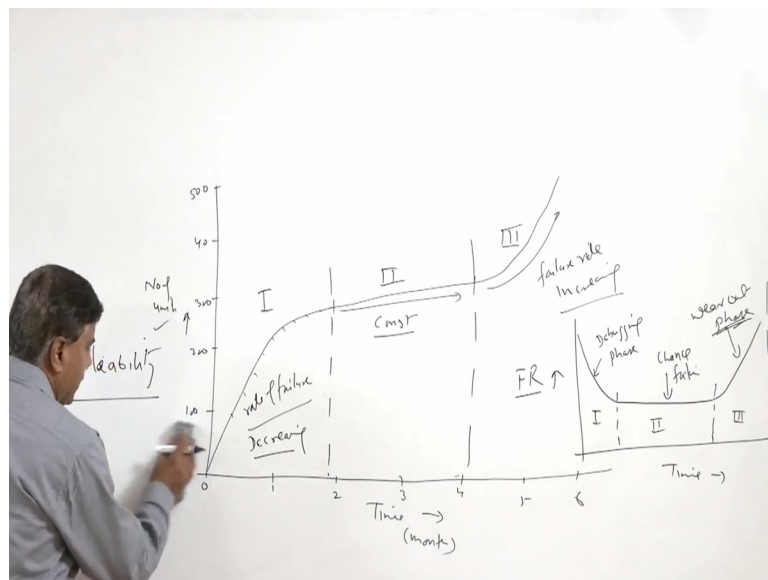


Failure Analysis & Prevention
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Lecture - 17
Industrial Engineering Tools for Failure Analysis: Reliability II

Hello, I welcome you all in this presentation related with the subject failure analysis and prevention and we are talking about the soft tools or industrial engineering tools which will be useful from the failure analysis point view and we are talking about the reliability related aspects regarding the failures.

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So, we know that whenever a component is put in use we will find that gradually its performance will keep on decreasing as a function of time and eventually it fails, but when the same type of components are put in use in very large numbers and then if they show the failure tendency as a function of time is noticed then what we can see here.

If in the x-axis we have the time and in the y-axis we have the number of units which are there, you can say initially if you see 0 time there are 500 number of units of a particular product which are working like say 100, 200, 300 and 400. So, as a function of time like say 1 month, 2 months, 3, 4, fifth, sixth a time in say we can write here months.

So, if we try to see initially the in which way the failure data for this number of units as a function of time if is recorded in what we find that, the failure rate actually in terms of the number of units the failure rate initially are decreases. So, what this trend shows in like initially the rate of the failure will be high and it will keep on decreasing as a function of time and then the rate of the failure will become constant and then again rate of the failure will become will be increasing.

So, here if you see in the y axis we have the number of units in x axis we have a time in if we see the slope of the curve will indicate the rate of failure and the rate of failure slope is actually negative it is, it is decreasing slope is decreasing and here the slope is constant. So, the rate of failure is also constant and the slope is positive. So, we can see the failure rate is increasing.

So, since in this case we have the number of units and the time. So, the slope of the curve will indicate the rate of failure. So, here in the initial stage we have the decreasing failure rate, constant failure rate, and increasing failure rate. If this curve is plotted it differently where in the y axis if we have like failure rate and in x axis if there is a time, then since the failure rate here slope is getting negative as a function of time. So, the failure rate will be decreasing.

See y axis we have failure rate and then failure rate becomes constant and then failure rate it starts increasing. So, this type of the curve if you see initially the failure rate is high then it will keep on decreasing then it will become constant and then it will be increasing. So, if you see either both these curves if you see there are the 3 distinct zones the zone 1, zone 2 and zone 3 hear the slope is negative. So, the rate is decreasing slope is constant and the slope is positive means their failure rate is increasing.

