Automatic Control
Dr. Anil Kumar
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture – 05
Laplace Transform and Transfer Function

So, welcome to the lecture on Automatic Control System. Today we will discuss in this lecture about Laplace Transform and Transfer Function. So, in previous lecture we saw that, in the design process of a control system we need to make block diagrams and those block diagrams contain transfer function and to calculate the transfer function we need to use the Laplace transform.

So, yesterday we saw that the block diagram.

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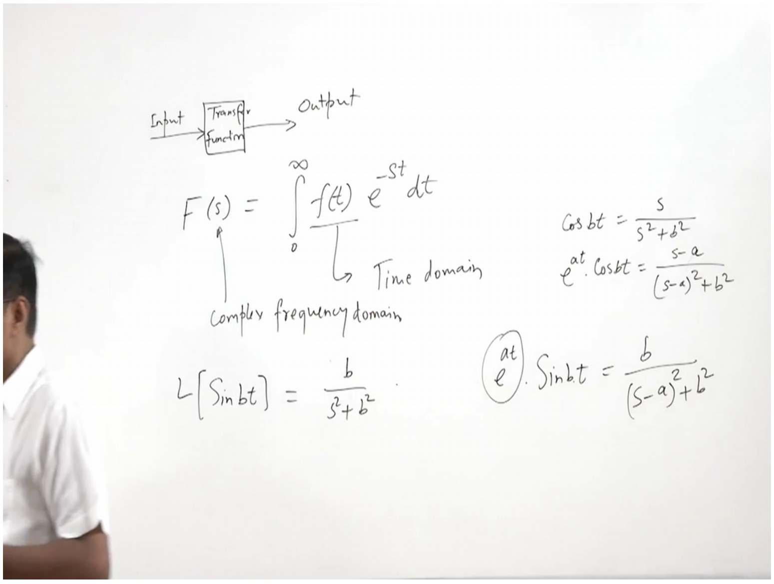

Can be represented here as; input, output and here is the transfer function. So, to find the transfer function we need to use the Laplace transform because Laplace transform is a technique that can be used to convert the differential equations or integral differential equations in time domain to algebraic equations in complex frequency domain.

So, when we have to solve a differential equation we can take the Laplace transform, we can shift our time domain to s domain, s is the complex frequency domain and then we can come back by taking inverse Laplace from s domain to time domain and we get the solution of that differential equation.

So, Laplace transform is defined. So, here F s is defined as integral 0, if there is just some function f t e power minus st dt. So, here this is a function f t that is the time domain function and we are getting here function F s that is the Laplace transform of f t and this is in complex frequency domain the inverse Laplace transform is also available.

So, if we have a function in ash domain we can take the inverse Laplace and we can get the time domain.

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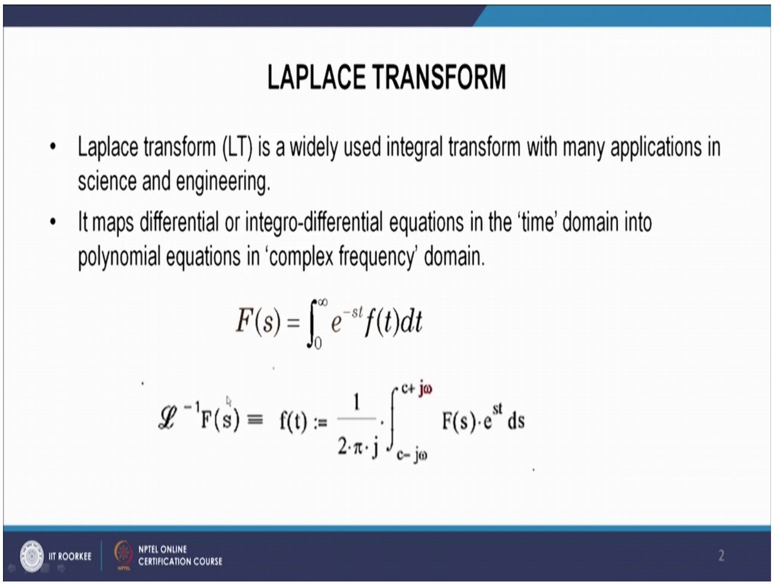

So, here we can see that the inverse Laplace transform a is it gives the f t and that is 1 by 2 pi j integrals c minus j omega c plus j omega F s e power st ds. So, here there are several functions in time domain and there are already available formula; that we must know because we can use them directly if there is some such function in differential equations.

