Mechanical Measurement Systems
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Lecture - 13
Generalized Model of a Measuring System

Hello, I welcome you all in this course on Mechanical Measurement Systems. Today, we shall discuss about the Generalized Model of a Measuring System.

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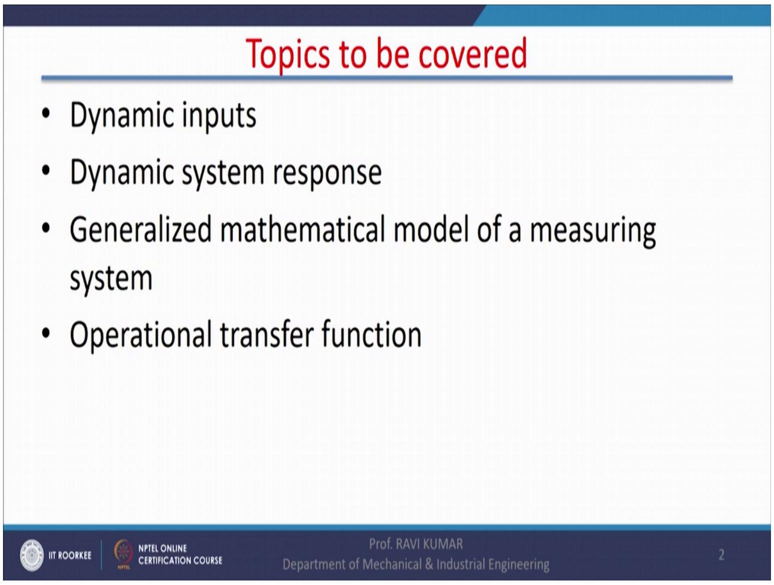

Topics to be covered in this lecture are Dynamic inputs, Dynamic system response, Generalized mathematical model of a measuring system and Operational transfer functions. Now we will start with dynamic characteristics of a measuring instrument. So, far we have discussed the static characteristics. This we have amply discussed.

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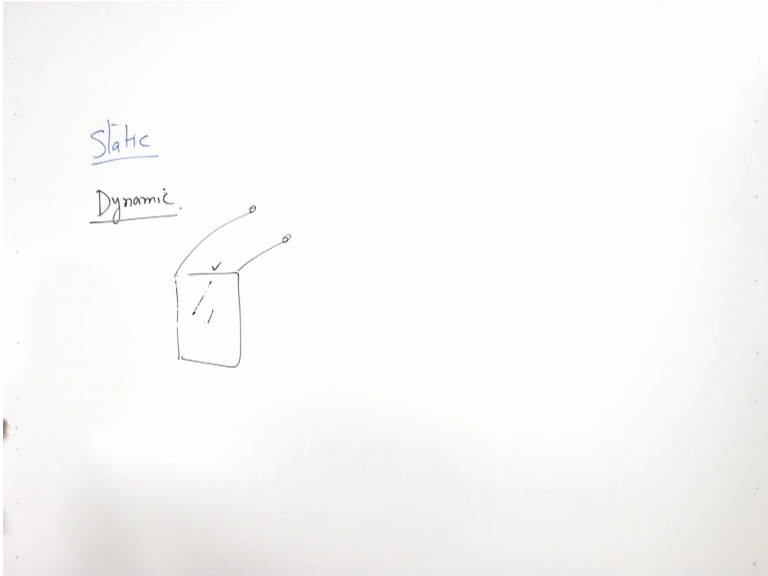

In the static characteristics, there is was do function of time right, but now we are going to discuss the Dynamic characteristics of the system; Dynamic characters characteristics means, the response of the system as a function of time.

For example, I will give you an example. We take a voltmeter. Let us start with the voltmeter. When voltmeter is connected to the line, immediately it will give certain output but you must have noticed the needle is starting from 0 to the particular output take some time that is a dynamic response of the instrument. And you must have also observed in some of the machines or some of the voltmeters, the response of this needle is very sluggish, slowly it will move from 0 and will be stabilized at a certain value. Second type of response is this is a sluggish response. Second type of response is the needle will move and it will overshoot the input and it will start oscillating.

And after certain time, it will be damped and it will indicate a certain value right. So, it takes certain time to stabilize. So, that is also not very good, the very good dynamic response of the instrument. Third type of response, you must have noticed that the moment you give input, there is no output. There is a time, they can all of a sudden, there is a movement in the needle.

So, you can have variety of outputs, variety of responses of the system for a given input. The input may be constant input, may be time variant. So, there are certain terms which we have to understand, before we before going through the dynamical study of the, dynamic characteristics of the measuring instrument.

The first is Periodic signal. Here, so dynamic input can be of many forms as I said earlier, it can be periodic.

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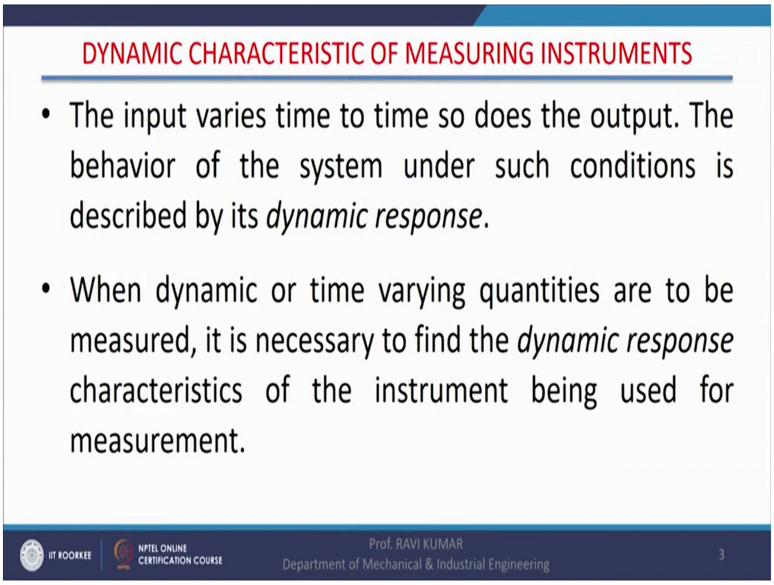

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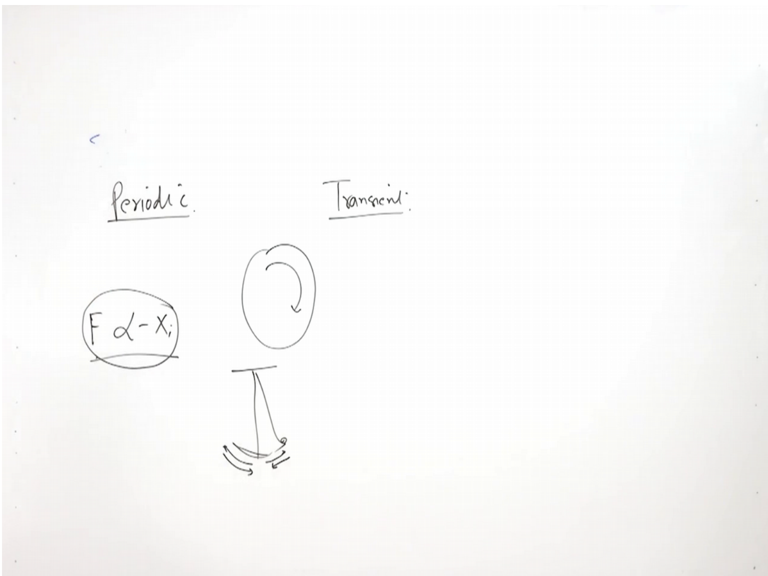

