Automatic Control
Dr. Anil Kumar
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture - 40
State Space Method

So, welcome to the lecture on application of MATLAB in automatic control, in this lecture we will discuss about state space method how to use MATLAB to solve the problems related to state space method.

So, we have already discussed that a state space method is a time domain time domain method that is applicable to a system that is time varying non-linear systems. And we can write the state equations and output equations for a system to model any system.

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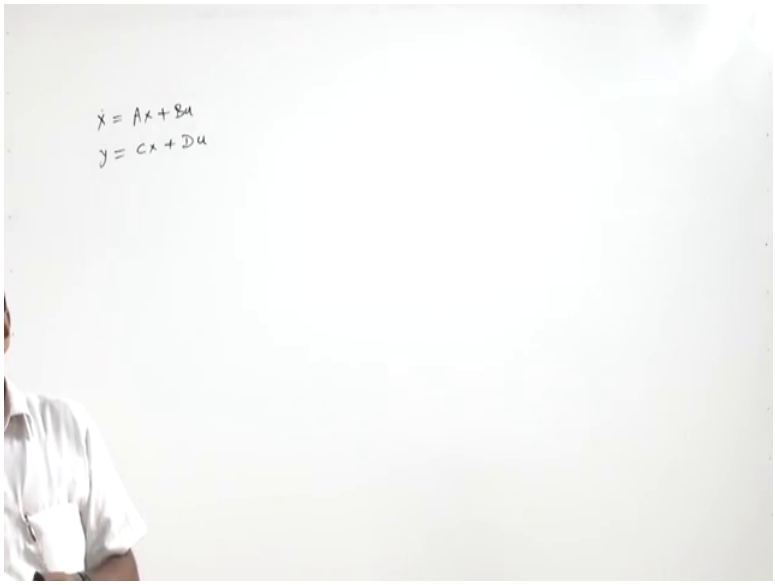

So, state equations we write like x dot equal to Ax plus Bu and y equal to Cx plus Du, here we know that different type of matrices A, B, C and D. We discussed in the lectures of week 7 about state space and how to transform from transfer function to state space, how to transform from state space to transfer function, then how to design a controller and how to determine the controllability of a system.

So, these aspects we will discuss today with numerical problems and how to solve these problems in MATLAB.

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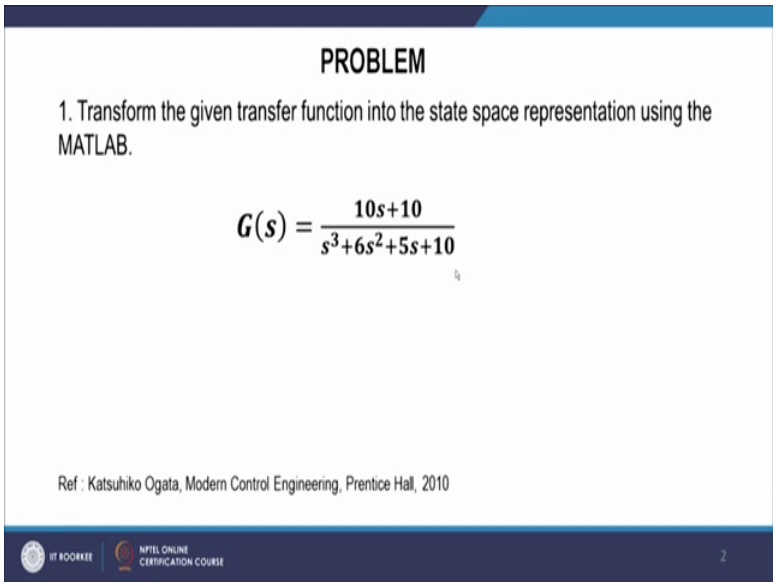

So, let us start here we have a problem here that transform the given transfer function into the state space representation using the MATLAB. So, we know that the transfer function a system is represented at as output upon input in the s domain.

So, that is given 10 s plus 10 by s cube plus 6 s square plus 5 s plus 10. So, we have this system 10 s plus 10 by s cube plus 6 s square plus 5 s plus 10. So, we will write the MATLAB code.

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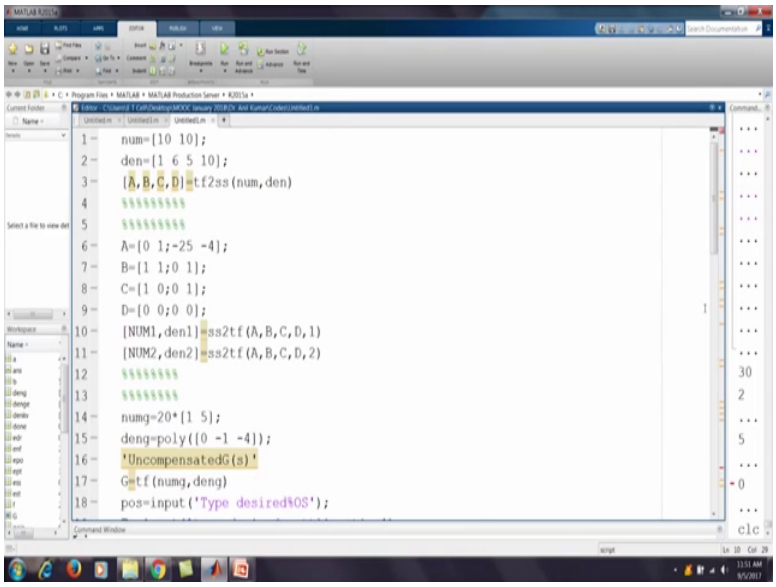

