Automatic Control
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Lecture – 06
Translational Mechanical System

So, welcome to the lecture on Mathematical Modeling. Today we will discuss in this lecture, the modeling of translational mechanical system. So, we know that a system can be modeled in different ways, different methods. So, the first and basic and very common method is the differential equations.

So, we can model a different type of systems using differential equations. Then we can also model the using the transfer functions or we can model in state space domain. So, here, we will focus more on modeling in differentially using differential equations and then converting them to transfer functions.

So, in this lecture we are focusing the mechanical system that has the translational motion. So, they have only the translational degree of freedom. We are not considering a mechanical system with rotational degree of freedom in this lecture. We will consider them in the next lecture so, the Translational Mechanical System. So, a mechanical system comprises several components. And a mechanical system can be represented in terms of basic elements of mechanical systems like spring, Mass and Damper.

So, here we can model a system, mechanical system using these elements because mass is inertia; it represents inertia element spring is high energy storing element and damper is energy dissipating. So, in the mechanical system these 3 phenomena occurs and these 3 elements represent those phenomena. So, let us take a very simple system and that is a single degree of Freedom System which has only one translational degree of freedom in which the mass can move. And how to write the differential equation for this system and then we convert this differential equation to find the transfer function of the system or we can find the block diagram for that system.

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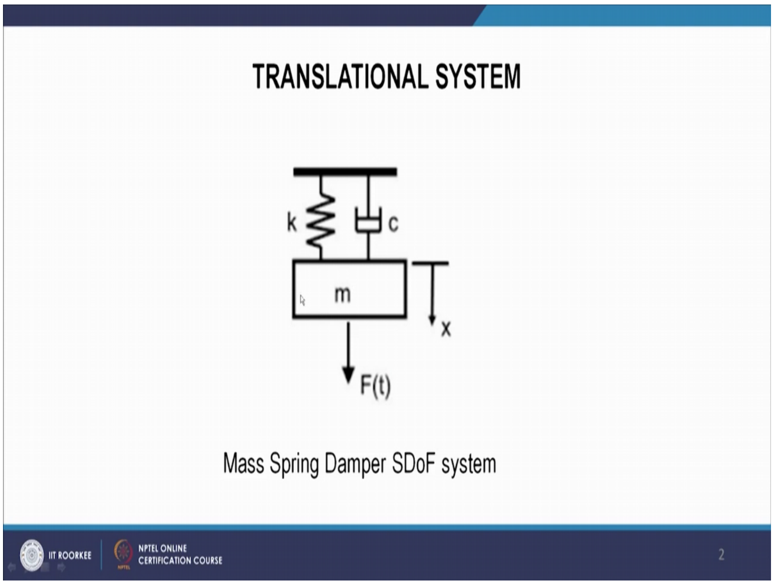

So, this is the system, single degree of Freedom System; Mass, Spring and Damper system.

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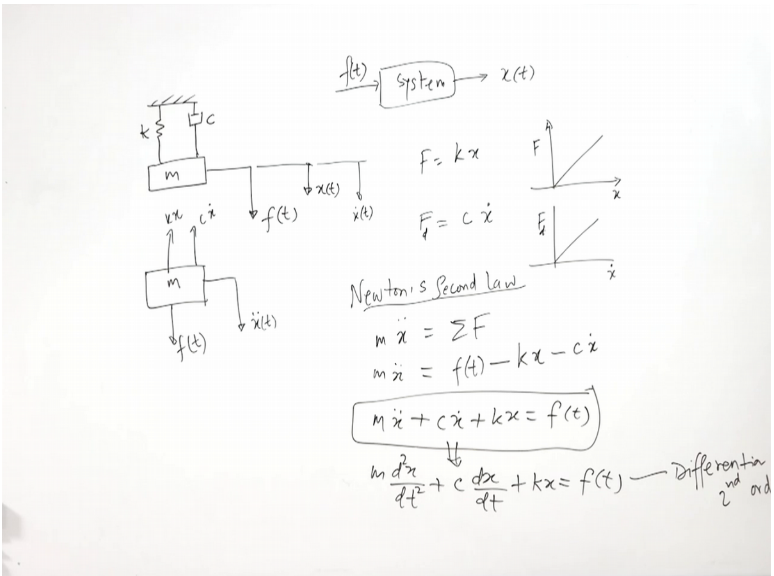