Now, periodic input means cycle is repeating itself. I mean after a certain time interval, the cycle is repeated. For example, a rotating table or rotating disc it is a periodic motion it is not a simple harmonic motion or a harmonic motion. Harmonic motion is a kind of periodic motion where the restoring force restoring force is proportional to displacement or yes it is proportional to displacement. So, that type of motion is known as harmonic motion and there a.

Very good example of simply harmonic motion is the pendulum because when the pendulum is moving in this direction, restoring forces working in this direction; when the pendulum is moving in this direction, the restoring forces working in this direction. Another example can be spring, if you compress this spring, the spring will push the weight right or restoring force will be proportional to the displacement in the instrument. Now third type of input can be Transient, Transient input. So, Transient input is the input for a very short time. It is non-cyclic right.

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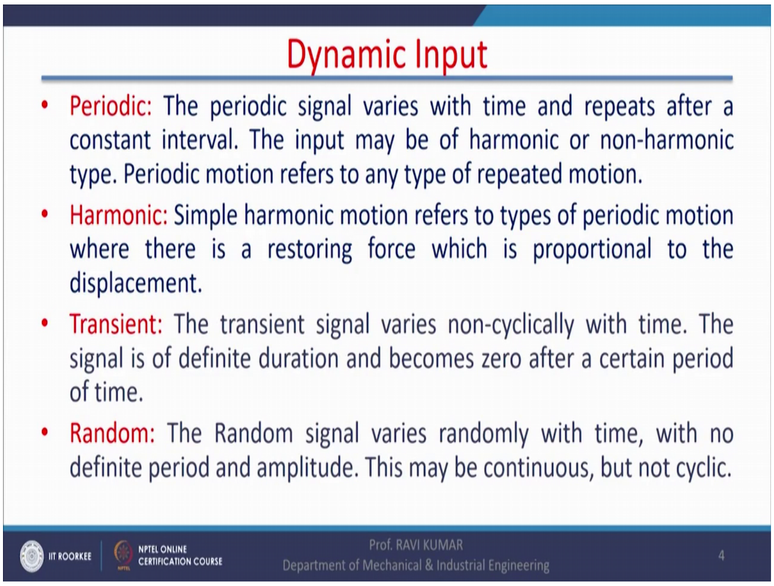

And it now it is for a very short time. So, transient input may be like this.

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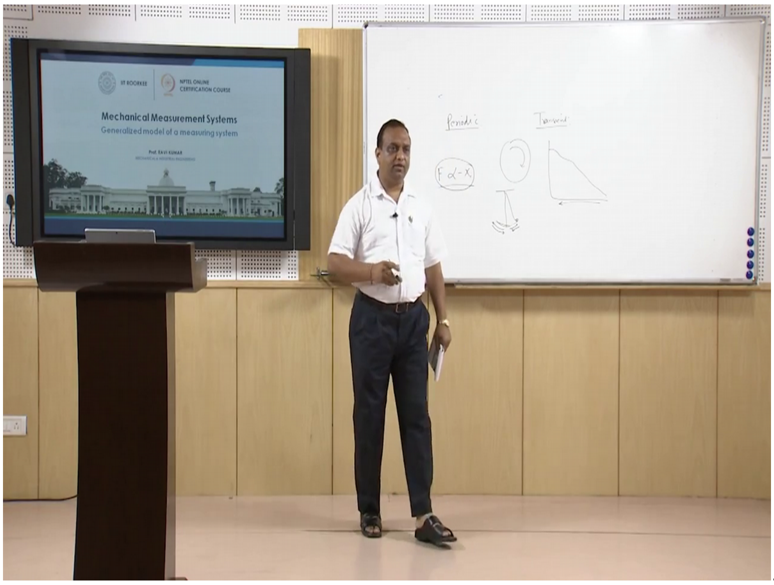

After certain time interval, it will die out. For example, in the area of mechanical engineering like quenching of steel rod, when you are putting high temperature is steel rod in water then input temperature variation is stabilized after certain time and it becomes almost equal to the temperature of the media. That is a one sort of transient input right and third one is Random input.

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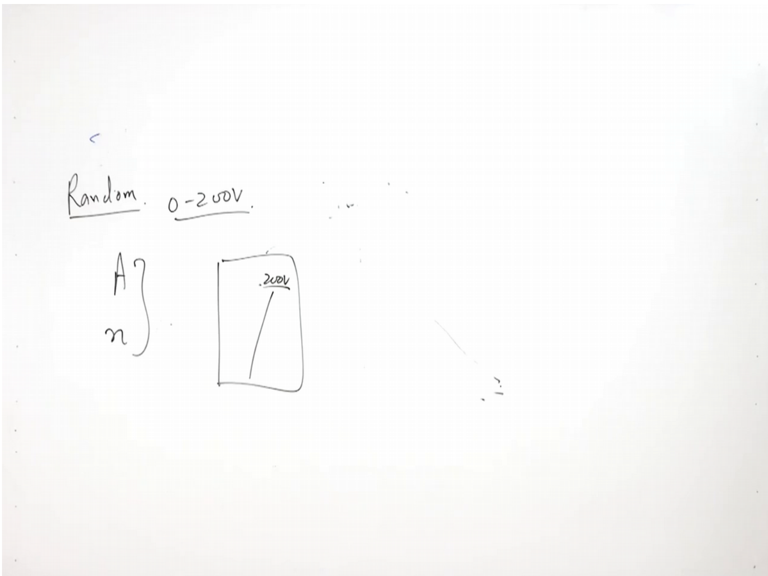

So, the last one, this is the last one. Forth one it is Random input.

So, as it is implicit from the name itself, it can vary with time, it can have any amplitude or oh no definite amplitude or frequency right. The input is completely random. So, random input is also one kind of input which can be considered. It may be continuous also random input may not diode with time, but certainly random input is not a cyclic input. It is not a cyclic input. So, these are the certain type of inputs we have in dynamic systems and they are certain more definitions like speed of response.