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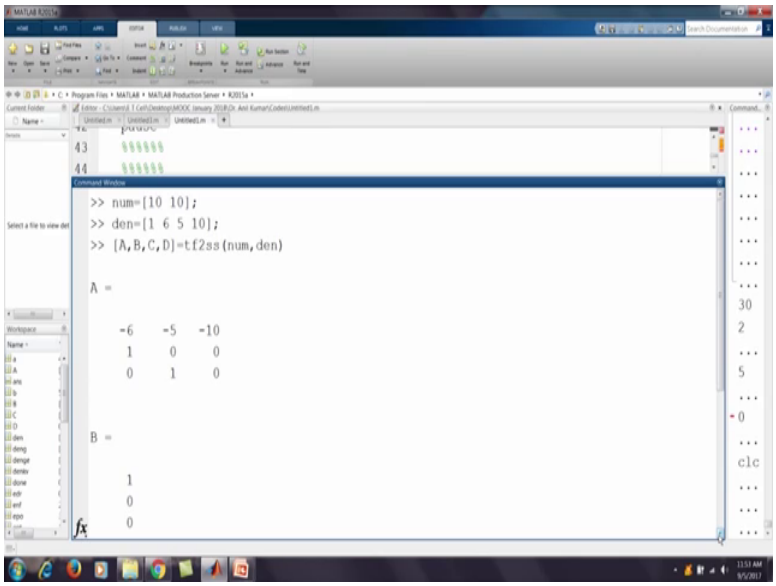

So, here so, this transfer function 10 s plus 10 so, we will define this numerator s; equal to so, we write the coefficients of s 10 and den 10 the constant term so, then denominator then equal to so, this is s cube plus 6 s square plus 5 s plus 10. So, the coefficients are 1, 6, 5 and 10 so, we will write 1, 6, 5 and 10 so, we define these 2 variables. Now, we want the A, B, C, D when we say the system in state space means we are interested in these 4 matrices A, B, C, D and so, A, B, C, D is tf2ss.

And then we have num and den so, so here we can see we first define the transfer function with the numerator and denominator and then here we use the command transfer function to state space.

So, tf2ss that is the command defined in MATLAB and we give the numerator of the transfer function and denominator of this transfer function and so, we will get these 4 matrices and once we get 4 matrices means we got the state space model x dot equal to Ax plus Bu and y equal to Cx plus Du so, we enter.

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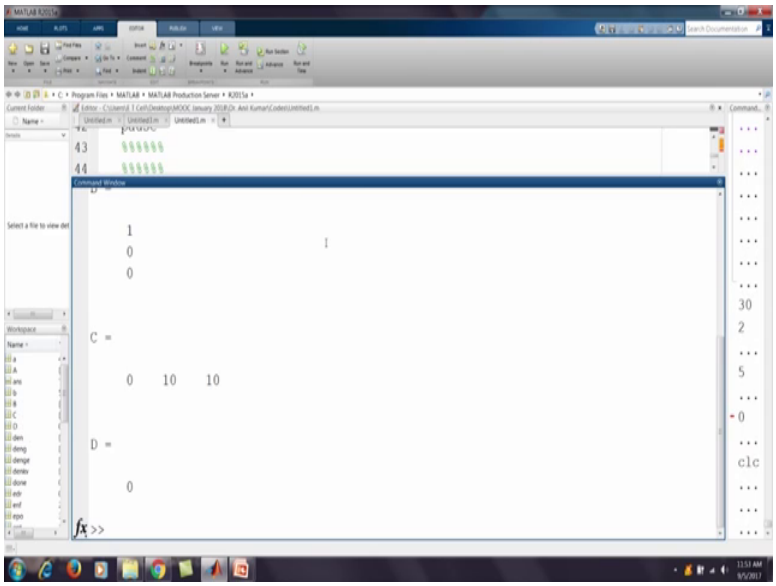

We got these matrices so, we got this matrix A minus 6 minus 5 minus 10, 1 0 0, 0 1 0. So, 3 by 3 matrix because the system is of third order s cube in the denominator; So, we got third order matrix B is 1 0 0, C is 0 10 10 and D is 0.

So, now we solve this problem 1, now we come to the problem next problem.

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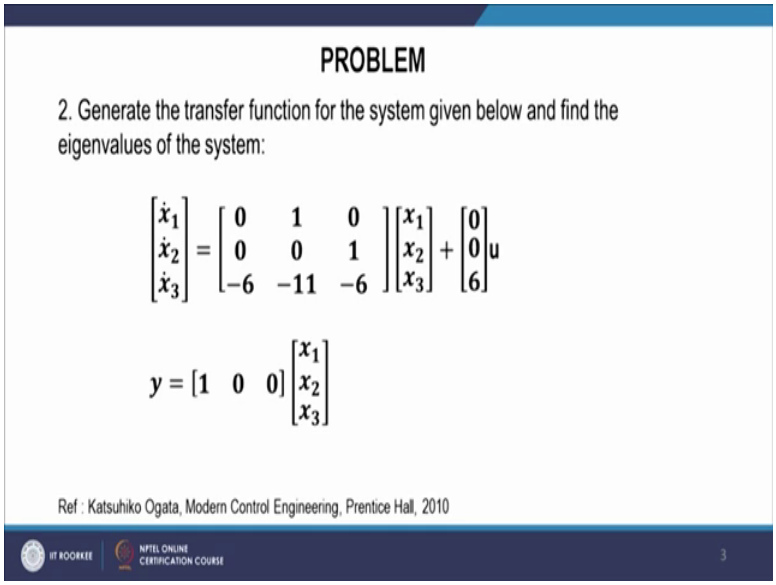

That is problem 2 so, here generate the transfer function for the system given below and find the Eigenvalues of the system. So, here we see this system is given in state space and we have to find the transfer function for the system. So, here A matrix, B matrix, C

matrix, we are given D matrix is 0 because it is not given. So, how to solve these in MATLAB? So, here we have to write this in MATLAB so, here we do clc.

