Automatic Control
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Lecture – 08
Electrical System

So, welcome to the lecture on mathematical modeling. In this lecture we will discuss on electrical system. So, as in previous two lectures we learned how to model mechanical systems translational, as well as rotational systems. In this lecture we deal with electrical systems. So, we know those electrical systems, the basic elements or resistance inductance or inductor and capacitor. So, these systems can be modeled using the, these elements and we applied mechanical. In mechanical system we applied Newton's law to write the differential equation of the system. In electrical systems we used Kirchhoff's law. Kirchhoff's two laws; that is current law or node law and voltage law or loop law. So, what is current law?

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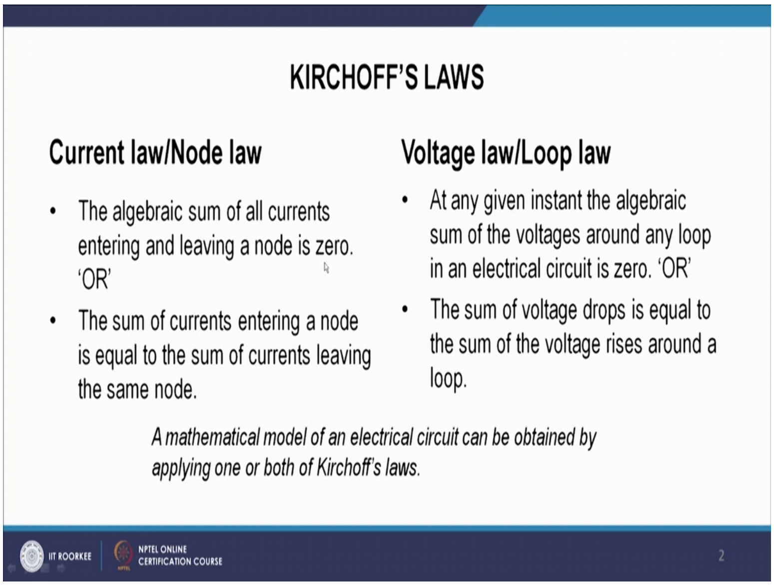

So, here the current law tells that the algebraic sum of all currents entering and leaving a node is zero or the sum of currents entering a node is equal to the sum of currents leaving the same node.

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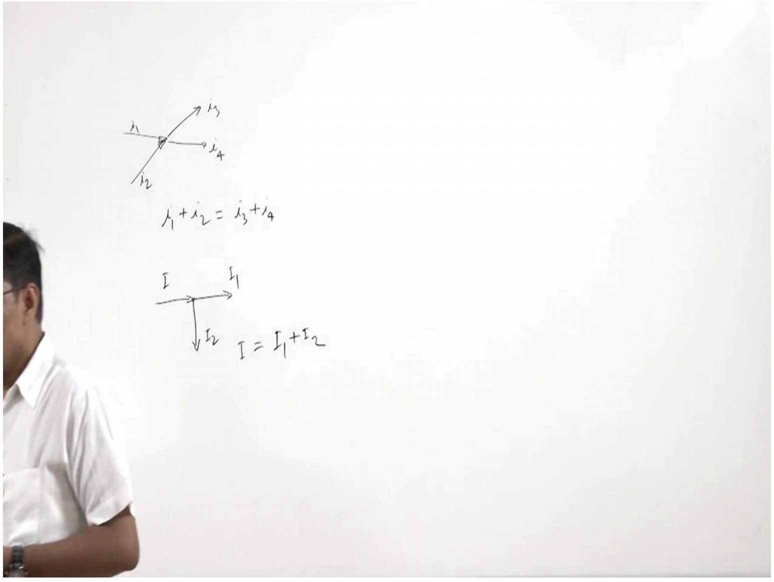

So, if we have some electrical circuit and there is some point in the electrical circuit, and there are currents coming like i 1 i 2 and these currents leaving here i 3 i 4.

So, according to this current law, the currents entering to this node is equal to currents leaving. So, i 1 plus i 2 equal to i 3 plus i 4. So, let us say if we have some point, where current I is entering and then here is I 1 I 2. So, here I equal to n I 1 plus I 2. So, this is the current law. Now the voltage law or loop law tells that, at any instant the algebraic sum of the voltage around any loop in electrical circuit is zero or the sum of voltage drops is equal to the sum of voltage rise around a loop.