So, for example, we have the Laplace transform of here a unity function 1 is.

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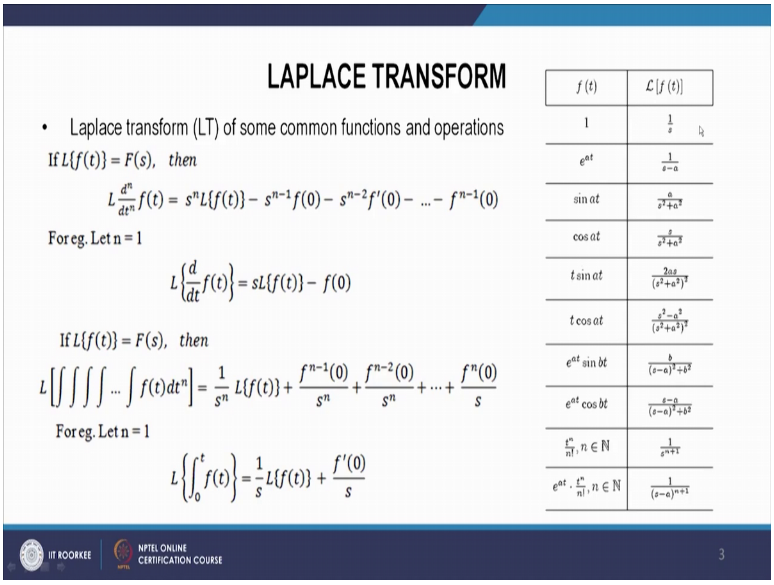

1 by s the Laplace transform of exponential function e over a t is 1 by s minus a and Laplace transform of sin function sin a t is a by s square plus a square, Laplace transform of cos a t is s upon s square plus a e square and here the Laplace transform of t sin a t is to a s upon s square plus a square whole square, Laplace transform of t cos a t is s square minus a square upon s square plus a square whole square.

Now, here we can see that the Laplace transform of sin b t into exponential function that is e power a t. So, we are getting b upon s minus a square plus b square. So, here we should note one point that if we have sin b t. So, the sin b t the Laplace transform of sin b t is b upon s square plus b e square.

However if we take the Laplace transform of sin b t into exponential a t. So, it will be b upon s minus a whole square plus b square. So, here whenever exponential function like a power at is multiplied simply we can the Laplace transform will be s minus the a s minus a of like this. So, similarly if we have Laplace transform of cos b t equal to s upon s square plus b square.

So, Laplace transform of e power a t into cos b t equal to s minus b sorry s minus a upon s minus a whole square plus b square. So, only we have to subtract this coefficient a to the s and we will get the Laplace transform of e power a t. So, Laplace transform of t power n of 1 factorial n is 1 by s n plus 1. Now, we come to the more use more applicable system like differential equations.

So, because differential equations are used widely and so, we should see what how we can take the Laplace transform of differential equations. So, here we can see that the Laplace transform of f t where f t is a differential equation f t is a function and so, Laplace transform of f t is F s then Laplace transform of the nth derivative of f t that is s n into Laplace transform of f t minus s n minus 1 f 0 minus s n minus 2 f dash 0 minus f n minus 1 0.

So, this is the nth derivative of f t and we are taking the Laplace transform so, for example, if we want the first derivative like L d by dt f t so, it is equal to so, any each one so, it is s and L f t minus f 0. So, L f t is F s so, we can write s F s minus f 0.