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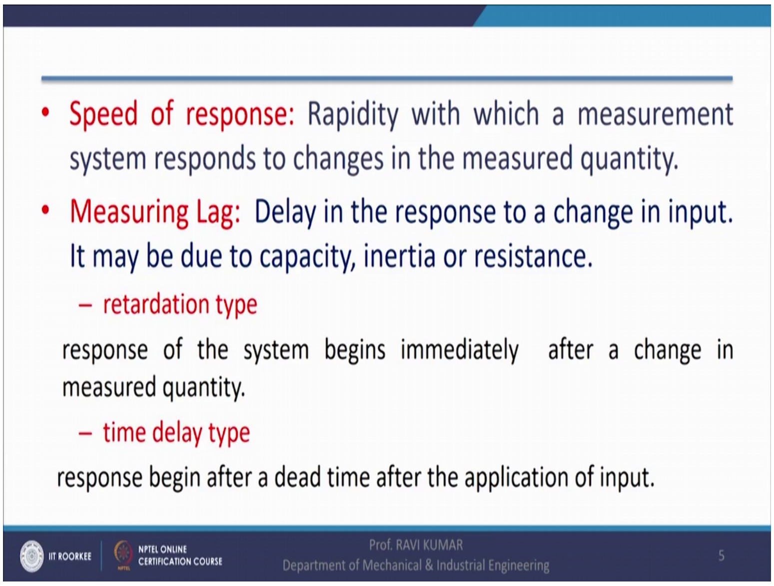

So, speed of response means as it is clear from the name itself, it is the rapidity with which the system responds to the input, change in input, there is a change. So, write I gave you an example of voltmeter suppose, there is a change in voltmeter 0 to 100 volt. How fast the output of the needle is moving and coming to the 200 volts is the speed of the response of the instrument. Then Measuring leg, delay in response then another one is Measuring leg. So, many of these terms are clear from the name itself. For example, Measuring leg.

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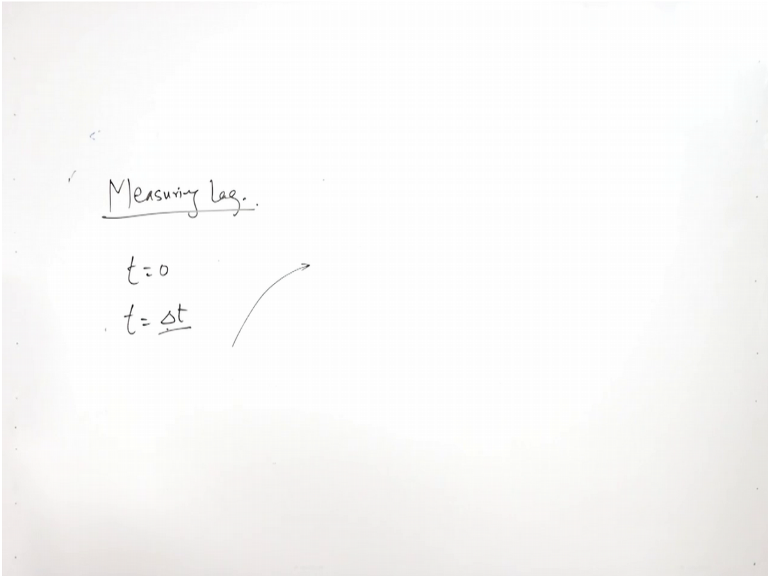

Suppose at time t is equal to 0, I give the input an exact measurement I am getting at t is equal to delta t.

So, delta t is measuring leg and measuring leg again, I have already explained it can be of two types; retardation type, when needle is slowly moving and attaining the value and another one is time leg delay; time leg means initially there is no deflection but all of a sudden the needle will come to the output right. So, there are certain terms regarding the dynamic response of measuring system. Now, derivative response can also be further classified. For further classification, one is Time response.

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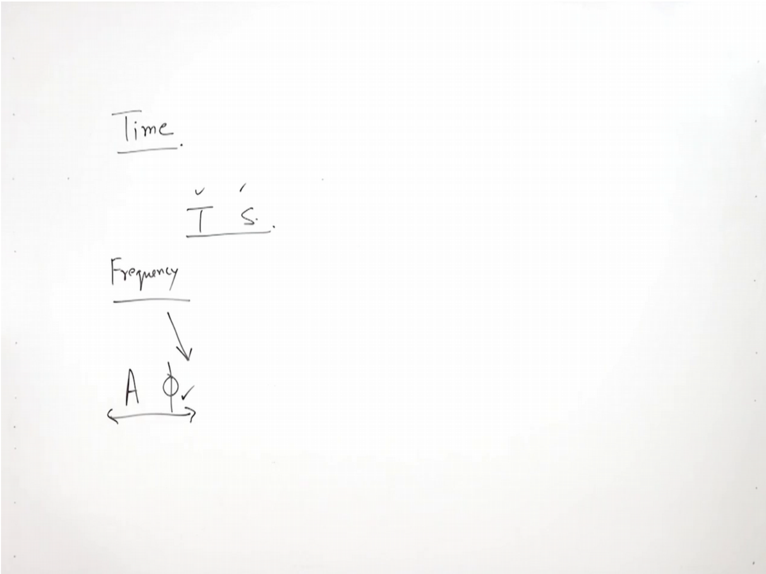

Another is Frequency response.

Now, Time response of the instrument; so, for time response of the dynamic system, there are certain differential equations for input and output. We will discuss those equations in subsequent slides. Their equations for input and output and solution of these equations has to be attained in order to find the time response of dynamic system and it has two parts, Transient and Steady State.

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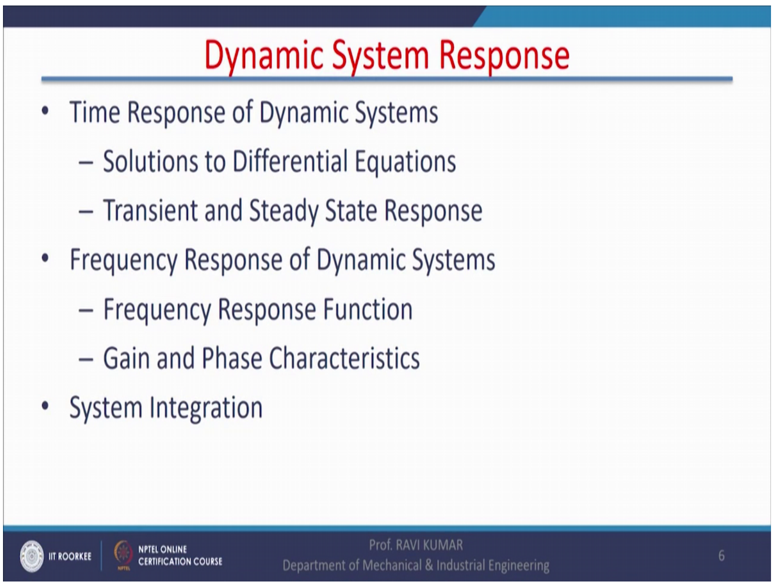

Transient and Steady State; Transient means, it is time dependent. Now, steady state response of the system is not dependent upon time. It is dependent on input quantity.

