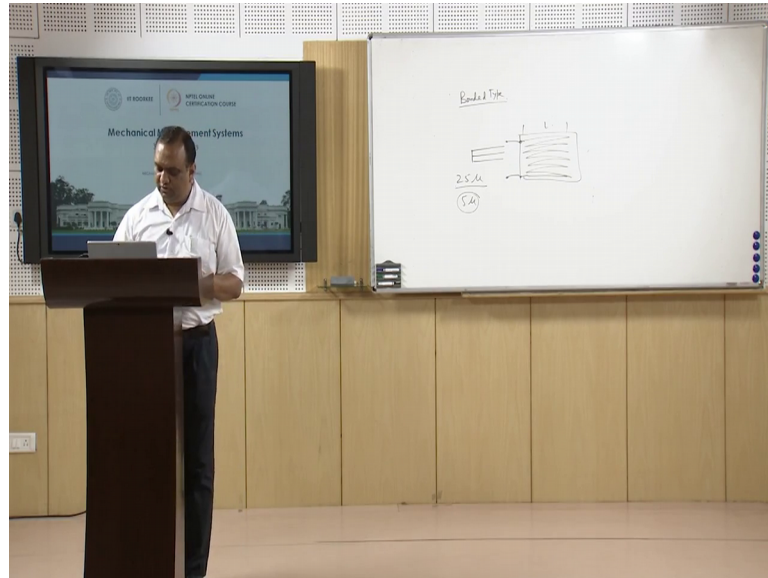
So, this is a moving part this is a moving frame, this moving frame will move in this direction in downwards direction. So, stress in these two wires will be released because they are pre stressed like spokes of a cycle tire or cycle wheel like spokes of a cycle wheel. So, these are pre stressed. So, when this is put pushed in this direction. So, there is going to be a relation between fixed frame and the moving frame. So, stress in these wires will be relieved. So, strain if they will develop some strain after relieving this stress and these wires will be stretched. In fact, resistance of all wires will change.

So, this is just to magnify the effect magnify the signal, this is arrangement is done just to magnify the signal and these four wires will take four arms of a wheat stone bridge right 1 2 3 4. And when the bridge gets unbalanced we can easily find the change in the resistance and subsequently the displacement in this direction.

So, this they seems to be very they are very efficient first of all they are very efficient, and they appears to be fragile I mean it appears that this is difficult to use them, but this strain gauges are very I mean efficient as a secondary transducer they are often used as a

secondary transducers in the systems. Then nowadays bounded type of strain gauge are being used, I mean widely they are being used bounded type of strain gauge it consist of consist of a bounded type of strain gauge.

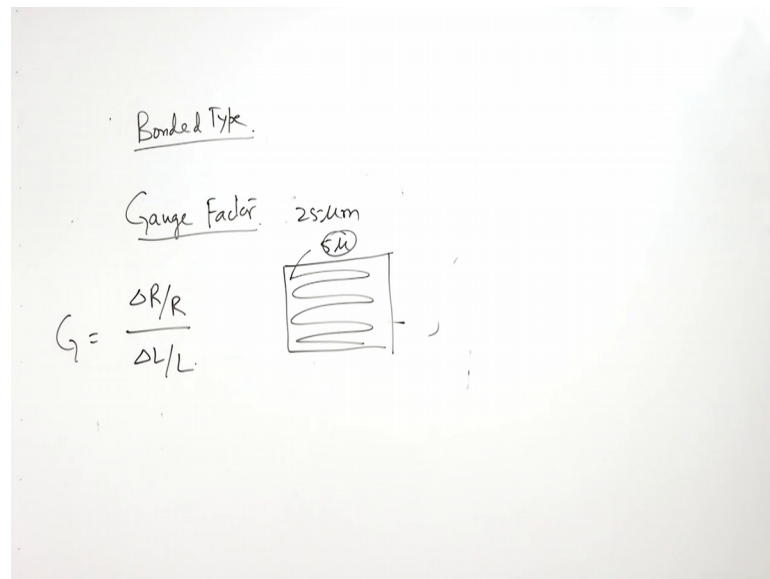
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Now, bounded type of strain gauge is means that it is clear for name itself. The strain gauge is bounded of the sub straight. So, there is a firm contact between sub straight and the strain gauge and strain gauge wire is very thin, it is a metallic wire and a metallic wire is pasted on a surface sub straight. The size of the strain gauge may be of say 2 mm or 3 mm and the wire is very thin, the wire thickness is approximately let us say 25 microns.