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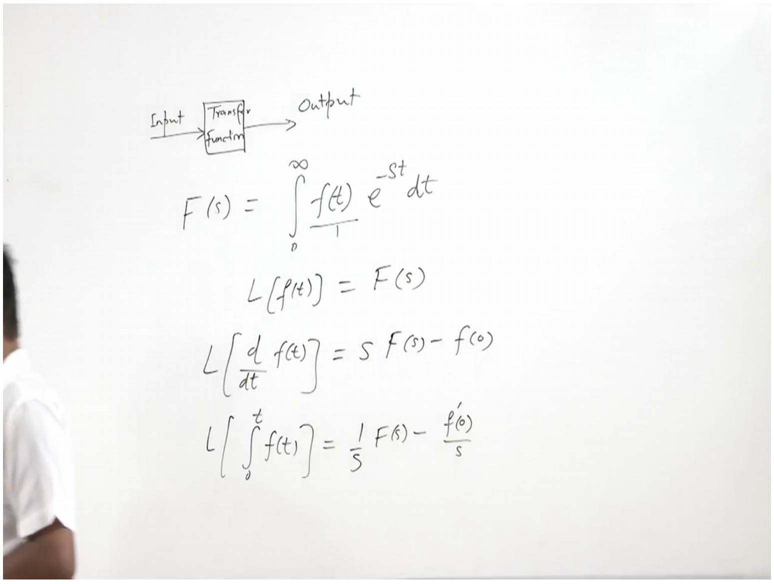

So, if L f t equal to F s so, L d by dt f t so, this is the first derivative of this f t that is equal to s, f s minus f 0.

So, this is the Laplace transform of the first order differential equation and similarly we can find the Laplace transform of the second order or higher order differential equations. Similarly, we have the Laplace transform for the integral equations so, here we can see the Laplace transform of nth integral of f t. So, we can see 1 by s and Laplace transform of f t plus f n minus 1 0 by s n plus f n minus 2 0 1 s n.

So, here we are seeing this for each function at the initial conditions are at t equal to 0 plus f n 0 by s so, these are the initial conditions at time t equal to 0. So, if we have n equal to 1. So, the first derivative we take the first sorry first integral so, 0 to t f t so, this is the first integral of f t. So, here we are getting 1 by s L f t plus f dash 0 by s so, L integral f t 0 to t.

So, it is 1 by s F s minus f dash 0 by s. So, these we see that we can find the Laplace transform of differential and integral equations because any system can be represented by differential equations; when we model mathematical models are given in terms of differential equations. And we can take the Laplace transform of these differential equations and we can enter from time domain to the s domain complex frequency domain and then these equations from differential equations they convert to algebraic equation and we can solve this algebraic equations we can factorize, we can simplify and then we can take inverse Laplace and we can come back to the time domain.

Now, as I told that the objective of the Laplace transform is to use in the to find the transfer function so, now we will discuss what is transfer function. So, we represented a block diagram assist we represented a system or a sub system through a block diagram a block diagram is.

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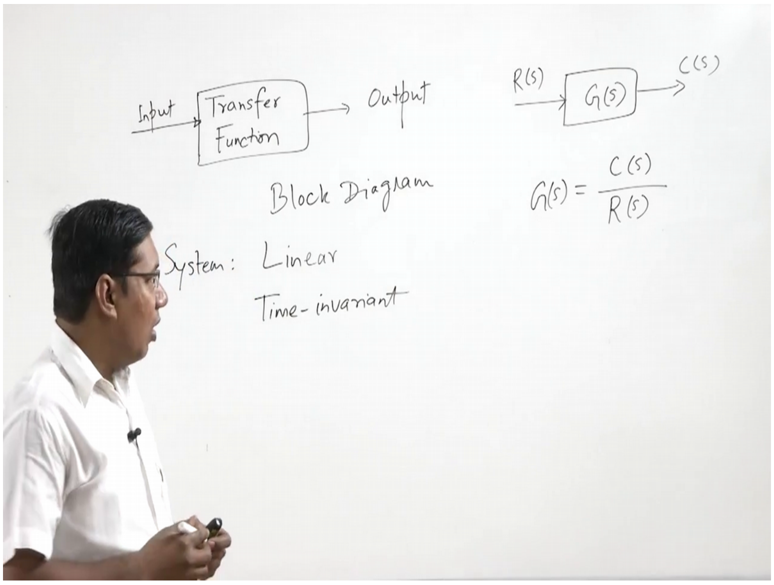

A block that takes the input so, it takes some input and it gives some output, because every system in physical any physical system that comprises with many subsystems and we give some specific input to the system and that input it passed to the subsystems. So, each subsystem has some input and then output and that output works as input to the next system.

So, any system we can represent our subsystem with Input Output relationship this is block diagram and here inside this is the transfer function. So, these transfer functions are they characterize the input output relationship of any such system. So, Input Output relationship is characterized through the transfer function and these transfer function are valid only for linear system, time invariant system.