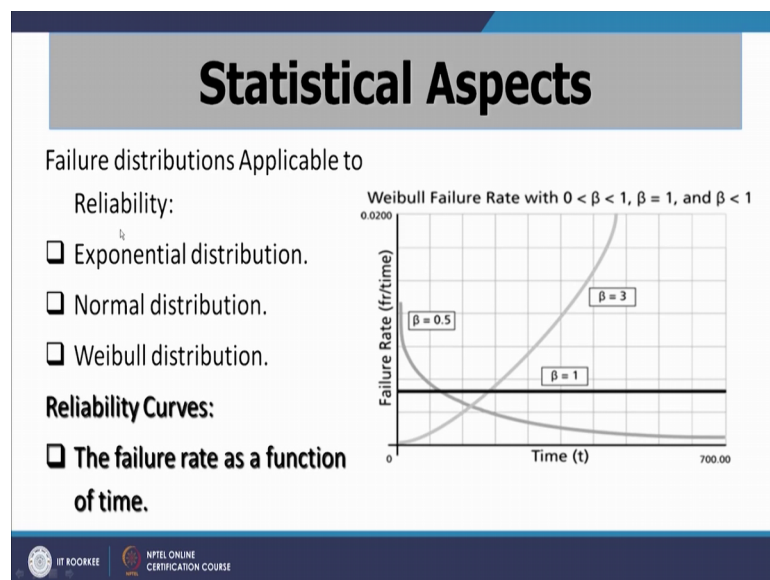
So, here the same thing when it is plotted in terms of the failure rate again we find 3 different zones like zone 1, zone 2 and zone 3. So, this is called this first zone is called debugging face, this the chance failure case and this is the wear out face. So, if you have to understand whenever a complex system is port in use initially it will post the number of the complications that is why and as soon as we start taking corrective actions the rate of failure will keep on decreasing.

Thereafter it will give us the useful life period that is where in the failures will be occurring by chance and that rate becomes almost constant and in the third phase is a

wear out phase where after completing the useful life of the product it the failure it starts increasing due to the wear and tear which will be experienced by the product. So, this the kind of failure rate as a function of time the failure rate for the different types of the products as a function time as a function of time varies.

So, the there are 3 kind of the variations switch are generally observed with regard to the failure rate as a function of time and these are indicated with the help of say this diagram where in the failure distribution which are normally applicable to the reliability calculations.

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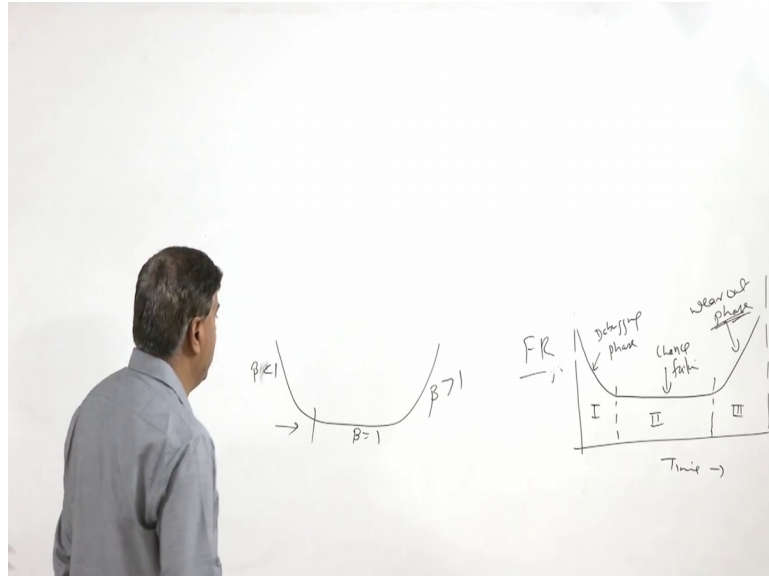


One is the exponential distribution, second is the normal distribution and the third one is the Weibull distribution. So, these 3 types of the distributions have been shown here, in one case the failure rate is decreasing as a function of time, in another case failure rate is increasing and in third failure rate is constant and the third case where the failure rate is increasing.

So, this either decreasing when the failure rate is decreasing as a function of time the beta value is found to be less than 1 and when the failure it is constant the beta value is 1 and with a failure rate is increasing as a function of time the beta value is greater than 1. So, in order to explain these failure rates as a function of time because initially the failure rate is decreasing, then it becomes constant and then it is increasing, this 3 phases related with the life curve of a product or of a system is represented with the help of this