Now this is not the wire this is the thickness of sub straight, the wire may be of 5 microns very thin wire and there is a I mean grid of wire, I mean not a grid of they are serially connected grid of wire it means the wires are like this very thin wire led on the sub straight like this ok. So, they offer very high resistance and here suppose they are terminated. So, they offer and this effective length of the strain gauge. So, they offer very high order of resistance to the current. So, there is a parameter in strain gauges that is gauge factor.

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Gauge factor of bounded type of gauge is quite high and gauge factor can be defined as the change in the resistance of the wire for per unit strain, this is known as gauge factor of the wire. Now in bounded strain gauges because these strain gauges are prepared by just first of all etching the surfaces of the sub straight right after etching the sub straight sub straight masking of the sub straight is done, because sub straight thickness is approximately 10 25 micro meter and then after etching vapor deposition is done on the surface.

And vapor deposition then results in the thickness of the wire as five microns as 5 microns right that is why for these type of volatile strain gauges, the G is high right and we can write this value of G also.

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$$R = \rho \frac{L}{A}$$

$$dR = \frac{A(\rho dL + L d\rho) - \rho L dA}{A^2}$$

$$G = \frac{dR/R}{dL/L}$$

$$\gamma = \frac{a}{b}$$

$$d\gamma = \frac{a db - b da}{b^2}$$

As we know that resistance of any wire is equal to specific resistance multiplied by the length of the wire and divided by the area right. Now if you want to have dR because as I said earlier gauge factor is dR/R divided by dL/L right; so change in fraction change in resistance ratio of fraction change in resistance and fraction change in length or the strain in the substance. So, dR is going to be equal to a square $A\rho dL$ let us take this as capital L plus $Ld\rho$ plus ρLdA right this is simple γ is equal to a/b then $d\gamma$ is equal to adb plus bda divided by b^2 . This is minus this is minus right. So, now dR we can take from here, now they are certain unknown terms here right.

So, for that we will take we start with the V is equal to A into L .

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$$\epsilon_0 L$$

$$R = \rho \frac{L}{A}$$

$$dR = \frac{A(\rho dL + L d\rho) - \rho L dA}{A^2}$$

$$G = \frac{dR/R}{dL/L}$$

$$V = AL$$

$$dV = A dL + L dA$$

$$= L(1+\epsilon) A(1-\nu\epsilon)^2 - AL$$

$$dV = AL \left[(1+\epsilon)(1-2\nu\epsilon) - 1 \right]$$

$$= AL \left[1+\epsilon - 2\nu\epsilon - 1 \right]$$

$$= AL \epsilon (1-2\nu)$$

This is the volume of the wire; volume of the wire is cross section area of the wire multiplied by the length of the wire. Now dv is equal to AdL plus LdA . dv is the change in volume. So, change in volume is going to be AdL plus LdA and is equal to L 1 plus epsilon this is change in volume, this is strain right multiplied by A 1 minus this is the orig final volume minus original volume. Original volume is L , final volume is final length; final length is length plus change in length because epsilon L will give the change in length area 1 minus ν epsilon whole square because this is ν is Poisson ratio right. So, this is π four d square that is why it is going to be square of that.

Now what we can do, we can take dv as AL right we will take AL common 1 plus epsilon multiplied by 1 minus 2 ν epsilon minus one sorry this. 1 not here 1 plus x to the power n 1 plus xn because higher term we are neglecting because they are their contribution in the final expression is insignificant. Now we can take product of this AL 1 plus epsilon minus 2 ν epsilon plus sorry minus 2 ν epsilon square minus 1. Now here also this term because epsilon itself is very less much low then I mean it is 0.0 something.