So, this is a system and here it is subjected to some force f t. So, here we want to know the x t. So, here is the system, we apply input f t and we are interested in output x t. So, x is the translational degree of freedom or displacement of this system the mass. And f t is your force as force is applied externally. So, it is input to the system.

Now, here we assume that this spring behaves linearly in the range of operation. So, here we have the relationship F equal to k x for this spring. Similarly, this damping is viscous damping. So, viscous damping relationship is represented by c equal to, F equal to the viscous force F equal to c into x dot. So, here this is the relationship linear relationship between the Viscous Force. Let us say this is damper force F d.

So, this relationship is linear. So, this is our assumptions in processing this example. So, now, first we have to find the differential equation of this system. So, when we apply certain force to the system and system is oscillating or in motion, let us say at time t, we the system has some displacement x t. So, in deep this instant of time, we make the free body diagram of this system.

So, this is a system. Here, this is m; here the force is f t. So, in a free body diagram, we will show all the forces acting on the system. So, action reaction force, action force inertia force. So, here f t, if we this mass is displaced with x, the spring will apply a reaction force that is k into x. And that will be opposite to the movement of this x. So, this is k into x.

Similarly, the Damper, if this is x; this will have also some velocity x dot t. So, if Damper will apply some force c x dot on this system. And there is some acceleration x double dot of this system and this will represent the acceleration will represent m; mass into acceleration will be present the inertia force. So, we apply the Newton’s Second Law to this system to find the equation of motion of this system.

So, Newton’s Law we apply m x double dot equal to sigma F. So, m x double dot equal to some summation of the forces. So, here m x double dot, so, forces here is f t, that is we represent in the same direction as x double dot. So, it is positive minus k x, k x is opposite, so, minus k x and then minus c x dot.

Now, this equation we can simplify, m x double dot plus c x dot plus k x equal to f t. So, here, this equation is m d square x by d t square plus c d x by d t plus k x equal to f t. So, we see that the equation of motion of this system is a second order Differential Equation.

So, this is Differential Equation of second order. Now, we have to find the transfer function for this system. So, we have to; to find the transfer function we know what? We have to do we have to take the Laplace Transform of both sides of this equation. So, if we take the Laplace Transform of both sides, what will happen so, m d square x by d t square plus c d x by d t plus k x equal to f t.

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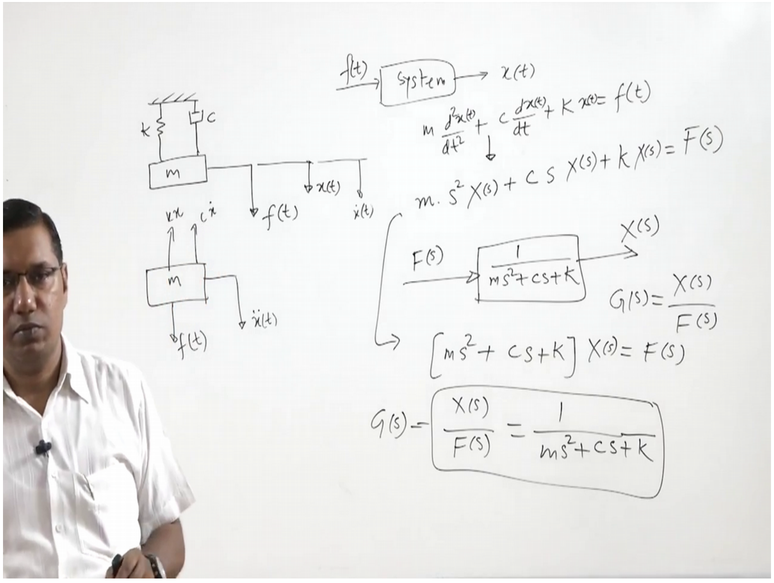