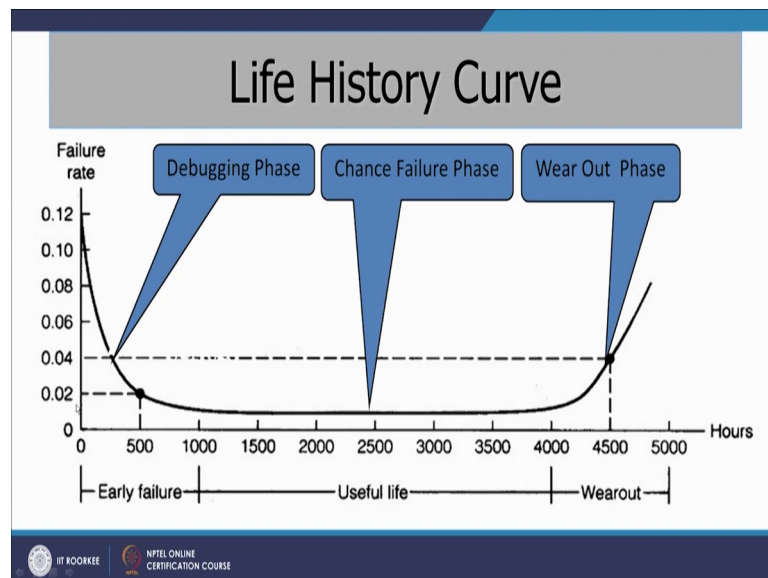
diagram which is also termed as a bathtub curve and which is this type of name is given because of its shape.

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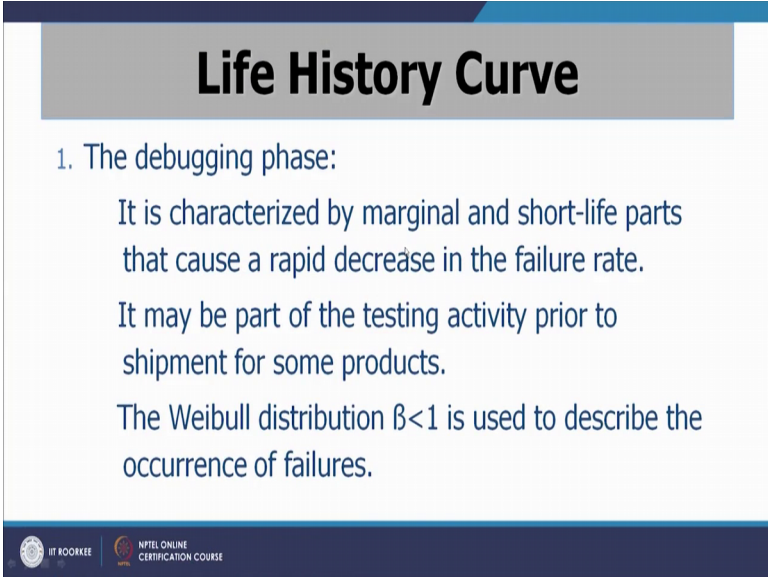
Here this shape is similar to the bathtub where in the initial zone is where the failure rate is decreasing. So, here our beta value is less than 1, here beta value is equal to 1 and here beta value is greater than 1 for the 3 different stages of the life of the product. So, this is what is represented with the help of this bathtub curve in this case what we have like the failure rate in y axis and the time of the service in the x axis.

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So, here we can see the failure rate is decreasing in initial stage this is termed as early failure or debugging phase where the minor issues are taken care of as a function of time. So, that system is start functioning as expected and then in the second stage the failures occur by chance and the failure rate is almost constant and it is minimum failure rate in this phase and after completing this useful life the portion wear out face is starts in where due to the loss of dimensions, loss of property is degradation in the size and shape, the product is starts malfunctioning and that internally to the failure of the component. So, this is what is known as the life history curve or bathtub curve for any kind of product.

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Life History Curve

1. The debugging phase:
 - It is characterized by marginal and short-life parts that cause a rapid decrease in the failure rate.
 - It may be part of the testing activity prior to shipment for some products.
 - The Weibull distribution $\beta < 1$ is used to describe the occurrence of failures.

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So, here in detail we can see what is the significance of the debugging phase, it is a characterized by the marginal and the short life parts that cause a rapid decrease in failure rate and it may be the part of the testing activity prior to the shipment of the product and we will distribution in this case is observed where in beta value is less than 1 is used to describe the occurrence of the failure in the debugging phase or the initial failure case.

And in the second case, where the chance failure phase existent the failures occur in random manner due to the constant of failure rate.