So, square of it again this can be taken as 0, this will be cancelled out and will be getting AL epsilon 1 minus 2 ν , ν is Poisson ratio. So, now, AL epsilon 1 minus 2 ν is change in the volume dv .

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The image shows handwritten mathematical derivations for the change in volume of a wire under stress. The derivations are as follows:

$$\frac{dL}{L} = \epsilon$$

$$R = \rho \frac{L}{A}$$

$$dR = \frac{A(PdL + LdP) - PLdA}{A^2}$$

$$\frac{dR}{R} = \frac{dP/P + dL/L - dA/A}{1}$$

$$AL\epsilon(1-2\nu) = AdL + LdA$$

$$dv = AL \left[(1+\epsilon)(1-2\nu\epsilon) - 1 \right]$$

$$= AL \left[1 + \epsilon - 2\nu\epsilon - 2\nu\epsilon^2 - 1 \right]$$

$$= AL\epsilon(1-2\nu)$$

So, change in the volume is AL epsilon 1 minus 2 ν is equal to AdL plus LdA it is always there. Now once we have this expression AdL and LdA and 1 epsilon is b

equal to dL because dL by L is equal to ϵ . So, $L \epsilon$ is equal to dL . So, AdL will come here.

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$$\begin{aligned} \epsilon L \\ \frac{dL}{L} = \epsilon \quad R &= \rho \frac{L}{A} \\ dR &= \frac{A \rho dL + L \rho dA - \rho L dA}{A^2} \\ AdL(1-2\nu) &= AdL + LdA \\ \cancel{AdL} - 2\nu \cancel{AdL} &= \cancel{AdL} + LdA \\ -2\nu \cancel{AdL} &= LdA \end{aligned}$$

Now, AdL and AdL now will write AdL minus $2\nu AdL$ is equal to AdL plus LdA . Now here AdL and AdL will be cancelled out, AdL and AdL will be cancelled out and we can say that $2\nu AdL$ is equal to LdA minus ok. Now after this we will put this value here in this expression here and we will get dR as $A \rho dL$ this $A \rho dL$ plus this multiplied by a right minus ρLdA . ρLdA can be replaced this LdA can be replaced by this LdA and here will be getting.

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$$\begin{aligned}
 \frac{dL}{L} &= \epsilon \\
 R &= \rho \frac{L}{A} \\
 dR &= \frac{A \rho dL + L d\rho + 2\nu A dL \rho}{A^2} \\
 \frac{dR}{R} &= \frac{\rho dL (1+2\nu)}{A} + \frac{L d\rho}{A}
 \end{aligned}$$

Plus 2 nu AdL multiplied by rho. We can take A as common and we can find d we can write d R is equal to rho dL 1 plus 2 nu because rho dL we can take from here, rho dL this will club these 2 right.

So, will be giving take rho dL 1 plus 2 nu a by a square. So, we can write divided by A plus Ld rho divided by A right. Now after these we will divide this d R by R d R by R.

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$$\begin{aligned}
 R &= \rho \frac{L}{A} \\
 \frac{dR}{R} &= \frac{dL}{L} (1+2\nu) + \frac{d\rho}{\rho} \\
 \frac{dR}{R} &= \frac{\rho dL (1+2\nu)}{\frac{\rho L}{A}} + \frac{L d\rho}{A \cdot \frac{\rho L}{A}}
 \end{aligned}$$

R is equal to as we know R is equal to R is equal to rho L by A. So, entire expression will divided by rho L by A this is also rho L by A. Now here rho will be cancelled out A will be cancelled out A will be cancelled out and L will be cancelled out and this will lead to

the expression dR upon R is equal to dL upon L 1 plus 2ν plus $d\rho$ upon ρ . Now you have to take dR upon R . So, dR upon R is equal to 1 plus 2ν plus $d\rho$ upon ρ divided by dL upon L sorry this capital L and this is sorry this is divided by dL upon L .