So, there are two parts of time response of dynamic systems, another is frequency response of dynamic system. So, again we will have a frequency response system, frequency response function and in frequency response; there are two things; one is change in amplitude right and another is change in the phase angle. In frequency response, the frequency is not altered. If I mean input is 50 hertz the output will be of the 50 hertz. There is a possibility or it normally happens then amplitude may change, amplitude of the signal may change or the phase angle may change or phase angles often changes.

So, these two, three, these two things are change in frequency response of frequency dynamic response of measuring system and the benefit of this type of analysis is, we can divide system in a number of components and we can have dynamic response of each component and then we can integrate and we can find the response of the measuring system in totality. This for this also, I will give example in later on.

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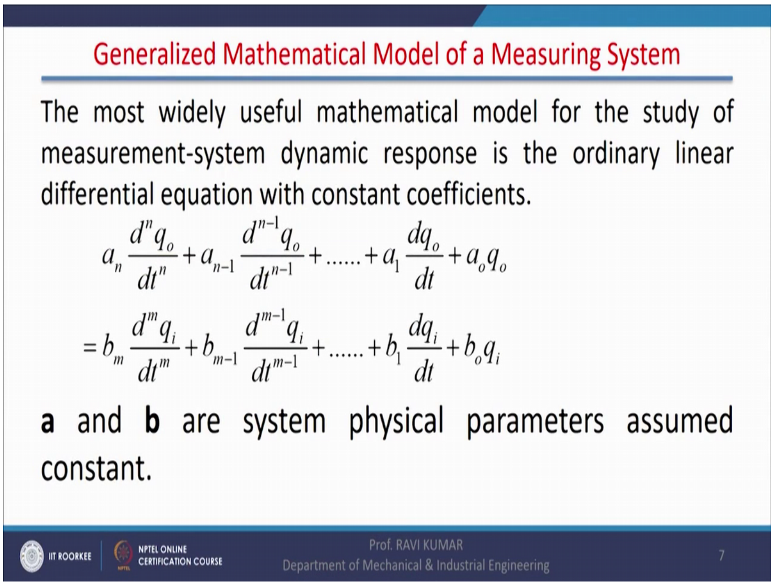

We will start with a generalized mathematical model of measuring system and generalized mathematical model starts with.

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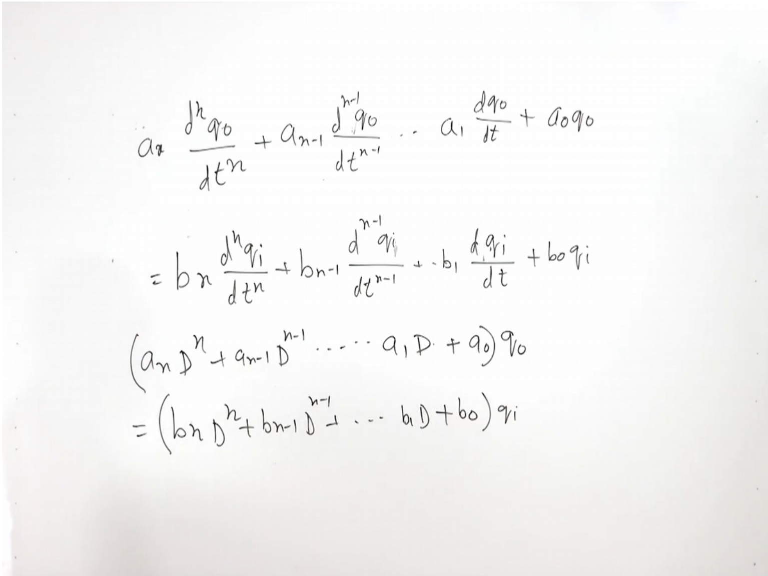

Suppose output is, we will start with the output a n d n q o by dt n plus a n minus 1 d n minus 1 q o by d t n minus 1 and so on and when we get a 1 d q o by dt right. And then a o sorry a o q o, this is a generalized linear equation for differential equation for output and for input also, we can take as b, b n d raise to power n q i d t n plus b n minus 1 d 2 power n minus 1 q i d t n minus 1 a sorry b a 1. So, b 1 b q i by dt plus b o q i.

This is a very generalized type of equation and for time dependent analysis, we have to solve this equation and these parameters are constants; they are all constants. Now this equation can further be written in the form of the operator because if we say, we can have solution if of this equation by two methods, what is d operator method another is a Laplace transformation right, but here we will adopt the classical method of solving these differential equations.

So, if I write this equation in the form of D operator, then it is going to be a and D n plus a n minus 1 D n minus 1 it is go to will go to a n a a 1 D plus a o q sorry, q o is equal to again b n then D n plus b n minus 1 D n minus 1 plus b 1 D plus b o q i right. Now, this solution of, we start with the output. We will take them one by one, we will start with output. So, output q o output, the solution of this equation will have two terms.

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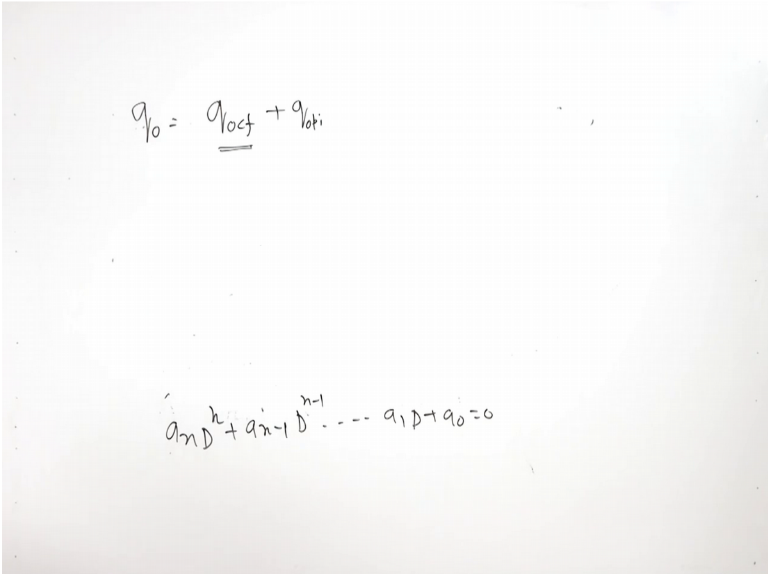

These two terms are q o, complementary function and q o particular integral fine. Now this complementary function will be a function of time, will is necessary a function of time and it has n arbitrary constraints. It has n arbitrary constants and, these constants the value of these constants can be evaluated by imposing initial conditions.

So, we will keep on imposing the initial conditions. We will get the value of all arbitrary constant of complimentary function. Particular integral does not have any arbitrary constant. So, we will start with complimentary function right and when we start the complimentary function we can take as this equation, this one can be taken as 0 or we can say that a and D raise to power n plus a n minus 1 D raise to power n minus 1 up to a 1 D plus a o is equal to 0. We will got this one. Now for this differential equation, we can have roots which are real and they are not equal.