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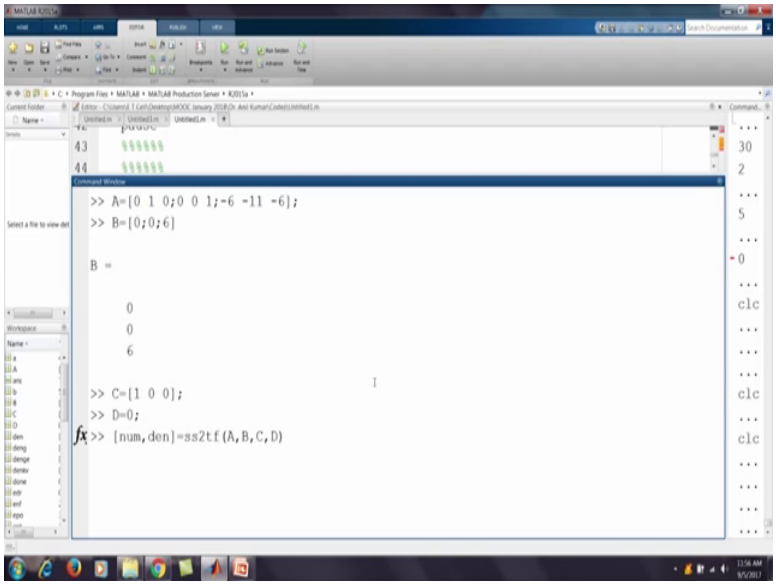

So, now, we define this matrix A equal to so, the first row that is 0 1 0, then second row 0 0 1, then third row minus 6 minus 11 and then minus 6.

So, this is the matrix A, matrix B is equal to so, 0 0 and 6 and then matrix C equal to 1 0 0 and matrix p equal to 0. So, we define this 4, then we use the another command that is state space to transfer function. So, we will get num, den variables of the transfer function that is equal to ss2tf and that is A, B, C and D so, we got num.

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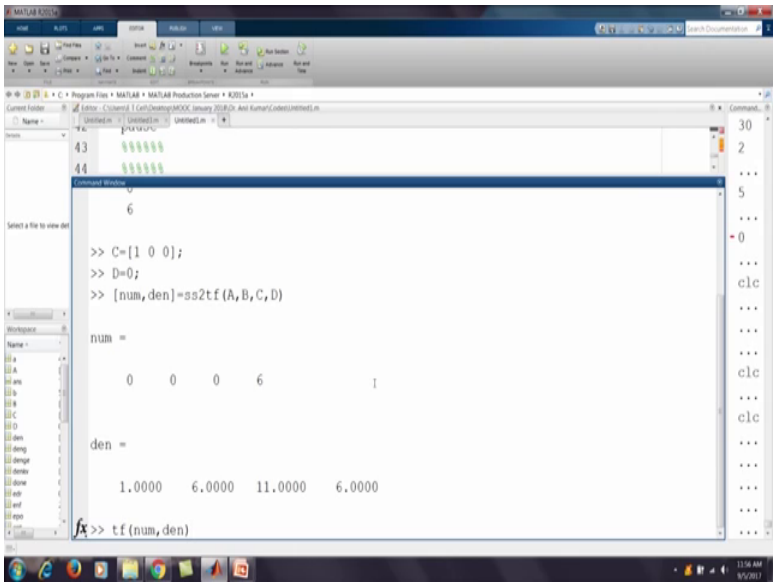

And den so, here we can see it is 6 and the this denominator coefficients are 1 6 11 6. So, we can also find the transfer function if we write tf num, den so num, den so, we can get the transfer function.

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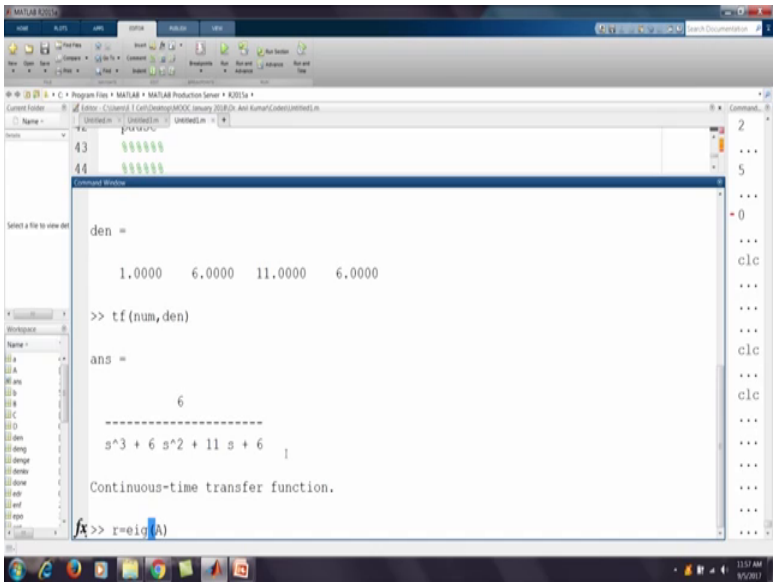

And this is 6 by s cube plus 6 s square plus 11 s plus 6, this is the transfer function so, we have converted the system from state space to transfer function. By using the ss2tf command and now we want the Eigenvalues. So, Eigenvalues of n state space is we we try to find the roots of the matrix A because, A is the system matrix and this shows the

represents the poles of the system. So, we write simply the r equal to eig, eig means we use Eigenvalues of system matrix A.