So, the system should be linear and time invariant. So, the differential equations that present the system should be linear and that should not vary with time the parameters should not vary with time of that system. So, the transfer function we can define transfer function often is defined for a linear time invariant system and it is defined as the ratio of Laplace transform of the output to the Laplace transform of the input under the assumptions that all initial conditions are 0.

So, suppose I have a system here and our R s is Laplace transform of input and C s is Laplace transform of output. So, the transfer function G s is Laplace transform of output upon Laplace transform of input. Now we saw that when we do the Laplace transform, there are several terms that contain the values at initial conditions like time t equal to 0. So, those conditions we have to assume 0 when we perform the Laplace transform of these functions.

So, let us take if we can represent a differential equation of nth order.

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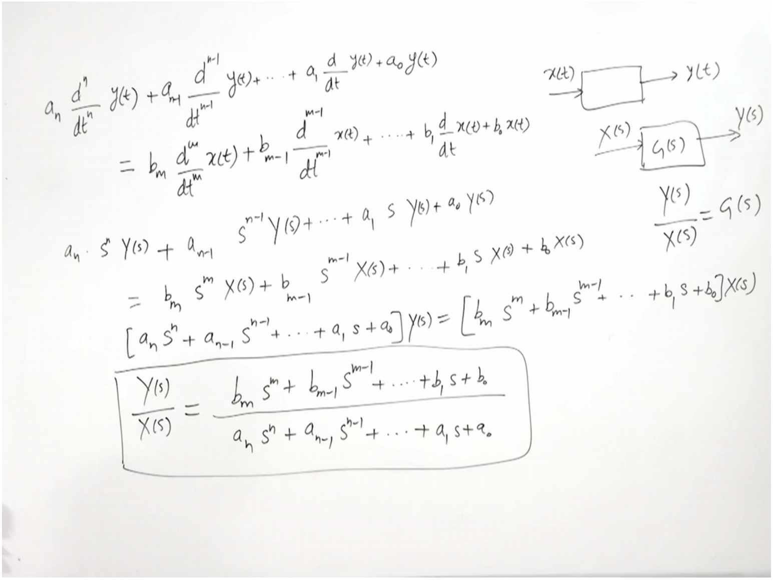

So, here I am writing the differential equation of general differential equation and in this differential equation we have y t, y t is the output and x t is the input. So, this is the so, it is this case so, we have in time domain this is input and here is the output and here is some function.

Now, we know that if we want to find the transfer function we need X s and Y s because we need to go in s domain. So, we can take the Laplace transform of this equation and we should try to find Y s by X s.

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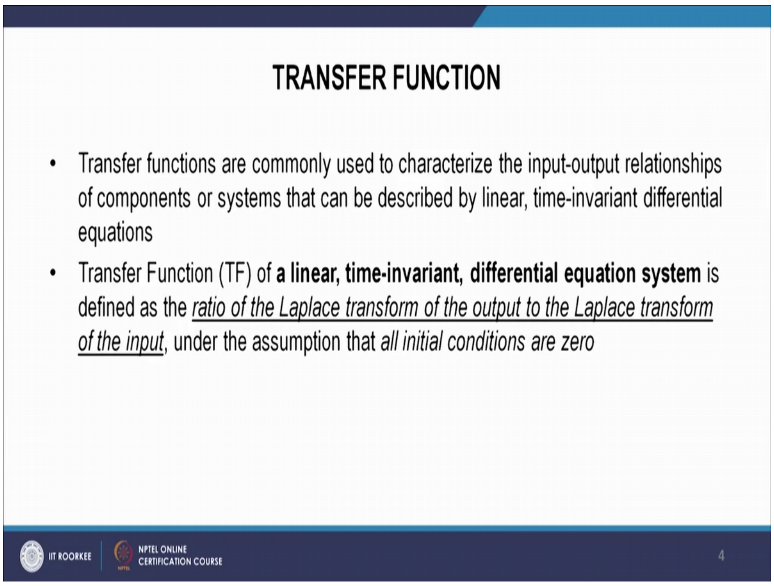

Because that will give you the transfer function for this general equation differential equation. So, we take the Laplace transform so, we know that the Laplace transform of nth order differential equation is s power n F s plus some terms that contain initial condition we can again see the formula, we can recall the formula, we can see here the d by dt n dn by dt n for f t equal to s power n and this is the Laplace transform of f t and minus these terms s n minus L f 0 minus s n minus 2 f dash 0 minus f n minus 1 0. So, we see that it accept these function s n L f t all the other functions they take the initial conditions their values is defined at the initial condition.