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Life History Curve

2. The chance failure phase:
Failures occur in a random manner due to the constant failure rate. The Exponential and the Weibull distributions $\beta = 1$ are best suited to describe this phase.
3. The wear-out phase:
Is depicted by a sharp raise in failure rates. The Normal distribution and the Weibull distribution $\beta > 1$ are used to describe this phase.



And the exponential and we will distribution for this situation is expressed through the beta equal to 1 is the best suited to describe this phase and the wear out face is depicted as shown by the sharp increase in the failure rate after the end of the at the end of the useful life portion at the second stage after the second stage and normal distribution and Weibull distribution for this kind of the face is expressed using the beta greater than 1 to describe this phase.

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Normal Failure Analysis

- The Normal distribution.

$$R(t) = 1.0 - \int_0^t f(t) dt$$
$$R(t) = 1.0 - P(t)$$

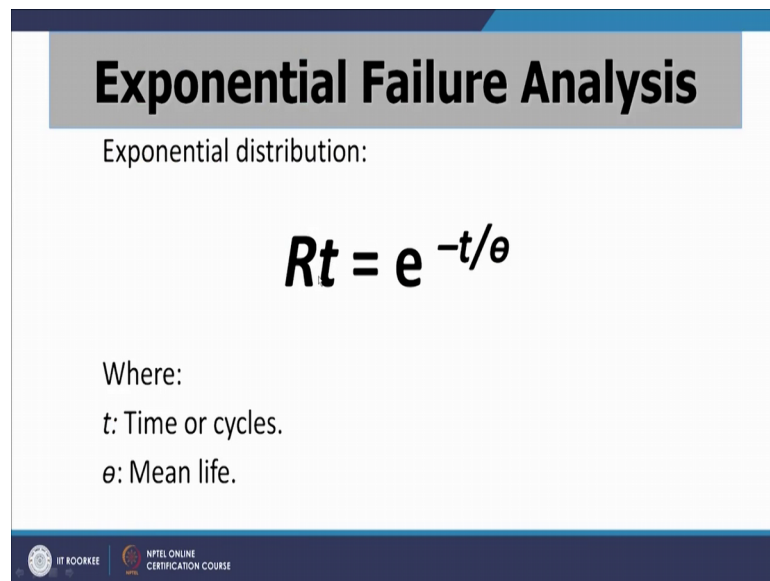
$R(t)$: Reliability at time t

$P(t)$: Probability of failure



In the case when the failure distribution is normal the reliability is expressed using these equations and where in the R_t is the reliability at a particular time and P is the probability of the failure at that particular time. So, R_t is obtained from the 1 minus P_t . So, the P_t here is P within the bracket t is the probability of the failure at the particular moment of the time.

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Exponential Failure Analysis

Exponential distribution:

$$R_t = e^{-t/\theta}$$

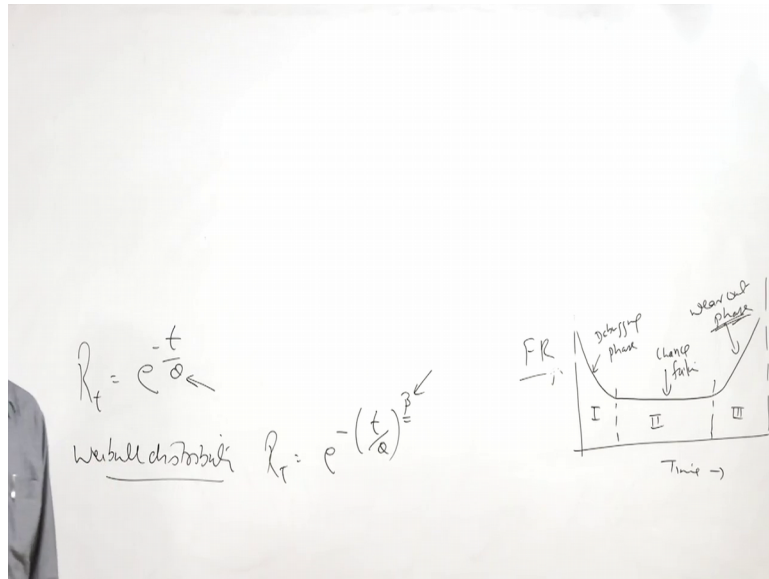
Where:

- t : Time or cycles.
- θ : Mean life.