I mean we have to take different combination of real and unreal and complex roots right. So, we will start with the first one. The roots are real and they are unrepeated sorry not equal I said not equal, it is unrepeated. So, real roots unrepeated.

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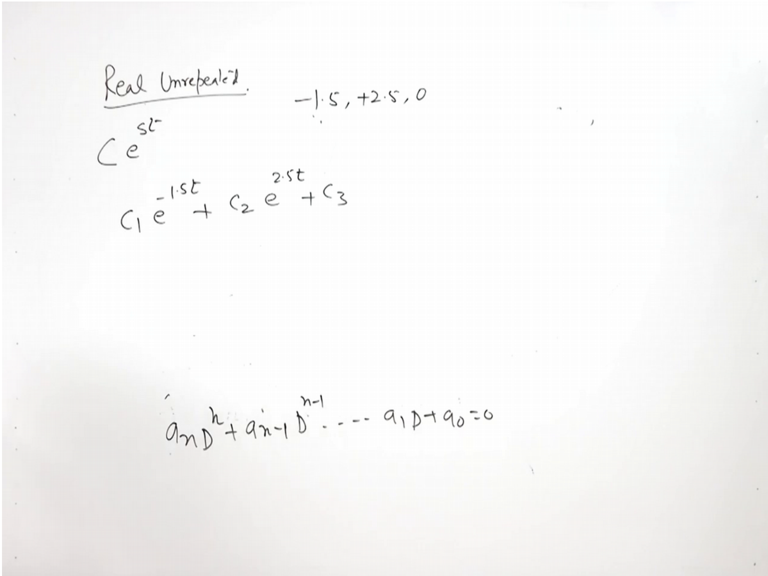

When roots are real and they are unrepeated, then solution is C e to power s t, when s is the root; for example,, the equation has this nth order equation suppose equation has three roots; minus 1.5 plus 2.5 and 0 in that case.

The solution is going to be C 1 e raise to power minus 1.5 t plus C 2 the raise to power 2.5 t plus C 3 because it is 0. So, this is the solution. Now imposing initial conditions, we can find the value of C 1, C 2 and C 3; that is one case.

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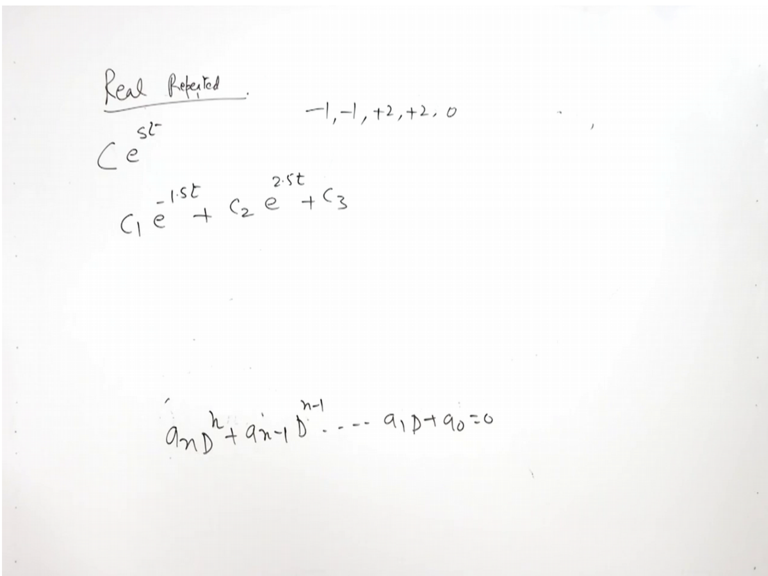

Now, suppose the roots are real and they are repeated; repeated means we are having roots like this, minus 1 again minus 1 let us say plus 2 again plus 2 and let us say 0.

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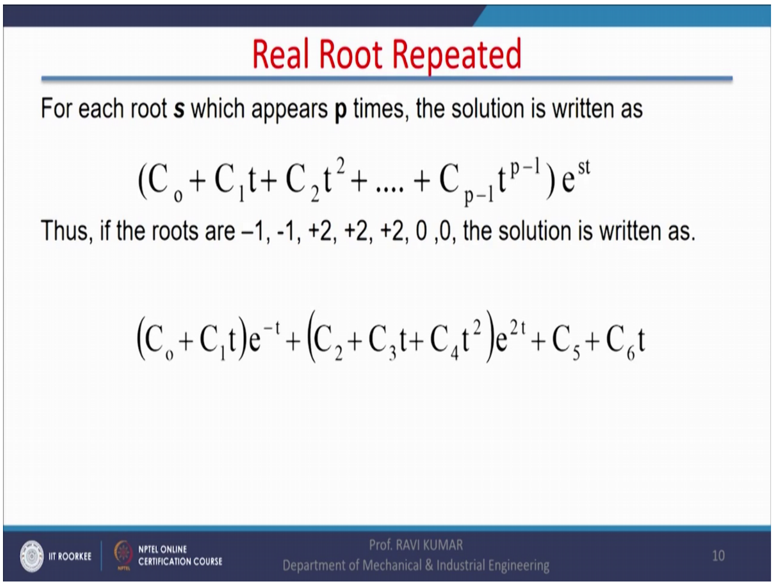

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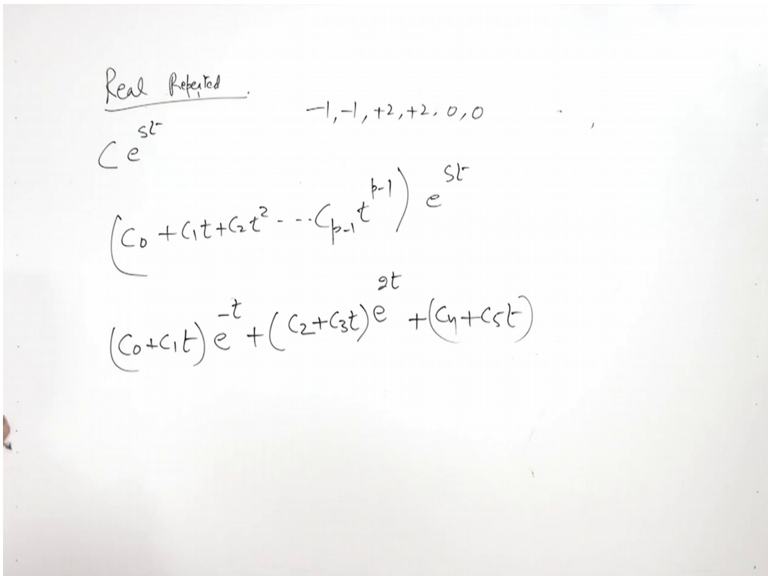

So, minus 1 and plus 2 are repeated. In the repeated root when the roots are repeated, the solution is C o plus C 1 d plus C 2 D square and so on up to let us say, there are p times repeated, so C p minus 1 and t raise to power p minus 1 because we have taken 0. So, p th term is going to be C p minus 1 right and this is multiplied by e raise to power s t. Now considering this configuration, we can have roots in this case is as C o sorry C o plus C 1 t right because it is repeated two times, p is equal 2. If it is repeated 3 times, then we will go for second order of t, e raise to power minus t plus C 2. This is not the c 2, this is different C 2 C 2 plus C 3 t e raise to power 2 t plus C 4 right. So, we can have this type of or suppose, this 2 is three times, then C 3 plus C 4 t square something like that.