And in the definition of the transfer function we already said that the initial conditions should be 0. So, we all these terms except the first term will be 0. So, the differential equations form of dn by dt and f t equal to s n and L f t and L f t is F s. So, we have s n into F s so, here we can write a n into s and F s. So, here we say why s because here our function is y t. So, we say Y s.

Now, we come to the next one so, this is our differential term with nth, n minus 1th order. So, here we will have s n minus 1 Y s plus a1 then here the first order. So, s Y s plus a 0 and this is y t. So, it is Laplace transform is Y s and this is equal to similarly this side the input side, the differential equation to the input side is b m here s power m X s plus b m minus 1 s m minus 1 X s plus b1 s X s plus b 0 X s.

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Now, we can take it y is outside so, we will have Y s and here a n s n plus a n minus 1 s n minus 1 plus a1 s plus a 0. So, here we have b m s m plus b m minus 1 s m minus 1 plus plus b 1 s plus b 0 and X s. So, now we have to find the Laplace transform G s and G s is Y s by X s. So, here Y s by X s is equal to b m s m plus b m minus 1 s m minus 1 plus b 1 s plus b 0 upon this term. So, a n s n plus a n minus 1 s n minus 1 plus a 1 s plus a 0.

So, this is the transfer function for this general differential equations so, we can see that this in this differential equation output part is nth order as well as input is also nth order. So, we learn how to find the transfer function from nth order differential equation and if we have some equation of finite order let us say second order differential equation, third order differential equation.

We can use the similar process and we can find the differential of that differential equation we can find the transfer function. So, for example, we take one example so, let us we have a funk equation like.

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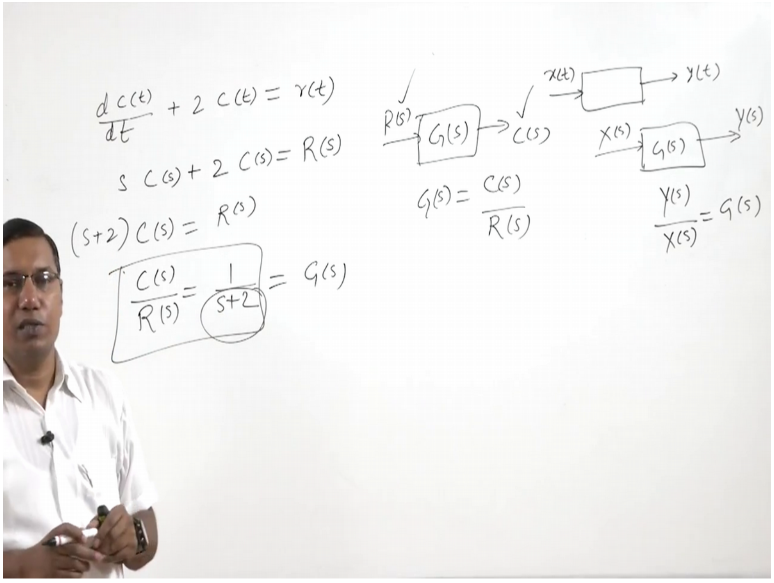

d c t by dt plus 2 c t equal to r t. So, this is the first order differential equation and we have to find the Laplace transform. So, from this equation it is clear that this is the input and this is the output.

So, we take the Laplace transform of both sides with assuming all in initial conditions 0. So, here it is s into c s because this is the first order differential equations. So, s power n so, s power 1 that is s and c t we have c s plus 2 c s.

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equal to r s, here capital letters we represent the Laplace transform function and here in a small letters we at present usually in time domain. So, from here we have to find so, it is like we have input r s and here output c s we have to find G s. So, G s is equal to c s output by input. So, here we take c s s plus 2 equal to r s. So, here is c s by r s equal to 1 by s plus 2 and this is the transfer function G s of the system.