So, this is the our equation Differential Equation. Now, let us take the Laplace Transform. So, here m, so, this is a second order Differential Equation. So, we remember that we take the Laplace Transform. So, s power n. So, S power 2 s square and f s. So, this is x, so x s plus c, d x by d t so, this is one power Differential Equation.

So, s into x s plus k and this is x. So, this is x s and that is equal to F s. Now, we should understand that here the d square x, x is function of t. Here, I have rewritten already, d x is function of time. So, we can write here also like x t. So, we are coming from time domain to complex frequency domain or s domain for this system and. So, this is our system. So, will be the in the transfer function. So, we have f t. So, here is F s and this is the output here is X s and what is here is G s, let us say this is the transfer function.

So, we have to find G s. So, G s is output upon input. And here already we have done initial conditions we have taken 0. So, G s equal to X s upon F s. So, we have to find from here X s upon F s. So, we can write m s square plus c s plus k x s equal to f s. So, X s upon F s equal to 1 upon m s square plus c s plus k and this is equal to G s.

So, we see that this is the Transfer Function. And we can write this in this block; inside the block, that is 1 upon m s square plus c s plus k. Now, from this Transfer Function, we need one we need a discussion on this Transfer Function. We can see that the highest power in s is 2. So, from Transfer Function digit it is clear that this is a second order system. And same is clear from here that the power of the differential equation that is second order.

The second thing I told in the last lecture that Transfer Function is a property of a system. It does not depend on input conditions. So, property of the system means what? It contains the, it is made of the parameters of the system. So, what are those parameter? What are the parameter of the system? They are m, k and c. Because the system physical system is represented in terms of it is basic elements Mass, Stiffness and Damping. And therefore, you can see here, this expression contains m, c and k and the other is no any other parameter of input nothing.

So, therefore, this Transfer Function representing the system because it is made of it contains the parameters of the system. If we change these parameters, the Transfer Function will change. So, it means the system will be changed. So, therefore, Transfer Function gives us the information about the system. So, now, this was a Translational Mechanical System of the first order.

Now, we go that is a single degree of freedom system not the first order, but it is second order system, but single degree of freedom system, only one degree of freedom. Now, we go to higher degree of freedoms like 2 degree of freedoms for the Translational Systems.

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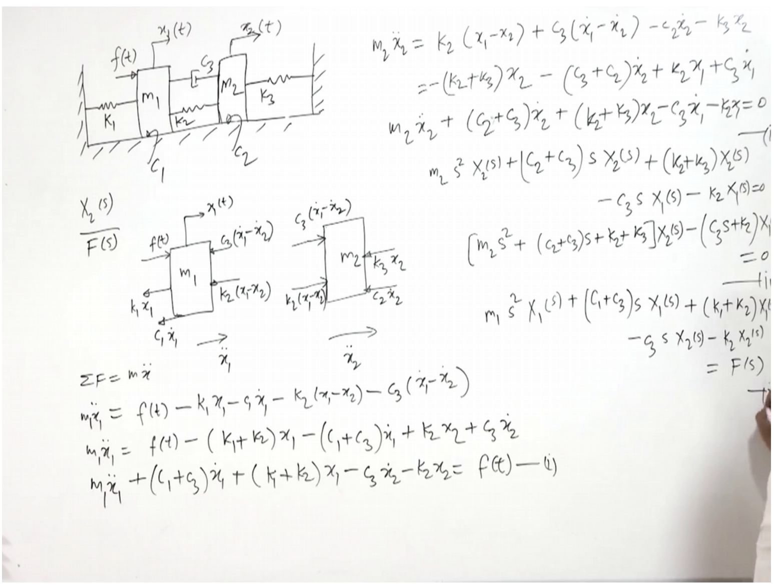

. So, now, we can see another mechanical system and this is the second order system; why? Because we have 2 masses and each mass has one degree of translation.