And suppose 0 is repeated two times this 0 is repeated 2 times, then it is going to be C 4 plus C 5 t. Likewise, we can have solution when there is a the roots are I mean real and repeated. Now roots can be complex root also.

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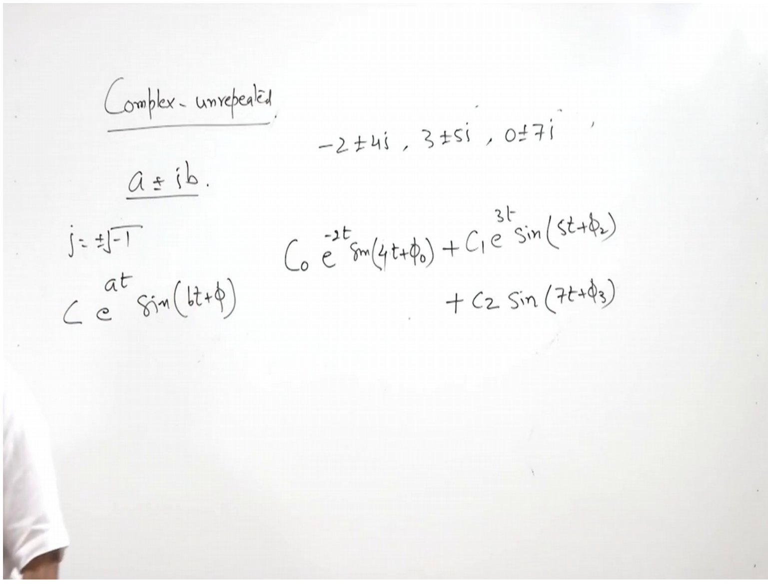

When the roots are complex, complex and unrepeated; complex means a plus i b, this is complex right and i is equal to plus minus under root a minus 1.

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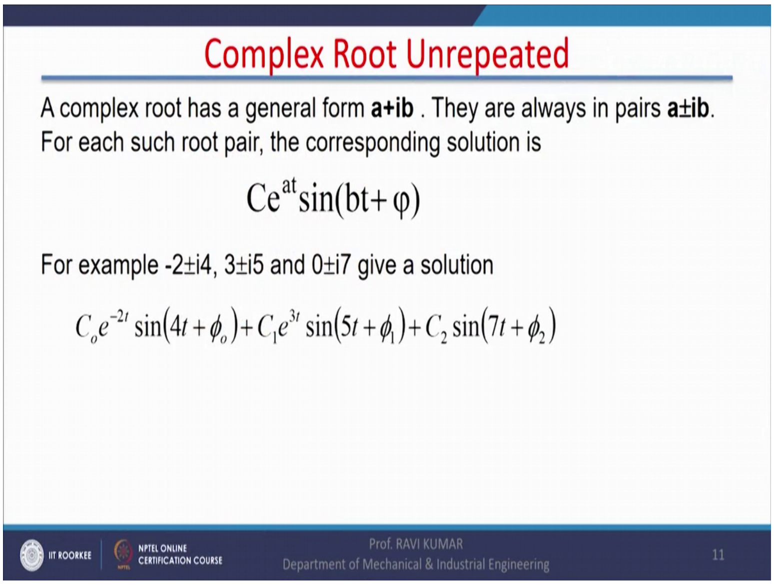

So, when the roots are complex and unrepeated thus and there in pair plus, minus; a plus i b.

So, when the roots are complex and unrepeated, the solution is C e raise to power 80 sin b t plus phi, phi is the phase angle phase change, change in phase. So, for example, I will take one example here. Suppose the roots are minus 2 plus minus 4 i that is the and an 3 plus minus 5 i and 0 plus minus 7 i, the roots are like this, but when the roots are like this, then we are going to have the solution as C o e raise to power a here the value of a is minus 2. So, minus 2 t sin b t b is 4, b t plus phi 1 or let us say phi o.

Some phi, we do not have the value of phi and then it is going to be next one is C one e raise to power 3 t similarly sin phi t plus phi 2. So, here we can make instead of b value is given. So, it is 4, 4 t. Now is 0, here a the value has the value of a 0. So, it is going to be equal to C 2 sin 70 plus phi 3 and so on we can find the solution of the roots. Now again, we can have complex and repeated.

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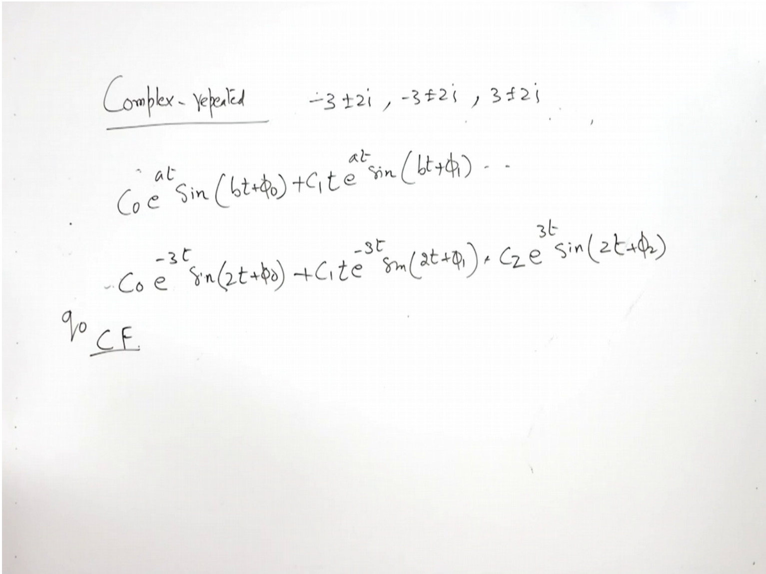

Also the roots are complex and repeated.