So, that is how we can find the transfer function of differential equation. Now there is some properties of transfer function you see here is input, here is output and this is the block you know that this block represents the system and. So, this block is made of system parameters, it represents the system property therefore, the transfer function is a property of the system and it is independent of the input it does not depend on the input this output will depend on the input as well as the system property that is transfer function.

But this transfer function is the system itself it is own property, now second point is that seeing a transfer function we cannot tell the physical structure of the system because several different physical systems may have the similar transfer function and second point is that the denominator of the transfer function will tell us the order of the system.

So, the highest power of s in the denominator will tell the power of the whether it is a la like here s power 1. So, this is the first order system and this we can also know from the this differential equation that this differential equation is the first order differential equation. So, the system is of first order this also we can get from just looking at transfer function the power of s the maximum power of the highest power of s will decides the order of the system.

And the last thing we should note that the transfer function is defined only for.

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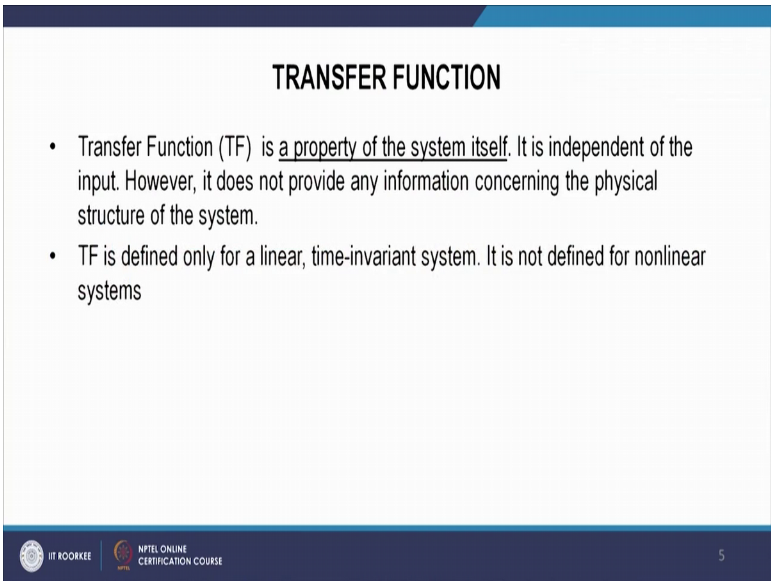

Linear time invariant system, it is not defined for non-linear system or systems that vary with time like we have missiles rockets. So, there is variation in time the mass of that.

So, the transfer constant is not defined for that so, I would like to stop here and so, some examples that we take we took from the reference books of Norman S Nise.

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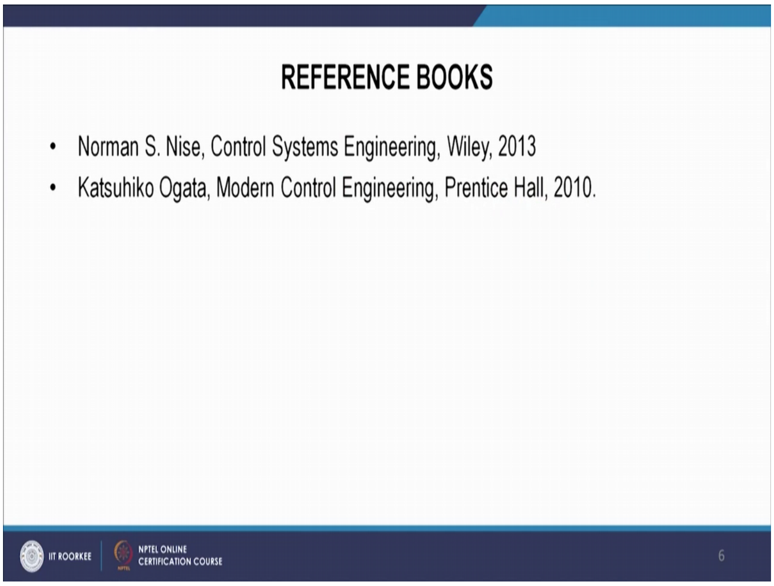

And K Ogata so, I would like to stop here and I.

Thank you for this lecture see you see you in the next lecture.