So, there is 2 degrees of freedom for this complete system. And the mass and mass is corrected from here this Spring, then between the 2 masses there is a Spring and Damper and there is this side one Spring. Here, k 1, k 2, k 3 the stiffness of these springs and the Damper has a damping viscous, damping coefficient c 3. There is friction between these mass and the surface when the mass moves and this friction is represented as equivalent viscous damping coefficient c 1 and c 2 for mass m 2.

So, this is mass m 1 and this is mass m 2. So, now, we have to find the Transfer Function for this system. And you can see here, we have applied the input force f t only under, there is only one force input applied that is on the first mass. We can also apply some another force on the second, but that will be then multiple input system. So, now, we have to find the Transfer Function for this system and we are more interested to find x 2 s by f s.

So, the Transfer Function of this x 2 s upon f s. So, let us write that first a differential equation. So, if we write the differential equation. So, we make the free body diagram bar for both the systems. So, here m 1 and here is m 2. So, now, let us say this is x 1 t and here is a force f t applied on this system. So, when we this mass moves in this direction, there will be a spring force up working here, that is k 1 x 1 and there will be a damping force working here, that is c 1 x 1 dot, here this mass is moving x 2 this is x 1. So, the relative movement of these mass in this direction is x 1minus x 2 and so, this is a spring will be compressed by the amount x 1 minus x 2.

So, it will apply a force in this direction that will be k 2 x 1 minus x 2. And similarly, a Damper will apply a force c 3 x 1 dot minus x 2 dot. So, and this mass is moving with x less than x 1dot, double dot. Now, if we apply the forces on the second mass, we can see this mass is moving x 2 here, this is compressing, this a spring the spring will apply a force opposite to this movement that is k 3 x 2.

There is the damping force here and this is moving this direction. So, this will apply a force in this direction that is c 2 x 2 dot. Now, here these forces will also act here. So, these forces will act here. So, these forces will act c 3 x 1 dot minus x 2 dot and here k 2 x 1 minus x 2. And this is moving as x 2 double dot.

Now, we apply the Newton's Second Law, sigma F equal to m x double dot for this mass. So, m x 1 double dot equal to the forces. So, the forces in this direction is f t ; others are in a opposite direction. So, f t minus k 1 x 1 minus c 1 x 1 dot minus k 2 x 1 minus x 2 minus c 3 x 1 dot minus x 2 dot. So, we can write. So, we collect the terms like x 1 term, so, minus k 1. So, this is x 1 and this is minus k 2 x 1. So, k 1 plus k 2 x 1 and this is minus c 1 x 1 dot and here minus c 3 x 1 dot.

So, this is minus c 1 plus c 3 x 1 dot and this is plus k 2 into x 2 and this is plus c 3 into x 2 dot. So, we can write this equation m x 1 double dot plus c 1 plus c 3 x 1 dot plus k 1 plus k 2 x 1 minus c 3 x 2 dot minus k 2 x 2 and that is equal to f t.

So, this is the equation of motion for the first mass and this is again the differential equation of second order. Now, the second mass, we write here. So, m 2 x 2 double dot equal to minus. So, in this direction, we have these 2 forces k 2 x 1 minus x 2 plus c 3 x 1 dot minus x 2 dot and minus c 2 x 2 dot minus k 3 x 2.