So, this voltage law is applicable for us a loop for a loop, that if in any loop we traverse through the loop and we sum the voltages with their signs that will be zero, or if there is some voltage drop and voltage rise. So, the voltage drops is equal to voltage rise around the loop. So, a mathematical model of an electrical circuit can be obtained by applying one or both of Kirchhoff's law.

So, we will see that how we can get the mathematical model for basic electrical system, that comprised three components resistance inductor and capacitor and how can we find the transfer function of an electrical system. So, let us take an L R C circuit.

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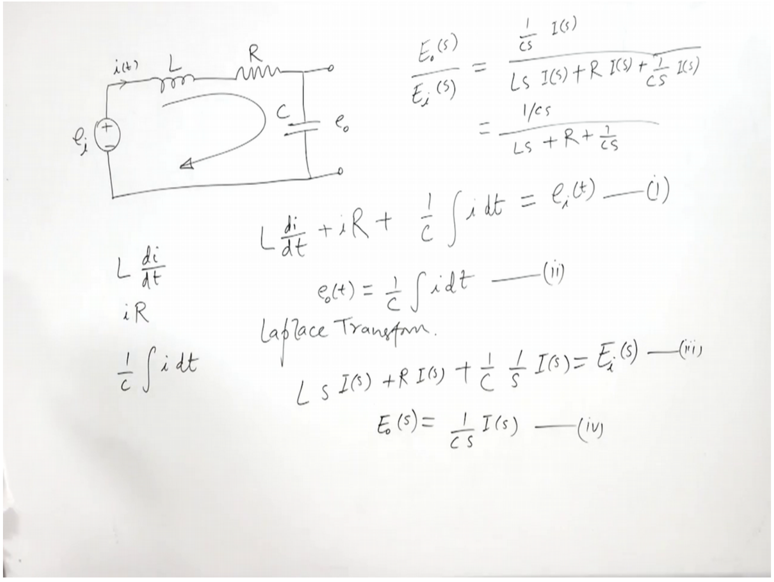

So, here we have this circuit here is this is inductor this is resistance and this is capacitor and here we are measuring the output voltage and there is a current I.

So, here is this input voltage this is output voltage we are measuring around the capacitor. So, this is output voltage we are measuring around the capacitor and. So, we want to write the mathematical model for this system and to find the transfer function of this system. So, we should try to understand that the voltage. We can write voltage drop in the inductor L d i by d t in a register voltage is proportional to the resistance and in and in capacitor, it is 1 by C i d t.

So, let us traverse this we apply the, because this current I is flowing through this loop there is no any diversion of this current. So, this same current is flowing through this loop. So, we do not need to apply the I mean node law, but we apply the voltage law. So, voltage now we apply. So, according to the voltage, now we traverse through this loop, we do the algebraic sum of the voltages drop and rise and then that is equal to 0.

So, voltage drop across the inductor is L d i by d t across the register is i R and across the capacitor is 1 by C i d t. So, this is the voltage drop that should be equal to voltage gain. So, here this is a source. So, we are getting I have a voltage gain e i. Now here this is output, we are measuring here output. So, output is equal to 1 by C i d t.

So, output we are taking the voltage across the capacitor. So, now, we can see, this is the equation for this system, it contains integration as well as differentiation, differential term. Now we have to find the transfer function for this, we take the Laplace transform. So, we take the Laplace transform of this equation number one, so we will get L s is plus R is here I function of time, because this current changes with time, when the volt voltage changes, changes with time and here is 1 by C this is integral.

So, 1 by s into Ei s because the Laplace of integral is 1 by s is and that will be equal to E i s for this equation 2. We can write E o s equal to 1 by C s is. Now, the transfer function E o s by E i s. So, 1 by C s is upon L s is plus R i s plus 1 by C s I s. So, this we can write 1 by C s upon L s plus R plus 1 by C s.