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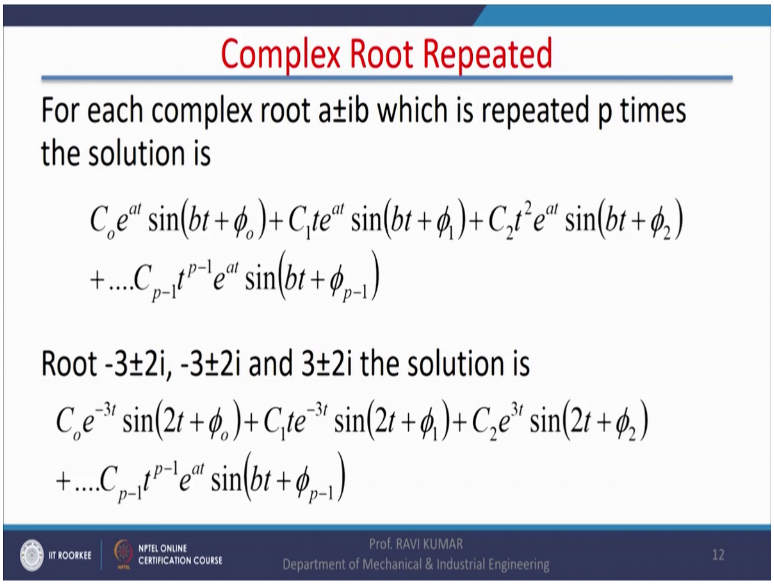

When the roots are complex and repeated, in this case the solution is going to be C o e raise to power at sin b t plus phi o plus C 1 t e raise to power a t sin b t plus phi 1 and so on.

So, here the phi is changing and this of t is added. Now here also we can take one example; one example the roots are let us say minus 3 plus minus 2 i, that is one root;

again minus 3 plus minus 2 i right and. Next one let us take 3 plus minus 2 i. So, these roots are repeated.

So, in order to express these roots; it is going to be C o e raise to power a is here minus 3. So, minus 3 t sin b is 2 2 t plus 5. Now, it is repeated. Since it is repeated, then we will take C 1 t; again e raise to power minus 3 t sin 2 t plus phi 1 and this is single, it is not repeated. So, this is going to be C 2 C 0, C 1, C 2, e raise to power 3 t sin 2 t plus phi 2 like this. So, we can address all type of roots if the roots are real, repeated and repeated. If the roots are I mean complex in that case.

In that case also repeated and unrepeated roots addressed in this case. So, this is about complementary function q o complementary function. Now, we have to find the particular integral also in order to have the solution of total solution of this differential equation q o p i.

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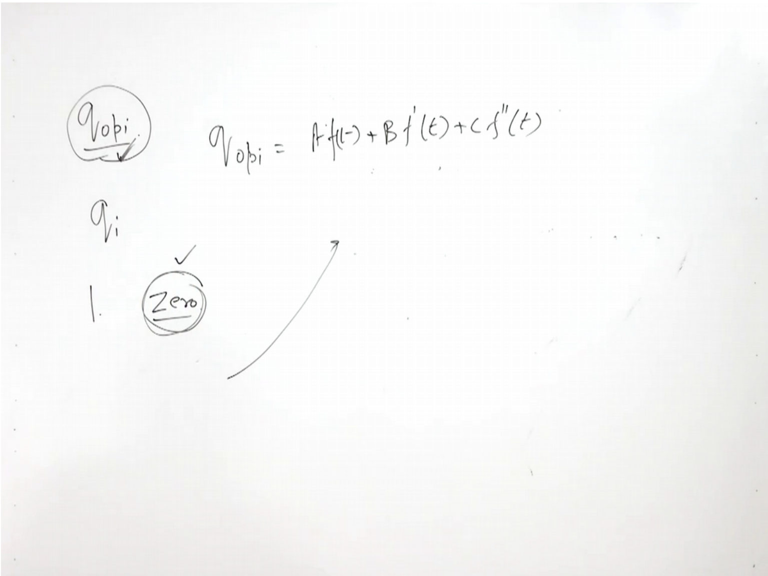

And q o p i, there is no universal method to find q o p i; the reason being need that this particular integral will depend upon the nature of input signal. It will entirely depend upon the nature of input signal. So, there is no universal method to find the particular integral that is first thing.

And input function, if it is a function of primary engineering interest, then we can have certain value of particular, suppose function is sinusoidal function or input function is a step function. So, then in those cases which we the functions which are of engineering interest, we can find the value of particular integral of output. So, there is no generalized method of finding out the solution for particular integral for the output there is a method of undetermined coefficient; this method can be used for finding out the solution for particular integral for the output.

Now, the method of undetermined coefficient again we will have to ask certain questions in this method. The first is, if after certain order of derivatives, after certain of derivatives, higher of order of derivatives are they 0. First of all higher order derivatives are 0 after certain order of derivatives. Second thing is, the higher order functions have seen functional form as lower order functions; that is also possible that higher order functions have same functional form is the lower order derivatives and the third is the functional form is continuous and it is rising.

If functional form is continuous and rising, then this method of undetermined efficient is not applicable but in the previous two cases, when it is higher order is 0, the functional the higher order derivatives are 0 and the second one is the higher order derivatives have functional form as the lower order derivatives. In those cases, we can find the solution of particular integral of with the help of undetermined coefficients and in that case the solution of particular integral is going to be A f t plus B f dash t plus C f double dash t.

This it will be in this form. If it is given, it will be in this form.

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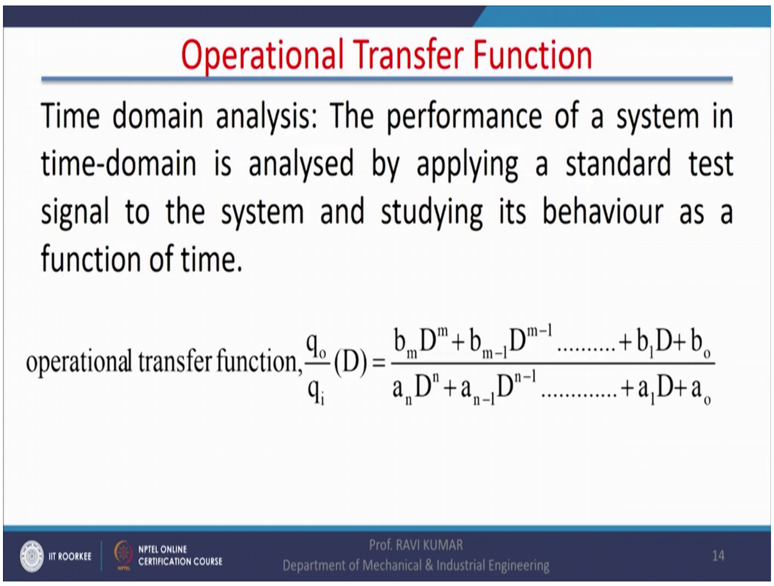

Now, after this will come to a very important parameter in dynamic analysis of measuring system that is operational transfer functions; because in subsequent lectures or subsequent analysis of dynamic measuring systems.