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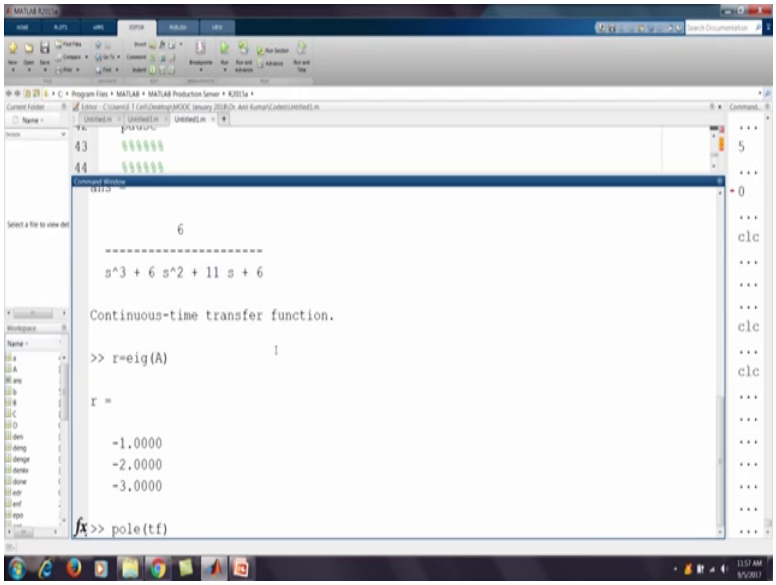

So, we get here the roots minus 1 minus 2 minus 3 and the same value we can because these are the poles of the system. Same value we should get also from the transfer function and so, if we give the command pole tf because this is a transfer function tf or sorry so, here we have pole tf num, den so, if we do pole tf num comma den.

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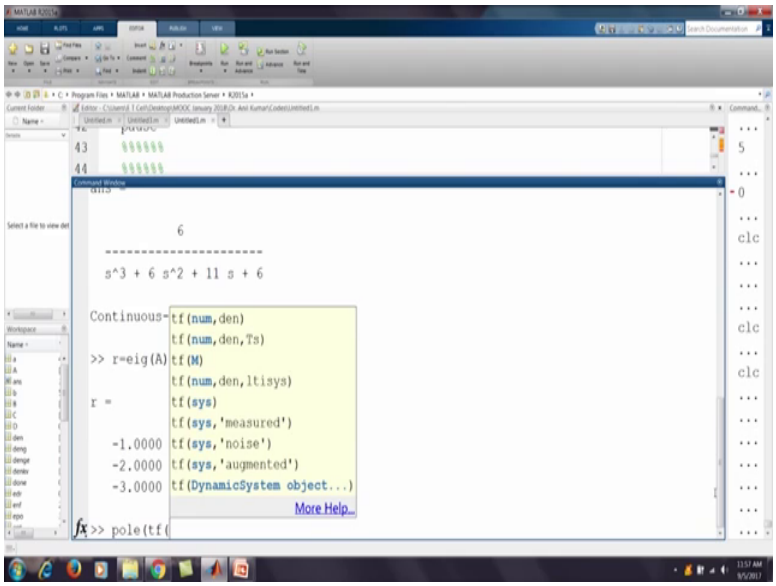

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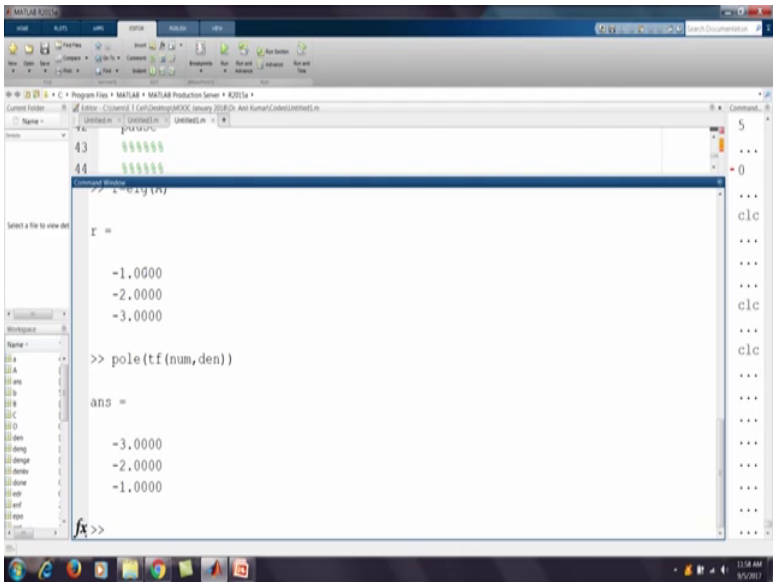

So, the transfer function is also another these this is the same system. So, we should also get the same pole so, here also we are getting minus 3 minus 2 minus 1 and here also minus 1 minus 2 minus 3. So, both the state space and transfer function models are giving the same poles.

So, now we come to the problem number 3.