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Handwritten derivation showing the relationship between the change in resistance (R) and length (L):

$$R = \rho \frac{L}{A}$$

$$\frac{dR}{R} = \frac{dL}{L} (1 + 2\nu) + \frac{d\rho}{\rho}$$

$$\frac{dR}{R} = (1 + 2\nu) + \frac{d\rho/\rho}{dL/L}$$

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Handwritten derivation showing the simplification of the gauge factor equation:

$$R = \rho \frac{L}{A}$$

$$\frac{dR}{R} = \frac{dL}{L} (1 + 2\nu) + \frac{d\rho}{\rho}$$

$$\frac{\frac{dR}{R}}{\frac{dL}{L}} = (1 + 2\nu) + \frac{d\rho/\rho}{dL/L}$$

$$= (1 + 2\nu)$$

So, dR upon R divided by dL upon L . Now this dR upon R and dL upon L it is gauge factor, it is gauge factor and this if this if we neglect this or this remains unchanged change in this specific resistance and change in with strain, then it is 1 plus 2ν . So, two

times Poisson ratio Poisson ratio for metals it varies between point 32.4 let us say 0.3. So, 0.6 for metal it is going to be 1.6.

But we have semiconductors though they are having gauge factor as 100 150 and 200 as well right. So, that is why nowadays semiconductors and piezoelectric materials they are becoming very popular for strain measurement or to work as a strain gauge. Now after this we will take up one in numerical, consider a single strain gauge of resistance 120 ohms.

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Handwritten calculations on a piece of paper:

$$\begin{aligned}
 R &= 120 \Omega \\
 E &= 200 \text{ GPa} \\
 \epsilon &= \frac{dL}{L} = 0.03 \\
 \frac{d\rho}{\rho} &= 0.003 \\
 \frac{dR}{R} &= \frac{d\rho}{\rho} + \frac{dL}{L}(1+2\nu) \\
 &= 0.003 + 0.03(1+2 \times 0.3) \\
 &= 0.051 \\
 0.051 / 0.03 &= 1.7
 \end{aligned}$$

So, resistance of the strain gauge is 120 ohms right and mounted on a axial direction on a axial load specimen of a steel, steel for a steel E is equal to 200 GPa. This is resistance of the strain gauge is 120 and E for steel is 120, the percentage is change in the length of the rod due to loading is 3 percent. So, strain dL upon L is equal to 0.03. So, the rod and they corresponding change in resistivity of the strain gauge material is 0.3 percent. So, change in the resistivity that is $d\rho$ by ρ is eq equal to 0.003 this 0.3 percent. So, $d\rho$ by ρ is equal to 0.003 estimate the percentage change in the resistance of a strain gauge and the gauge factor. So, first of all dR by R ; dR by R is $d\rho$ by ρ plus dL by L 1 plus 2 ν .

We have already done that. Now all the values are there $d\rho$ by ρ is 0.003 plus dL by L is 0.3 strain 0.03 and 2 ν is 1 plus 0.3 into sorry 2 into 1 plus 2 into 0.3 Poisson ratio is 0.3 it is given. So, with the help of these we can always find dR upon R and dR upon

R in this case is 0.051 that is or we can say point 0.5 5 1 percentage change in the resistance. Now for gauge factor we will divide this ΔR by R by strain that is dL by L . So, when we divide this ΔR by R 0.051 to 0.03 we get 1.7. So, gauge factor for this strain gauge is 1.7. Now the next part is little interesting if the strain gauges connected to a measuring device (Refer Time: 23:47) of determining the change in the resistance with an accuracy of 0.02 percent. So, it is connected to a device which is capable of measuring the strain with accuracy of plus minus 0.02 ohm. So, accuracy of the measurement is plus minus 0.02 ohms.

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Handwritten equations on a whiteboard:

$$R = 120 \Omega$$

$$E = 200 \text{ GPa}$$

$$\epsilon = \frac{dL}{L} = 0.03$$

$$\frac{\Delta R}{R} = 0.003$$

$$\pm 0.02 \Omega$$

$$\sigma = \epsilon \cdot E = \frac{\Delta R}{R} \cdot \frac{E}{S_g}$$

$$\frac{\Delta R}{R} / \epsilon = S_g$$

So, this strain gauge is connected to a device to a measuring device (Refer Time: 24:13) of determining change in resistance with an accuracy of plus minus 0.02 ohm what is the uncertainty in the stress? So, uncertainty in the stress because stress is strain multiplied by E right or is equal to ΔR over R multiplied by E divided by strain gauge that is a strain, because ΔR upon R divided by strain is equal to ss gauge factor. So, here we have taken used this expression and we have replaced strain by ΔR over R divided by S_g .