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And for the exponential failure district if the distribution of the failure is exponential in that case the reliability is calculated using this simple equation like the reliability at the moment of the time is obtained through the equation e raise to the power minus t by θ .

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So, t is the time at which the reliability is being checked and the θ is the mean value, mean life or the mean target value for which reliability is being assessed. So, this is how we can calculate the reliability for the time for the condition when the distribution of the failure is exponential. So, θ you can say the target value or the mean value and that t is the time at which time or the cycle or the value for which the reliability is being calculated.

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Exponential Model - Example

The response time X at a certain on-line computer terminal has an exponential distribution with expected response time equal to 5 sec.

(a) What is the probability that the response time is at most 10 seconds?

(b) What is the probability that the response time is between 5 and 10 seconds?

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So, here now we will see one typical example related with the case where the distribution of failure is exponential. So, here what we can see here, if the response time X at a certain online computer terminal has an exponential distribution means the response time for a particular computer terminal has the exponential distribution and if that time is X with the expected or target value of the time of response is say 5 second.

So, the average or the target value here is 5 seconds and the response time shows the exponential distribution which is say X . So, what we need to calculate is, what will the probability that response time is at the most 10 seconds? So, the probability to have the response in 10 seconds is calculated this is the one case, what will the, how to calculate the probability for the response time of the 10 seconds and the second case is what is the probability that response time will fall between the 5 and 10 seconds?

Here the target value or the theta value is 5 so, for this case if you have to calculate the reliability that for calculating the reliability first of all we have to calculate the probability of the failure.

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Exponential Model - Example



The $E(X) = 5 = \theta$.

The probability that the response time is at most 10 sec is:

$$\begin{aligned}
 P(X \leq 10) &= F(10, 0.2) \\
 &= 1 - e^{-10/5} \\
 &= 1 - 0.135 \\
 &= 0.865 \\
 \text{or } P(X > 10) &= 0.135
 \end{aligned}$$

The probability that the response time is between 5 and 10 sec is:

$$\begin{aligned}
 P(5 \leq X \leq 10) &= (1 - e^{-2}) - (1 - e^{-1}) \\
 &= 0.233
 \end{aligned}$$



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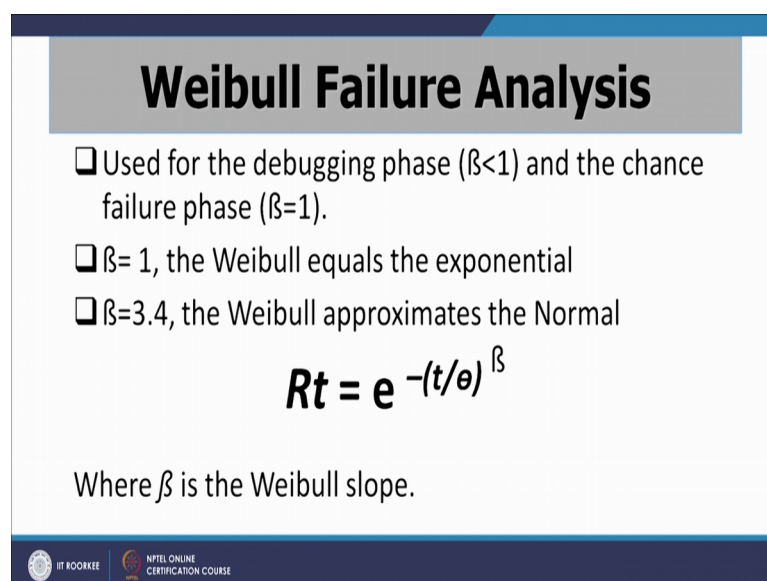
So, probability that a response time is at the most 10 second is obtained through the we know that reliability is equal to 1 minus probability of the failure and the probability of failure here is 1 minus t by θ , t here is the value for which we are trying to see the probability to achieve that or t is the value that is in the that what is the probability to have a complete that activity in 10 seconds with respect to the target or average value of

like say the 5 seconds. So, here 1 minus e raised to the power 10 by 5. So, 10 is the time in seconds for which the probability is being assessing average time value is 5.