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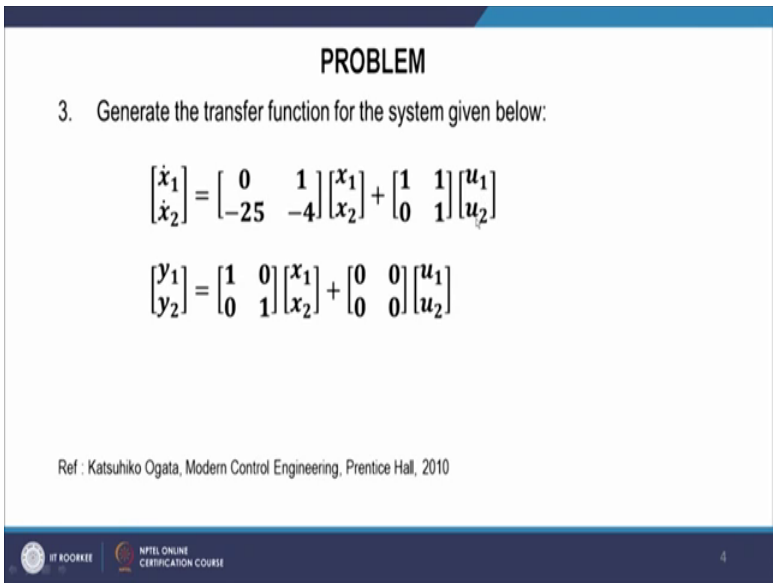

So, here generate the transfer function for the system given below. So, we have this state space model, but here we can see is the are the 2 inputs u1 and u2. Earlier we had here

only one input u, but now here we have 2 inputs u1, u2 so, we have to obtain the transfer function. Now, we know that when we have 2 inputs the transfer functions will be 4 because the output is y1 and y2.

So, one transfer function with respect so, 2 transfer function with respect to 1 input; So, y1 s by u1 s y1 s by y2 s by u1 s and then y1 s by u2 s and y2 s by u2 s. So, there will be 4 transfer functions so, let us do this.

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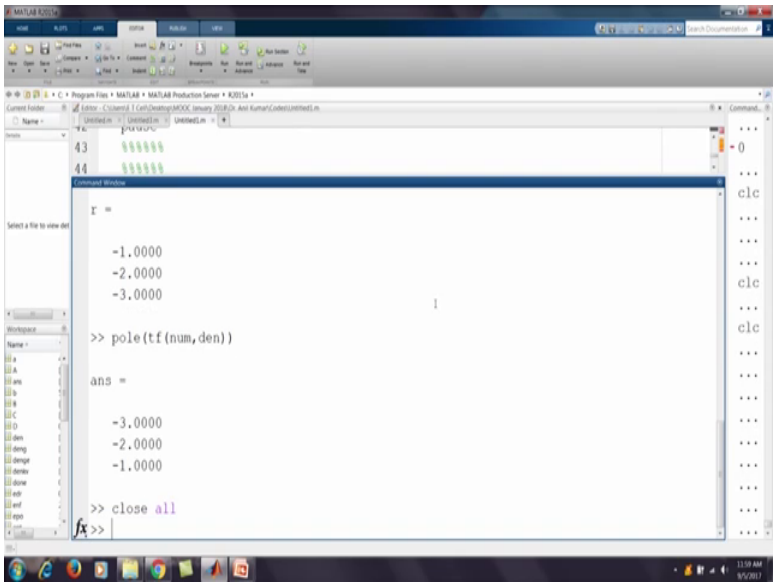

In MATLAB so, here close all clc.

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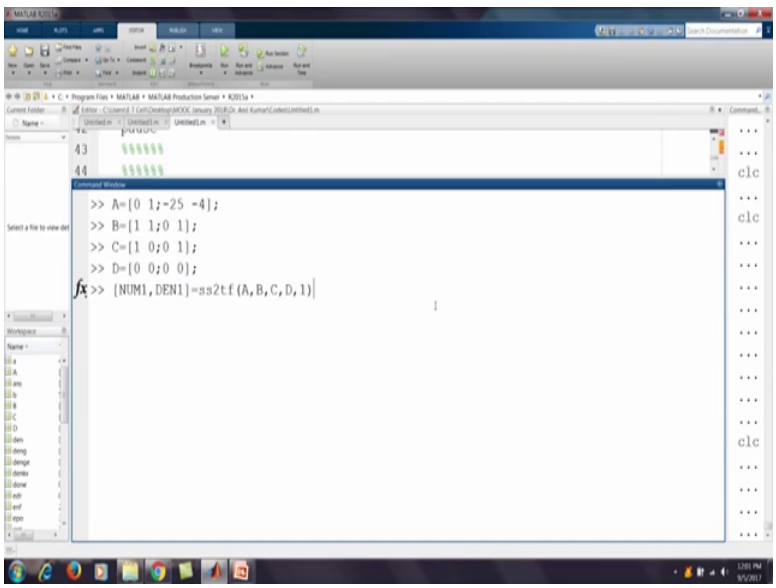

So, now we have to define this matrix A equal to 0 1 minus 25 minus 4 so, this is A matrix. Then matrix B we have 1 1 0 1, then matrix C 1 0 and 0 1 this is matrix C and matrix D D equal to 0 0 0 0. So, this is the matrices of the state space model we have defined. Now we want to get the transfer function so, let us say NUM1 comma DEN1 equal to ss2tf, here we have A B C D and 1, 1 means with respect to input 1.