So, we can simplify this. So, here is we collect x 2. So, k 2 this is minus x 2 and this is. So, k 2 minus k 2 plus k 3 x 2 and here is x 2 dot and this is x 2 dot. So, minus c 3 plus c 2 x 2 dot, plus k 2 x 1 plus c 3 x 1 dot. So, we can write m 2 x 2 double dot plus c 3 c 2 plus c 3 x 2 dot plus k 2 plus k 3 x 2 plus c 3 x 1 dot plus k 2 x 1 equal to 0. This is the differential equation for the second mass course. Here, it will be c 3 x 1 dot, this will be minus sign and here k 2 x 1 this will be minus sign because this will go to the other side.

Now, we can take the Laplace Transform of these both equations. So, if we take this. So, this is m 2 s square x 2 s plus c 2 plus c 3 x 2. Here, we have the first order. So, s x 2 s plus k 2 plus k 3 x 2 s minus c 3 s x 1 s minus k 2 x 1 s equal to 0.

So, here we can write m 2 s square plus c 2 plus c 3 s plus k 2 plus k 3 x 2 s minus c 3 s plus k 2 x 1 s. This is equal to 0. So, let us say this is equation number 3. Similarly, we can write this equation, equation number 1. We can write we take the Laplace Transform of this equation number 1 and we can write. So, this here it is m 1. So, we can write m 1 s square plus and this is x 1 s plus c 1 plus c 3 s x 1 s plus k 1 plus k 2 x 1 s minus c 3 s x 2 s minus k 2 x 2 s equal to F s.

So, this is equation number 4. So, now, we have this equation 3 and 4. We can I rewrite this equation 4 like we can write m 1 s square plus c 1 plus c 3 s plus k 1 plus k 2.

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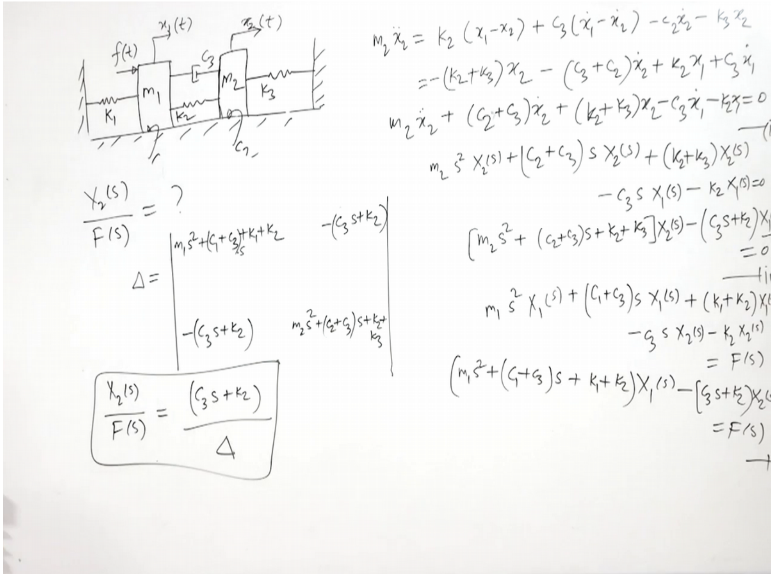

And this is x 1 s minus c 3 s plus k 2 x 2 s equal to f s. So, this is, let us say this is equation 4.

Now, there is Cramer's Rule. We can apply Cramer's Rule to find the solution of the, we to find X 2 s by F s because our objective was to find this Transfer Function. So, we can apply the Cramer's Rule.