So, this is how we can calculate and the, what we can see here reliability is coming out 1 minus 0.135. So, this is the reliability of the failure and 0.865 this will give us the reliability to achieve the response time at the most in 10 seconds and similarly the probability to have the response time between the 5 and 10 second can be obtained in the same way like the probability to get this response time in 5 seconds or in 10 seconds what will have here the probability for having the response in 10 seconds minus the probability to have the response time in 5 seconds. So, this is what we can see the probability to have the response time in 10 seconds that is been obtained through 1 minus e raise to the power minus 2 and minus 1 minus e raised to the power minus 1.

So, here minus 2 and minus 1 are coming from like say minus 2 is by 10 by 5. So, that will be giving us the minus 2 and here the 5 by 5 that because here this is the range. So, minimum value is being placed here. So, the average value is 5 and the, that time for which we are looking at is also 5. So, this will give us the value of minus 1 and on calculations we get this the probability that response time is between 5 and 10 second will be the 0.233.

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Weibull Failure Analysis

- Used for the debugging phase ($\beta < 1$) and the chance failure phase ($\beta = 1$).
- $\beta = 1$, the Weibull equals the exponential
- $\beta = 3.4$, the Weibull approximates the Normal

$$Rt = e^{-(t/\theta)^\beta}$$

Where β is the Weibull slope.

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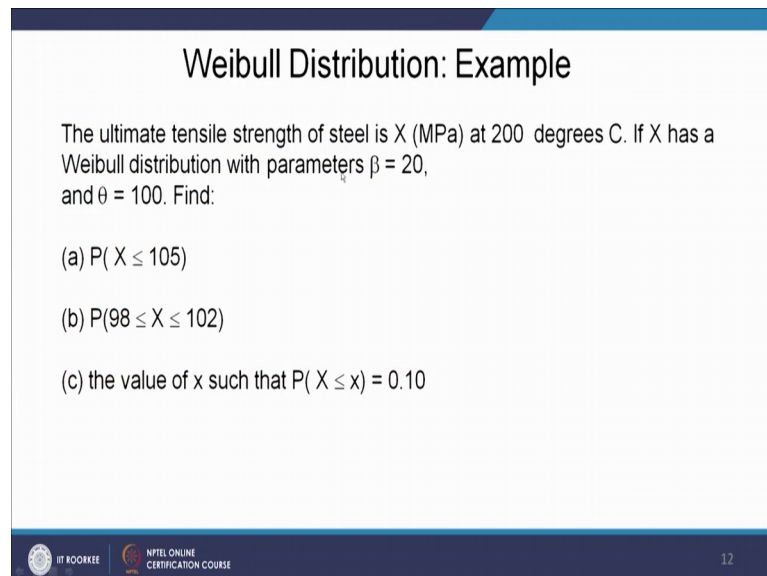
Now instead of having the normal distribution and exponential distribution if the failure distribution is of the Weibull type. So, in that case Weibull distribution for that we know

that for the debugging phase the Weibull distribution the in the debugging phase for the weibull distribution the beta value is are less than 1 and for the chance failure this value is equal to 1 and the when the beta value is equal to 1 the weibull distribution equals to exponential distribution and when the beta value is equal to 3.4 then weibull distribution approximate equals to the normal distribution.

So, to for having the reliability at a particular time we need to calculate relative for the weibull distribution the failure rate here R_t is obtained through the e raised to the power minus t by θ , t by θ this component is same as that for the exponential distribution additionally it has one more coefficient and this is the beta. So, depending upon the phase of the life of a product whether it is debugging phase or the second phase that was a chance failure case or the wear out phase we have to put the suitable value of the beta.

So, the beta basically is obtained from the slope of the curve where the failure rate is available as a function of time. So, here say this is another example for the when the failure distribution follows the weibull distributions. So here the in this example if the ultimate strength of the steel is X at a 200 degree centigrade and if X follows the weibull distribution with the parameter beta is equal to 20.

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Weibull Distribution: Example

The ultimate tensile strength of steel is X (MPa) at 200 degrees C. If X has a Weibull distribution with parameters $\beta = 20$, and $\theta = 100$. Find:

- (a) $P(X \leq 105)$
- (b) $P(98 \leq X \leq 102)$
- (c) the value of x such that $P(X \leq x) = 0.10$

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Then for this situation the average value is the 100 MPa that is the strength then we need to find, what is the probability to get the value less than 105? And what will be the probability to get the value between the 98, to 102? And what will be the value if the

value of X for the probability of the 0.1? So, we will take up the first 2 issues where in we can see the probability to have the value less than 105.