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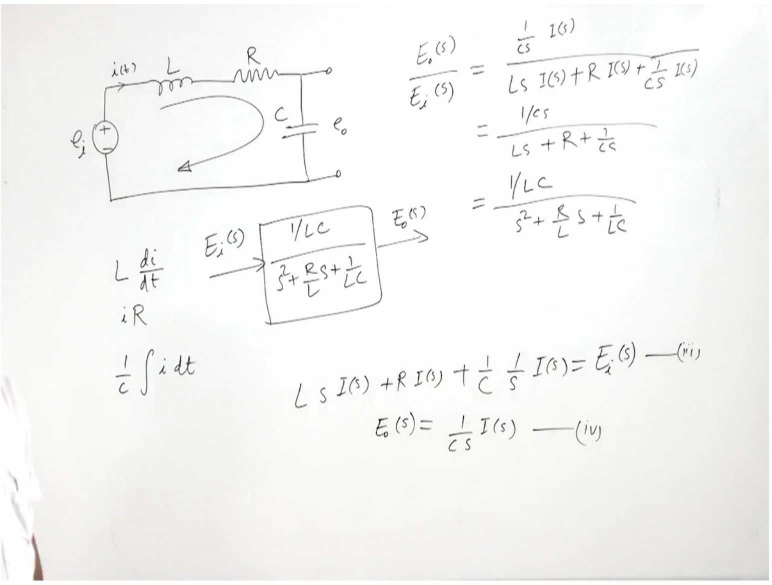

So, here again we can simplify this expression. We take L here and S here, so S square plus R by L S plus 1 by L c. So, we can represent this transfer function. So, here E i s and E o s and this is 1 by L C s square plus R by L S plus 1 by L C. Again we see that this is the transfer function that contains the properties parameters of the system; that is L R and C and this is again the second order system, because here the, there is a square term.

Now, we discuss the concept of impedance in case of electrical circuit. In electrical systems the concept of impedance can be used to find the transfer function and represent the system. So, in electrical system the impedance is defined as voltage upon current in s domain.

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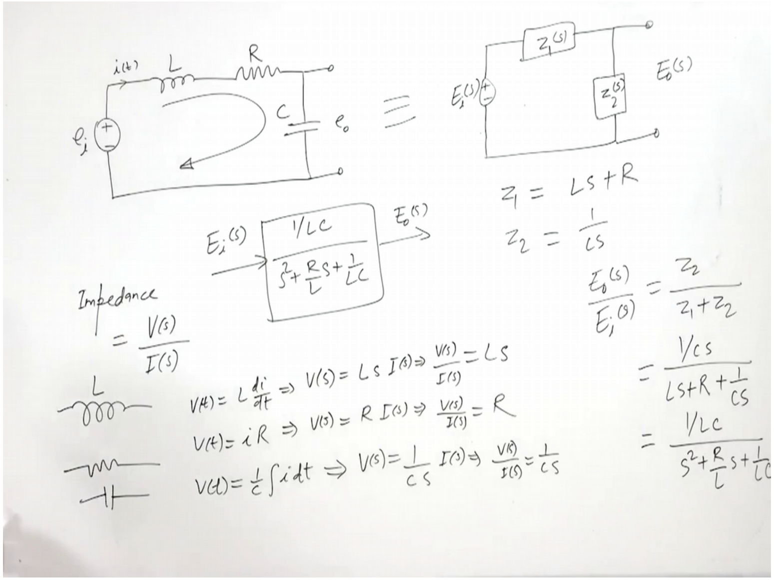

So, for inductor L were V t equal to L d i d t.

So, here V s equal to L into s I s. So, here V s upon I s equal to L S. So,. So, this is the impedance for inductor for a resistance V n equal to i R. So, here V s equal to R into Is. So, V s upon Is equal to R. Similarly for a capacitor we have V equal to 1 upon C i d t. So, V s equal to 1 by C s I s. So, this applies that V s by Is equal to 1 upon C s.