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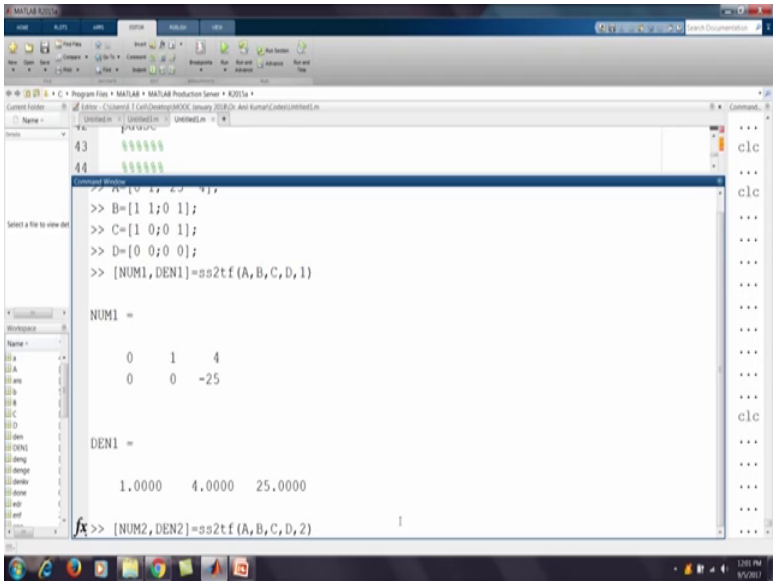

So, we are getting here you see we are getting these 2 numerators and 1 denominator. So, this is s plus 4 by, here s square plus 4 s plus 25 and this is minus 25 upon s square plus 4 s plus 25 so, these are the 2 transfer functions with respect to first input.

Now, we come to the second input so, we can write here NUM2 DEN2 and this is with respect to the second input. So, this is 2 and we will get here so, s plus 5 upon s square plus 4 s plus 25 and then s minus 25 upon s square plus 4 s plus 25.

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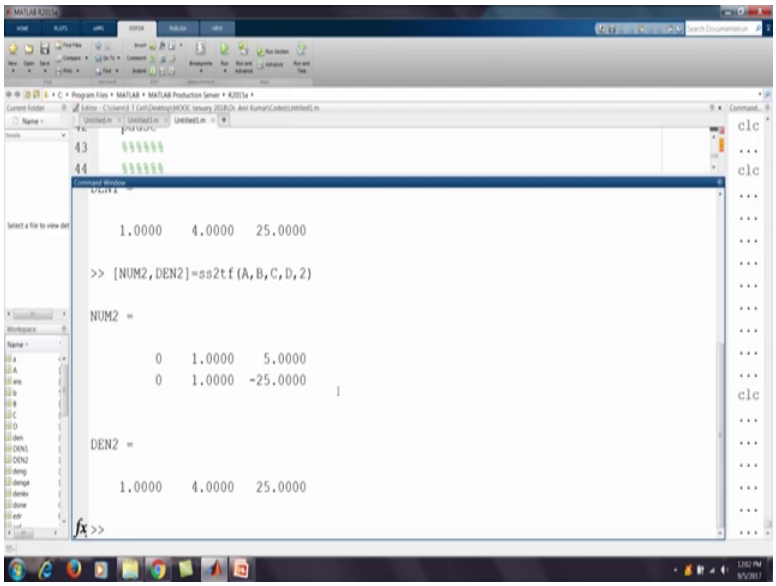

So, we can compare with respect to the earlier. So, we have 4 transfer functions, we because there were 2 inputs so, 2 transfer functions for each input. So, now, we come to the problem number 4.

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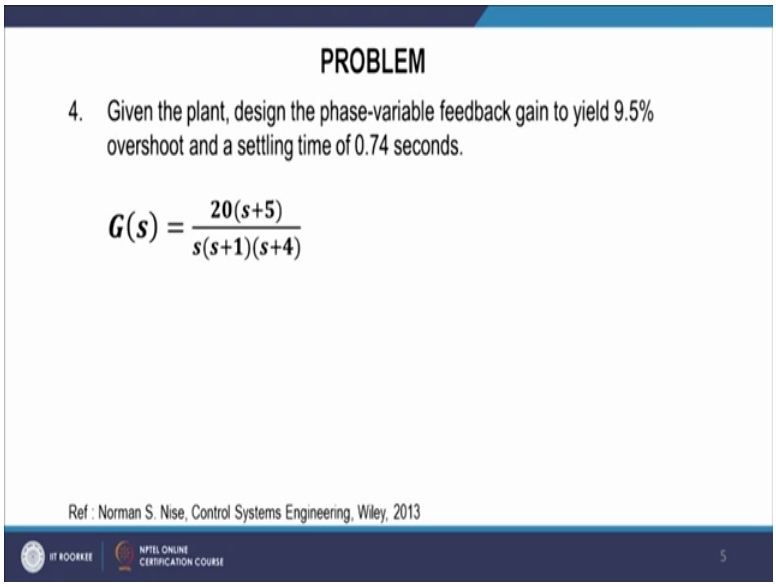

So, this problem is that we are given a plant design the phase variable feed by a gain to yield 9.5 percent overshoot and a settling time of 0.74 second. So, we have the transfer function Gs 20 s plus 5 upon s s plus 1 s plus 4. So, this problem in theory was discussed

already in the lecture in when we discussed state space in the 7th week, the last lecture that is the 7 lecture 7 5.

So, theory it we discussed and we discussed this problem. Now in MATLAB we have to solve this problem. So, we will here discuss about this problem. So, here we have 9.5 percent overshoot and settling time ts a 0.74 second and we have Gs is equal to 20 s plus 5 upon s s plus 1. So, we saw that in phase variable form we use the we designed feedback gains k1, k2, k3 so, that was able to change each parameter of the system to give this desired response.

So, now we go here.