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Weibull Distribution - Example Solution

(a) $P(X \leq 105) = F(105; 20, 100)$



$$= 1 - e^{-(105/100)^{20}} = 1 - 0.070 = 0.930$$

(b) $P(98 \leq X \leq 102)$

$$= F(102; 20, 100) - F(98; 20, 100)$$

$$= e^{-(0.98)^{20}} - e^{-(1.02)^{20}}$$

$$= 0.513 - 0.226 = 0.287$$



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So, what we have seen, the value for which the probability is being considered is 105. Beta is the Weibull distribution coefficient that is 20 and 100 is the average value. So, we have all these 3 values so, the probability for this situation is being obtained from the equation $1 - e$ raised to the power minus.

So, here this is the t is the value for which probability is being calculated that is 105 and 100 is the average or the target value, average value or the mean value and the beta is the coefficient that is the beta is the 20, 20 is the beta value which is given for this case and when we calculate this when we solve this equation what we get the 0.93 is the probability to get the value less than 105.

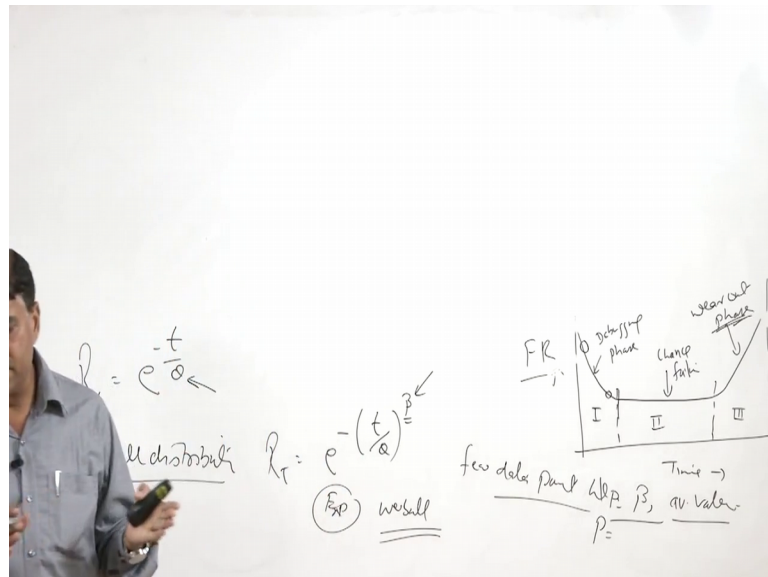
Similarly the probability to get the value of strength ranging from the 98 to 102 that is also can be calculated from this equation where in like say the probability for failure to get in this range 102 and 98 it is obtained. So, here what we can see 102 is the higher range and the beta is the Weibull distribution coefficient and 100 is the average value or the target value average value which is given. While on the 98 is the lower value and the beta is the, that same coefficient and the 100 is the coefficient value 100 is the average value.

So, this is how we can calculate the probability for the higher value minus lower values, that will give us the probability to have the value of the a strength between the 98 to 102 and this will be giving us like 0.28 with the probability to achieve the value of a strength in range of the 98 200 that will be like 0.287.

So, this is how I will summarise this presentation, in this presentation I have talked about the failure rate as a function of time and their 3 different kind of the distribution of the failure rate one is the nominal normal distribution, exponential distribution and weibull distribution. So, use of this failure distribution is that if you have the failure data for a particular phase of the product, then we can estimate whether what will is the what will be the probability to get a particular product to survive or what will be the particular you need to deliver the job as per the required function

And at the same time Weibull distribution is very useful in the sense that; if we have the data very few data also like in this phase few data points then this can be used to see what is the probability of the failure of the particular period of the time.

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So, even very few data points help to have the value of the beta with regard to the given average value we can calculate the probability for the survival or the reliability of a particular product to work or possibility of the failure a probability of the failure can be obtained through this kind of the failure distributions which can be like the exponential distribution or it can be the Weibull distribution so, from the failure analysis point of

view from the history of the failures of particular type of products if we have the failure rate and the slope of the curve for the failure rate.

Then we can slope of the failure rate then we can calculate the probability of the failure of a particular product or the system after a particular period of time. So, this will help us in making the suitable plans So, that preventive measures can be taken from the maintenance point of view.

Thank you for your attention.