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$$\begin{aligned}
 R &= 120 \Omega \\
 E &= 200 \text{ GPa} \\
 \epsilon &= \frac{\Delta L}{L} = 0.03 \\
 \frac{d\epsilon}{\epsilon} &= 0.003 \\
 &\quad \pm 0.0206 \\
 \sigma &= E \cdot \epsilon = \frac{\Delta R}{R} \cdot \frac{E}{S_g} \\
 \frac{\partial \sigma}{\partial \Delta R} &= \frac{E}{R S_g} \\
 U_{\sigma} &= \pm \frac{E}{R S_g} U_{\Delta R} = \frac{200 \times 10^9}{120 \times 1.7} \times 0.02 \\
 &= 19.6 \text{ MPa}
 \end{aligned}$$

Now, $\frac{\partial \sigma}{\partial \Delta R}$ is equal to $\frac{E}{R S_g}$ we have differentiated it with respect to ΔR . Now, So, uncertainty of measuring stress is equal to plus minus $\frac{E}{R S_g}$ that is multiplied by uncertainty in measurement of R and this is going to be equal to E is 200 into 10 to the power 9 divided by resistance is how much? 120 ohms 120 multiplied if you remember gauge factor 1.7 and uncertainty in the measurement of resistance is 0.02.

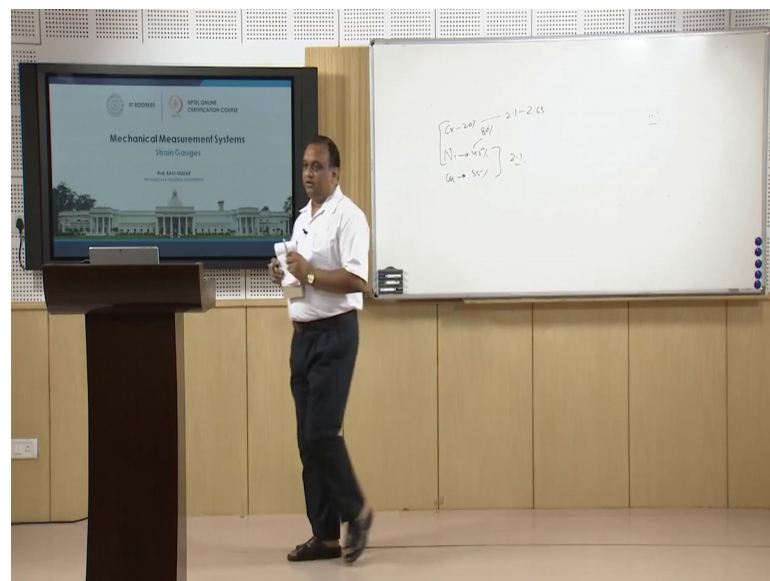
So, this will be multiplied by 0.02 and we will get 19.6 mega pascal of plus minus 19.6 mega Pascal in the error in the measurement of stress right and strain uncertainty in strain is also required error in the measurement of strain is also required. So, for finding out the error in the strain is uncertainty in stress divided by E that is it.

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$$\begin{aligned}
 R &= 120 \Omega \\
 E &= 200 \text{ GPa} \\
 \epsilon &= \frac{dL}{L} = 0.003 \\
 \frac{d\epsilon}{\epsilon} &= 0.003 \\
 &\quad \pm 0.0206 \\
 \frac{U_\sigma}{E} &= \frac{19.6}{200 \times 10^9} = \frac{98 \times 10^{-6}}{9.8 \times 10^5} \\
 \frac{\partial \sigma}{\partial R} &= \frac{E}{R S_g} \\
 U_\sigma &= \pm \frac{E}{R S_g} U_{OR} = \frac{200 \times 10^9}{120 \times 1.7} \times 0.02 \\
 &= 19.6 \text{ MPa}
 \end{aligned}$$