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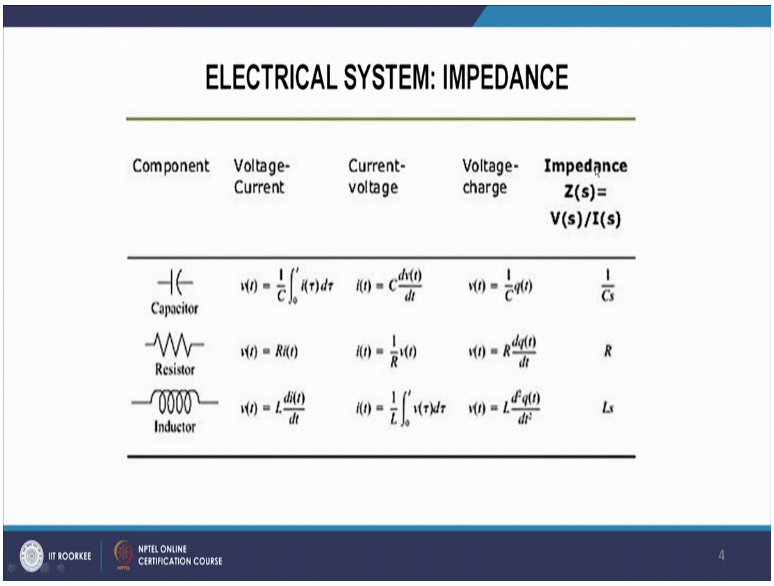

So, in this slide we can see, we have represented the impedance. So, voltage current relationship for these three elements capacitor resistor and inductor current voltage relationship, and then voltage charge relationship and then impedance. So, impedance is v s upon is. So, for capacitor it is one by C s, for resistor it is R and for inductor it is L s. Now we can find the this system we can represent as equivalent impedance form. So, here we have let us say this is Z 1 impedance and then here Z 2.

So, here e O s E i s and this is Z 1; that is of course, on of s, and because impedance is defined in terms of s. Now here Z 1 equal to that is the impedance of a inductor and impedance of register. So, that is L s plus R and Z 2 equal to impedance of the capacitor and that is one by C s. So, here we can find from here u s by e is equal to Z 2 s Z 2 upon Z 1 plus Z 2 Z 2 upon Z 1 plus Z 2.

So, Z 2 is 1 by C s and Z 1 is L s plus R and Z 2 is 1 by C s. So, we can again simplify it 1 by L C and s square plus R by L s plus 1 by L C. So, we can see that using the impedance method we can also find the transfer function, the same we can see, we find the same transfer function. So, therefore, this method is important for the systems. We do not need to go to the differential equations; we can directly find the transfer function.

So, we can take one more example and that may contain the nodes, where the currents will divide and, so it may form the more than one loop ok.

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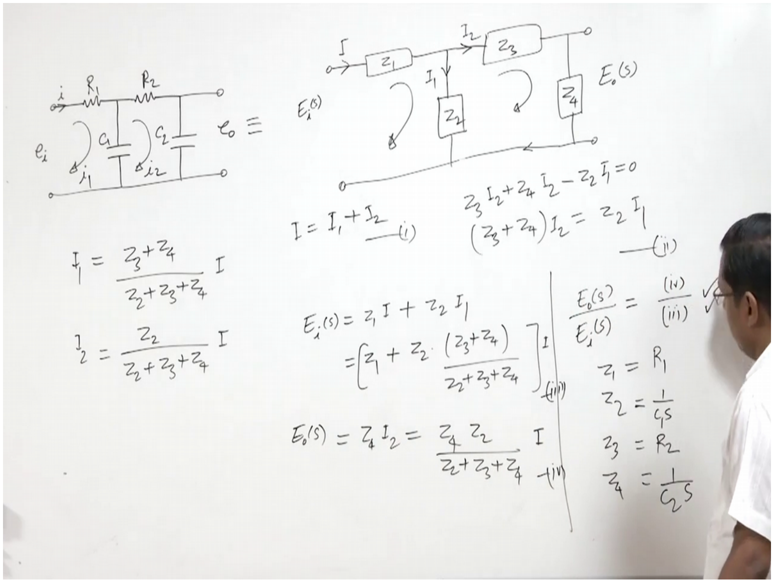

So, here we have, we have R 1 to R 2 to register and two capacitors. There are two loops here and we are reading the output here across this second capacitor. Now let us represent it in the impedance form. So, here we have E i s then let us we have current I here of course, it is function of s, then here a Z 1 impedance, this is here, we have capacitor impedance then here one more capacitor impedance and this is E o s.