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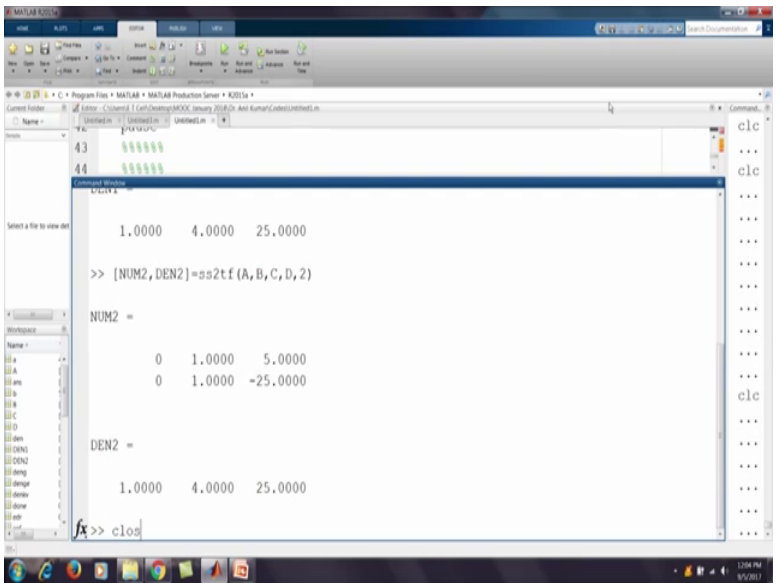

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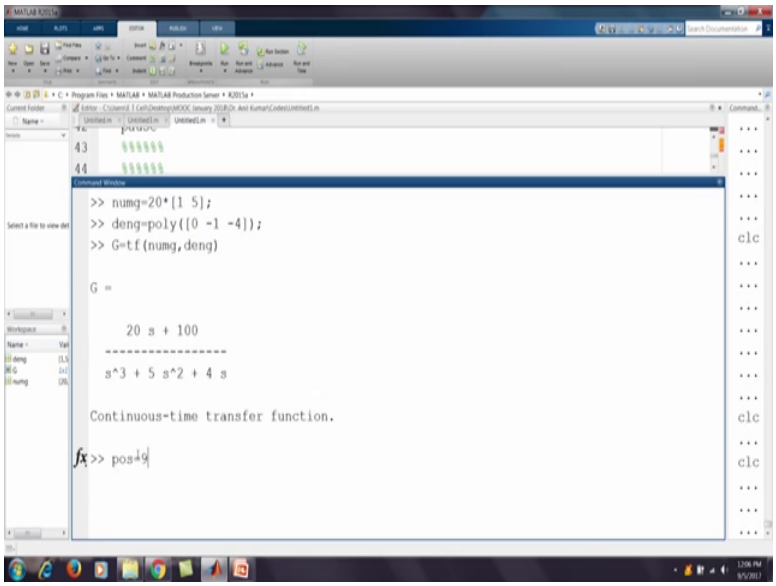

So, here first of all we will define the numerator so, numerator here equal to 20 into 1 5. So, this is the numerator because numerator is 20 s plus 5 so, 20 into s 1 means s plus 5 so, we define numerator. Now, denominator deng equal to so, it is a polynomial with here is the coefficient of polynomial are s that is 0 minus 1 minus 4 these are the roots. So, 0 and minus 1 and minus 4 so, this is the denominator. Now, we get the transfer function G equal to tf that is numg comma deng.

So, you see we got this transfer function 20 s plus 100 by s cube plus 5 s square plus 4 s. So, this is the transfer function now we say we give the percent overshoot pos that is equal to here we have 9.5 so, because this percent overshoot.

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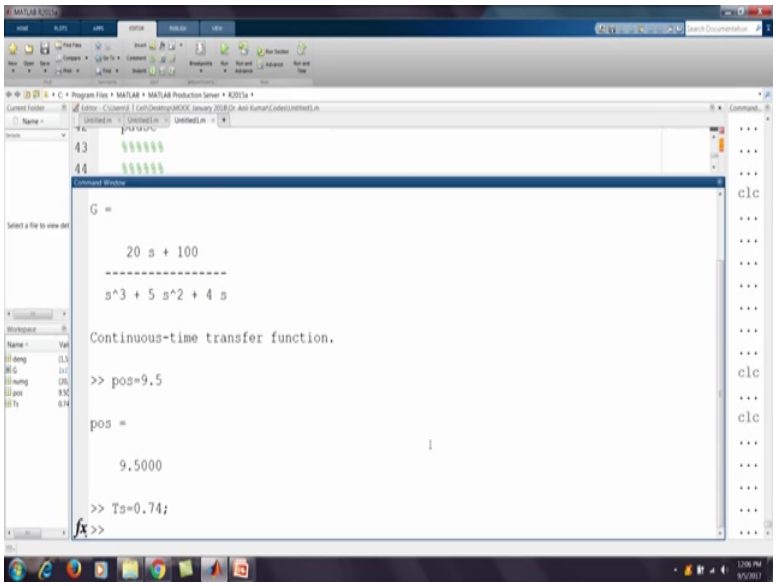

We are get giving 9.5 and settling time we want 0.74 so, give this settling time.

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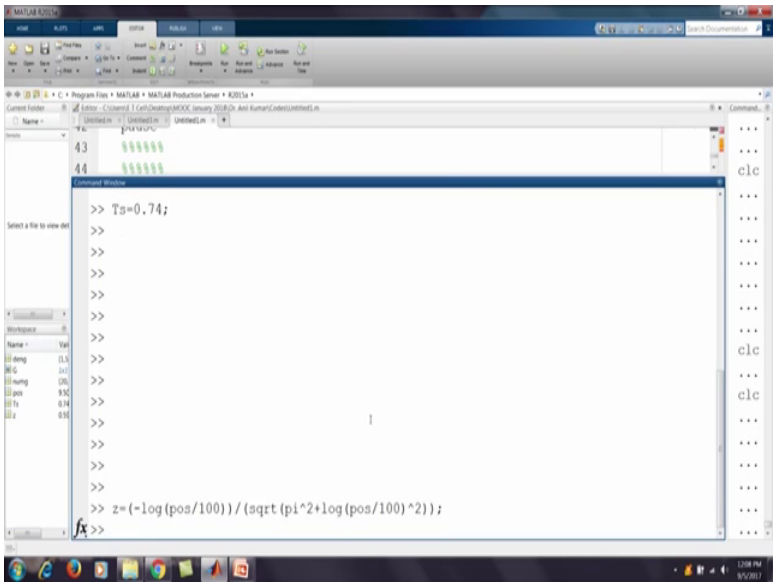

So, we give these values, now we want to calculate the damping so, damping equal to minus log pos by 100. So, percent overshoot by 100 by s square root.