So, uncertainty in this stress is 19.6 divided by 200 into 10 to the power 9 is equal to 98 into 10 to the power minus 6 or 9.8 into 10 to the power minus 5 right this is the uncertainty or it is we can say 98 into or 10 to or 9.8 into 10 to the power minus 5 ok. Now after this we will continue our discussions on bonded strain gauges. So, I will give you some idea about the gauge factors for constantan in strain gauge where wire is constantan the constantan the composition is nickel is 45 percent 45 percent and copper is 55 percent.

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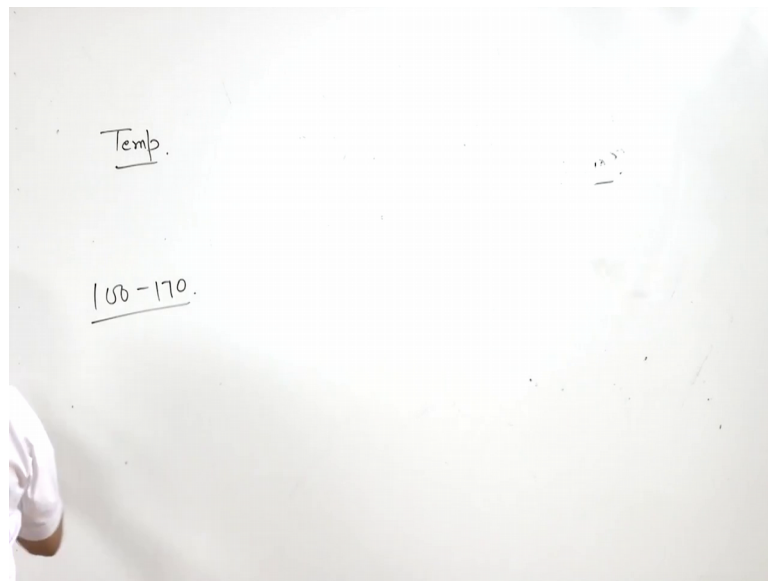


It is the gauge factor is a approximately 2.1. So, metals and alloys for metals and alloys the gauge factor is I mean it if we change the composition for example, if you take nickel

80 percent and chromium chromium 20 percent, nickel chromium alloy then nickel chromium alloy the gauge factor is 2.1 to 6 point sorry 2.63.

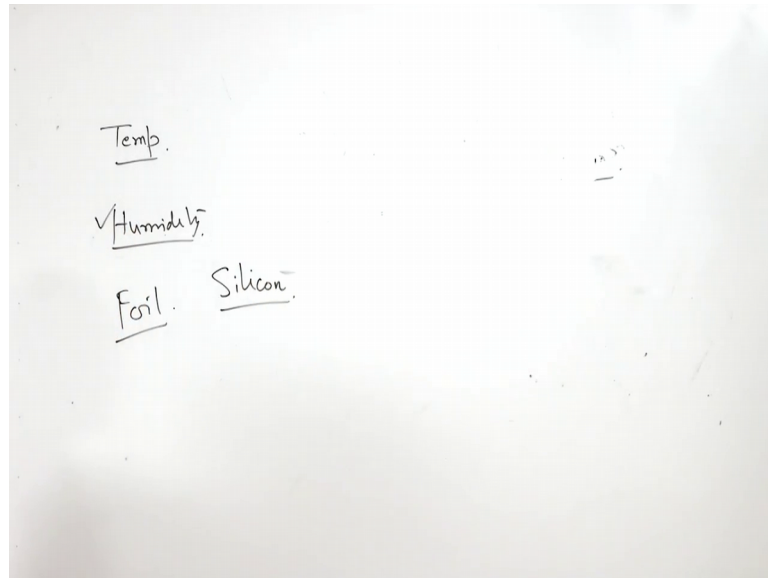
If you take single crystal semiconductor, if you take single crystal semiconductor the gauge factor they lie in a range of 100 to 170 the gauge factor in a single crystal may lie in a range of 100 to 170.