So, output voltage; now if we have current I, this current will be divided at this node, let us say this is I 1 and this is I 2. So, here we have node law; that is I equal to I 1 plus I 2. We can apply the node law and then these voltages. So, we go this loop. So, Z 3 I 2 plus Z 4 I 2 and here minus Z 2 I 1 equal to 0, because here we are going in the direction of current and here we are going opposite to the direction of current, so here minus Z 2 I 1. So, Z 3 plus Z 4 I 2 equal to Z 2 I 1.

So, we have this equation number 1 and equation number 2 from the two equations, we can find I 1 and I 2. So, we can find I 1 I 2. So, we solve this equations, we will find I 1 equal to Z 3 plus Z 4 upon Z 2 plus Z 3 plus Z 4 into I and I 2 equal to Z 2 upon Z 2 plus Z 3 plus Z 4 into I.

Now, E i s this first loop. So, E i s equal to Z 1 into I plus Z 2 into I 1, because there is the voltage drop and here is the voltage gain. So, this now we can put here this Z 1 plus Z 2 I 1, we can represent in terms of I. So, Z 3 plus Z 4 upon Z 2 plus Z 3 plus Z 4 I and E o s equal to Z 4 into I 2 equal to Z 4 into I 2.

So, here Z 4 into Z 2 upon Z 2 plus Z 3 plus Z 4 into I. Now we can find E o s by E is. So, E o s we can divide this term by this term. So, I will be cancelled out and these terms. So, Z 1 we can put. So, this is equation number 3 and this is equation 4. So, E o s by E is we can find by dividing equation number 4 by 3, this term the right hand side terms and there we could put Z 1.

So, Z 1 is a register. So, the impedance is R 1 Z 2 is a capacitor. So, it is impedance is 1 by C s and Z 3 is register. So, it is impedances is R 2 and Z 4 is again a capacitor, it is. So, here is C 1 and this is C 2 and of by putting these values on these impedances we can find the transfer function E o s by E i s.

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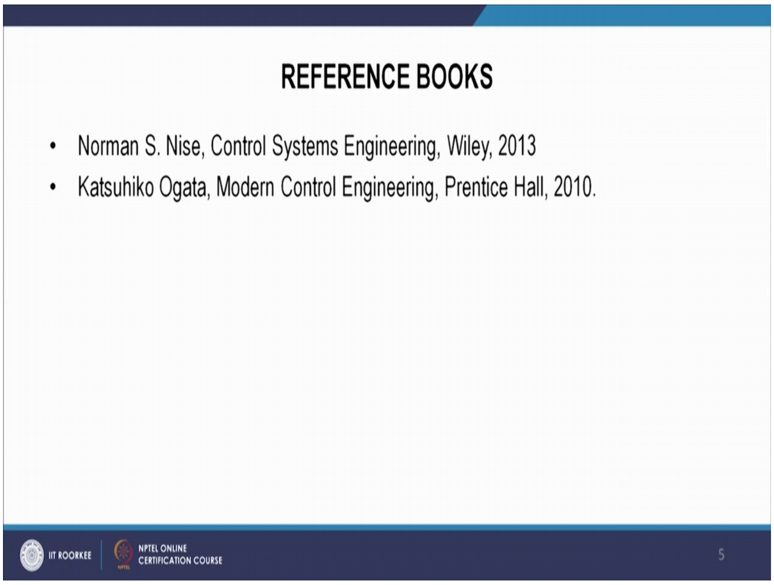

So, these examples for taken from the book of Norman S Nise control systems engineering. So, we learned that how to model the electrical systems. We use the basic elements like resistance capacitor and inductor and we can go apply two methods the different. First we write differential equations using the Kirchhoff's law and then we find the transfer function by taking the Laplace transform of that equation. Second method is that we represent the circuit in terms of impedance circuit impedance and we can directly apply these laws on the impedances, and we do not need to write the differential equations in this case, we can directly find the transfer function. So, I thank you for attending this lecture and see you in next lectures.

Thank you.