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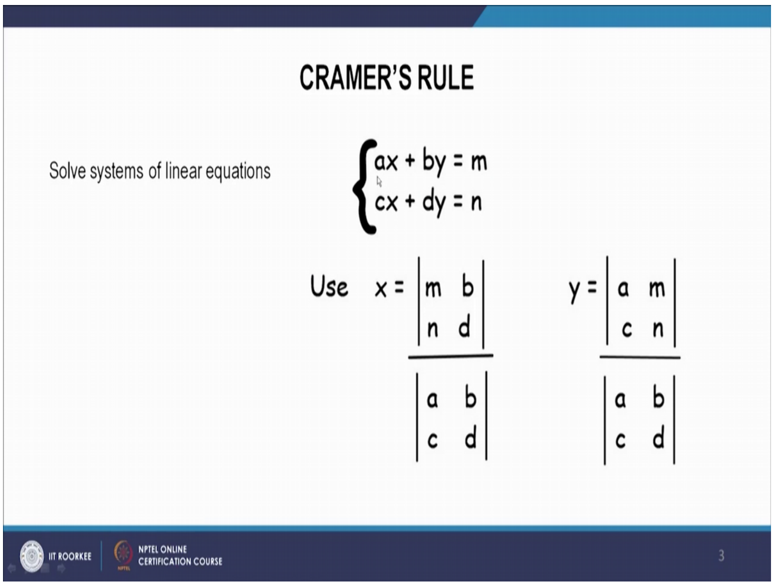

So, we can see here the Cramer's Rule tells that if we have the linear equations in the form a x plus b y equal to m and c x plus d y equal to n, we can find x and x is equal to.

So, x is the first variable. So, we replace this. So, we have a determinant a c b d here and we replace a c with m and n. So, m n b d this determinant and y equal to we replace this b and d with m n. So, this term.

So, now, we have the similar equation here, we have here x 1. So, a x 1 plus b y 1 x 1 plus b x 2; here c x 1 plus d x 2 and these. So, this is the similar equation and we can find from this, if we say the determinant delta. So, determinant delta is equal to m 1 s square plus c 1 plus c 3 plus k 1 plus k 2 and minus c 3 s plus k 2 and here also minus c 3 s plus k 2 and this is m 2 s square plus c 2 plus c 3 s, here is s into s c 2 plus c 3 s plus k 2 plus k 3 plus k 2 plus k 3.

So, this is determine delta. So, if we want to find X 2 s upon F s, we have to we can find it X 2 s upon F s equal to c 3 s plus k 2 upon delta. So, here, this term, we can find from the Cramer's Rule from we can see, we can apply this Cramer's Rule and we can find this term and here is this delta. So, we can find this Transfer Function. So, again, these examples were taken from the books of Norman S Nise.

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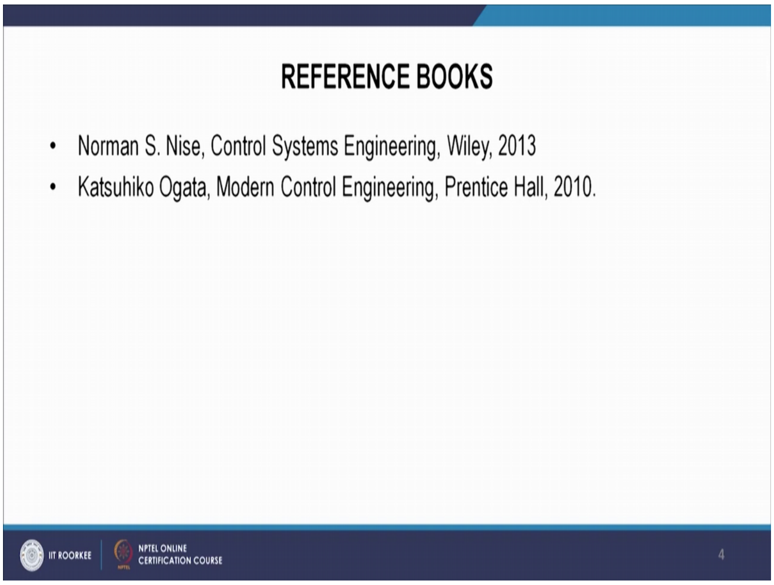

So, we learned here how to write the differential equations for a translational mechanical system and how to find a transfer how to find a transfer function solving these differential equations for say a single degree of freedom system as well as the 2 degree of freedom system.

So, I thank you for this lecture and see you in the next lecture.