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So, you can compare where we are going for the metallic type, and in metallic type it is not exceeding even 3, but here for gauge factor for semiconductor is lying between 100 and 170. The certain factors which effect the performance of strain gauges first of all temperature variation and second is humidity.

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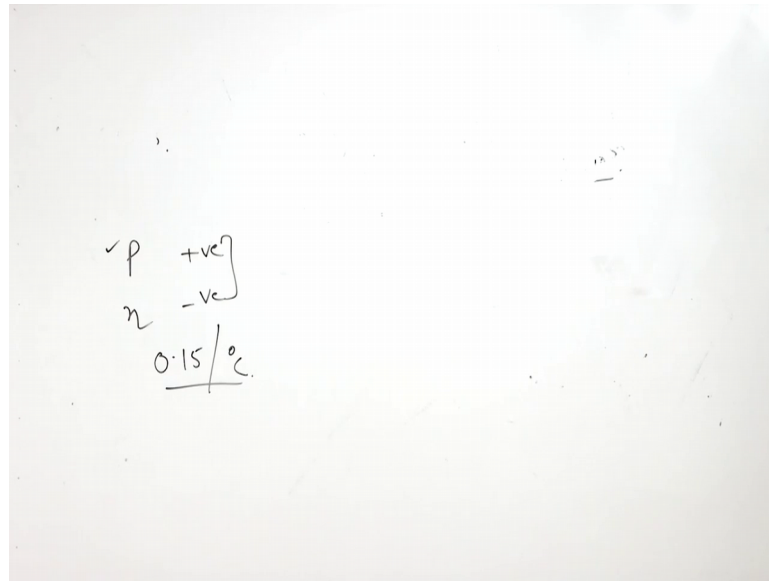


They are two main factors which affect the performance of the strain gauge, because when there is a change in temperature there is a differential movement between substance and the strain gauge, they may not have same coefficient of expansion of contraction. So, this stress or if the force of or I mean the relative motion between these two causes the slip of the strain gauge, and that case the right can the reading will be effected and will get the erroneous reading. So, that is one thing and in the humidity insulation breaks between down and the strain gauge insulation breaks, and it causes corrosion of the wire also sometimes.

So, due to humidity also the strain gauge has to be saved first of all against the humidity, and second there should not be much temperature variation in environment. And there are another type of strain gauge which is known as foil type of strain gauge, they are also becoming very I mean popular nowadays. And regarding the semiconductor strain gauge, they are normally made from silicon crystal silicon crystal and for designing the strain gauge is the orient thing in strain semiconductor orientation and doping is important right. So, if you do if you increase the doping, then it causes it effects the resistivity of the strain gauge resistivity is reduced right.

So, right kind of doping is required germanium, germanium also used as a can be also be used as a strain gauge.

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And P type of semiconductors have positive gauge factor and n type of conductor has have negative gauge factor. So, sometimes negative gauge factor is beneficial because if there is a variation in temperature that can be compensated, but if you want linearity if you want linearity then we should go for p type of semiconductor definitely they are made of semiconductors, they are quite fragile so, they have to be handled with care. And they have very high temperature coefficient, coefficient can go up to 1.15 degree centigrade sorry 0.15 degree centigrade this is the temperature coefficient.

So, doping is a very important parameter in designing of a strain gauge, if there is a light doping then gauge factor is high.

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If we do the high doping we increase the doping, in that case gauge factor will reduce at the same time temperature dependence of the sensor will also reduce of the strain gauge is also reduced.

So, a designer has to strike balance between two and has to decide right kind of doping. So, that we maintain high gauge for the strain gauge, at the same time the temperature dependence is as minimum as possible. So in fact, these type of a strain gauges semiconductor strain gauges they are inherently non-linear they are not very linear they are and if we decrease the doping in the strain gauge, we will get high gauge factor and high non-linearity also. But, in order to address non-linearity nowadays ic's are available which can make the non-linear signals in to the linear signals.

So, this is all about the strain gauges which are widely used in for research and industrial purpose. In the next class we will start with the piezoelectric transducers.