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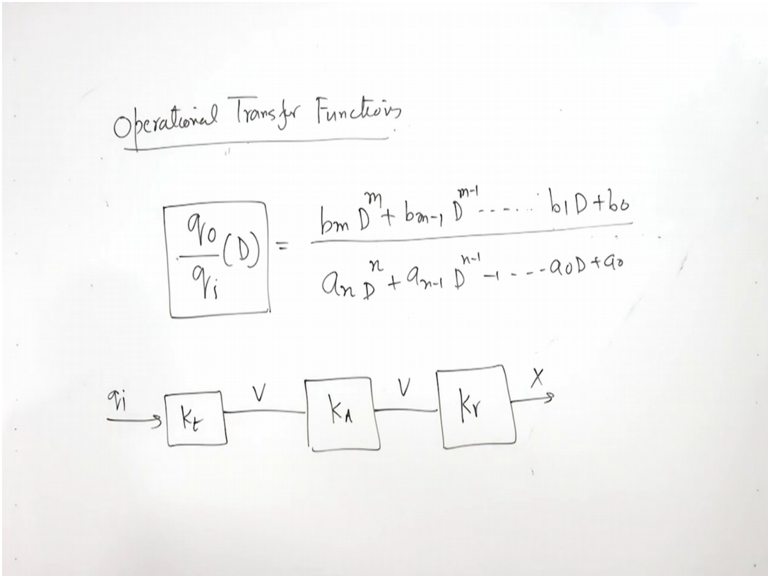

We will be frequently using this term, Operational Transfer Functions. Now, operational transfer function, it is depicted by q o by q i.

And this is equal to right b m D m plus b m minus 1 D m minus 1 and so on, q o like q i and this is b 1 D D plus b o divided by input is a n D raise to power n plus a, n minus 1 D to power of n minus 1 plus a o D plus. This is different from the output input ratio, output input ratio is instantaneous and this is a function, this is a function which shows the relationship of output with input. It is entirely different from output input ratio.

So, this operation function transfer function shall be used in future analysis of dynamic systems. And this operation function is very helpful suppose, I will give you an example. Suppose I want to measure output of certain transducer, there is the transducer I want to measure some there is some input q i and I want to measure certain output. So, output of the transducer is in volts and it has some transfer function, let us say it is K t transfer function, K t. For transducer, now transistor output signals are very low. So, they often go to the amplifier. So, when it is going to the amplifier, again amplifier has certain transfer function.

And it is again given voltage, but higher voltage, it is amplifying the voltage right then it is going to a recorder; there is a mechanical recorder where on a paper the input is traced. So, there is a recorder and it has transfer friction K r and output is displacement, movement of the pen all displacement right. So, movement of the pen is rated with q i. Now we have transfer functions. So, we can simply say x by q i is equal to the transfer function is going to be in this case because output of this is input of this.

So, transfer function is going to be product of this, this and this. So, this is how a complicated system, a complicated system can be divided into small components right and these components simply, the transfer function of these components is multiplied to get the final output and input relationship. So, that is operational transfer function, we will discuss in details in operational transfer function when we will go for the further analysis of dynamic response of measuring systems. Now there is a sin input also.

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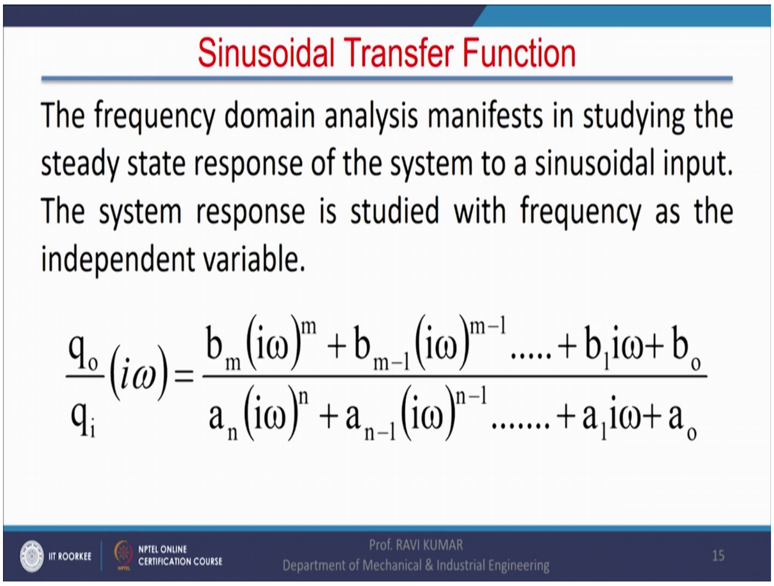

Frequency input, this is a linear input, time domain input and that is frequency domain input

So, when there is a frequency domain input, the frequency domain has also transfer function that is q o by q i.

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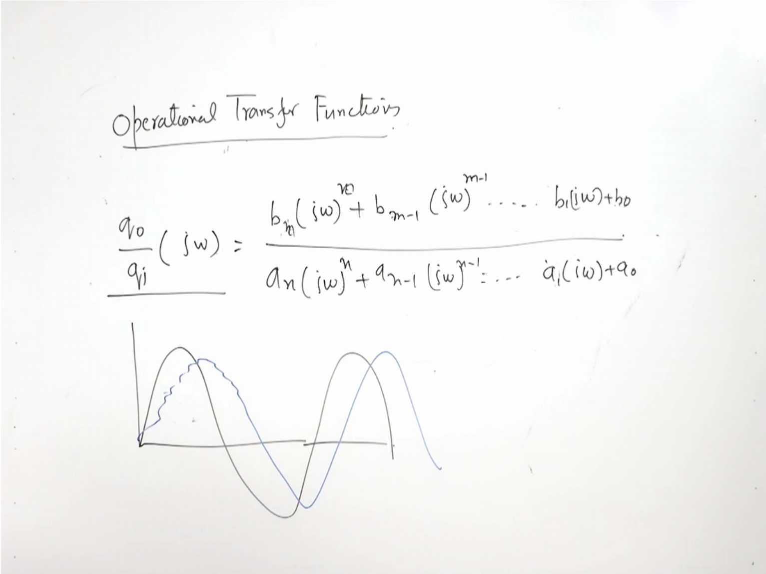

And it is denoted by i omega, omega is frequency 2 pi n right and is equal to again b o i omega sorry this is b n i omega n plus b, this is m b m minus 1 i omega m minus 1 and so on. And it is b 1 i omega plus b o divided by input that is a n i omega, this is m n plus a n minus 1 i omega n minus 1 and so on and then b 1 sorry a 1 i omega plus a o. So, this is how a transfer function is expressed for sinusoidal input.

So, because when we come across the measurements, there are I mean many instances when we have sinusoidal input to the instrument. Now, responsible, suppose that is a sin input to the instrument, there is an instrument and it has to measure sin waves. The response of the instrument may be like this that initially some dynamic response but later on the output will stabilize right.

So, all these responses on the system, dynamic responses of the system under the sinusoidal input also we will be discussing in the subsequent lecture. So, this is the introduction of dynamic response of measuring system and from the next lecture, we will start taking particular cases for dynamic response of measuring system. That is all for today.

Thank you very much.