So, here we use this formula minus ln percent overshoot by under root pi square plus ln percent overshoot by 100 square. So, we get the damping so, here if we want to know.

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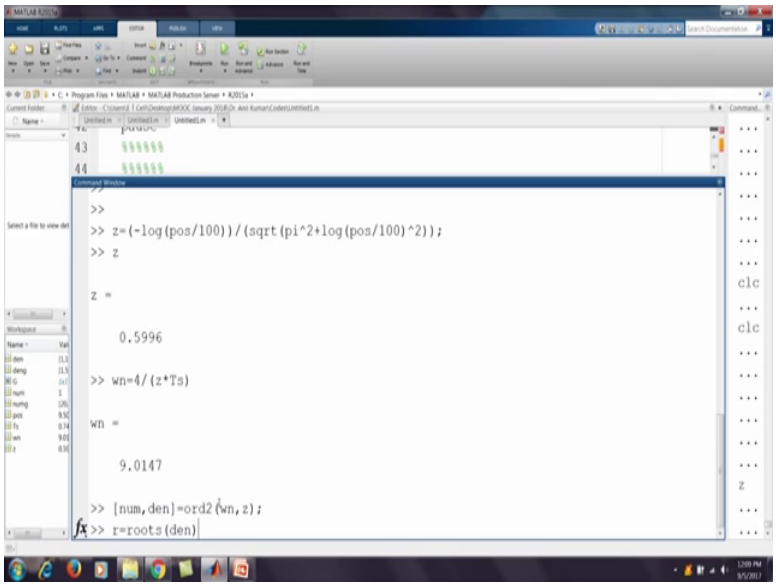

What damping we are getting about 0. 6. Now, omega n that is natural frequency we will get 4 by so, z into ts. So, we will get 4 by because natural frequency we can get by 4 by zeta into settling time. So, we get this natural frequency that is 9.0147.

Now, we are going to define a second order system that is omega n, z. So, with natural frequency this and damping this we are going to get a second order system; So, that is the desired pole location of the complex poles and r equal to roots of denominator. So, this will give the poles and we can see these are the poles.

So, these are the desired pole locations to obtain the given settling time and percent overshoot. Now we want to give the put the third pole so, because there is 10 at 5 so, we decided to put the third pole near to the 0. So, that the 0 can cancel pole can cancel the 0 so, there should be cancellation.

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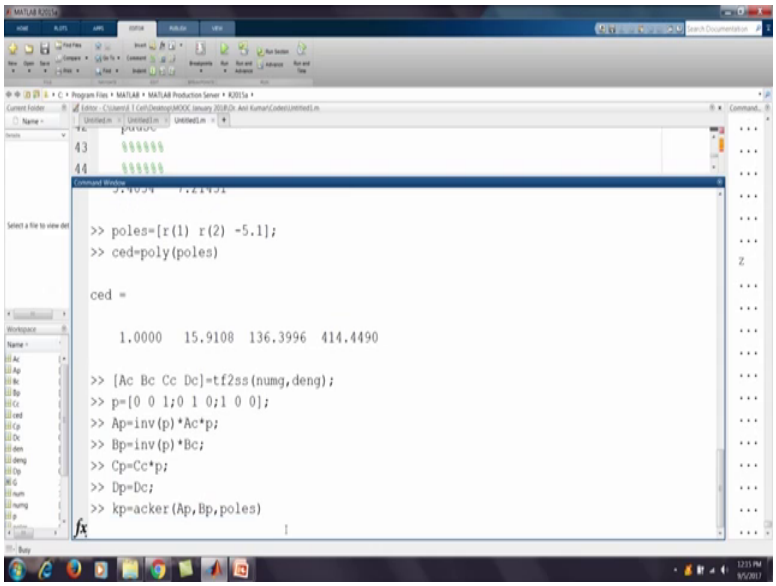

So, we put the third pole at 5.1 minus 5.1 so, we get poles that is poles equal to, here r1 so, r1 is this pole minus 5.4 plus 7.2 I then the second pole is r2 r 2 and then the third pole is minus 5.1 so, here minus 5.1. Now, we want the desired characteristic polynomials with these poles. So, ced equal to polly poles. So, this is the polynomial so, s power 4 plus this s cube plus this s square plus this value that is the polynomial and we can find the values of state space matrix.

So, we convert from transfer function to state space for this earlier system numg and deng. Now we define because what we are getting here this state space, we have to convert into the phase variable form. So, phase to transform from a state space to phase variable. So, that is this is canonical form and we have to convert into phase variable form. So, we need a transformation matrix so, here p equal to we define a transformation matrix 0 0 1, then 0 1 0, then 1 0 so, 0 so, this is the p the transformation matrix. Now, we will define the phase variable of this matrix.

So, here we converted from canonical form to phase variable form these matrices and now we will apply to find the feedback gain parameters k1, k2, k3 for this form so, kp equal to we use acker command for this. So, this command this is Ap Bp is this matrix and poles are these desired poles. So, in order to achieve these poles what will be the feedback gain parameters case? That should be applied in this Ap and Bp to change these to these poles.

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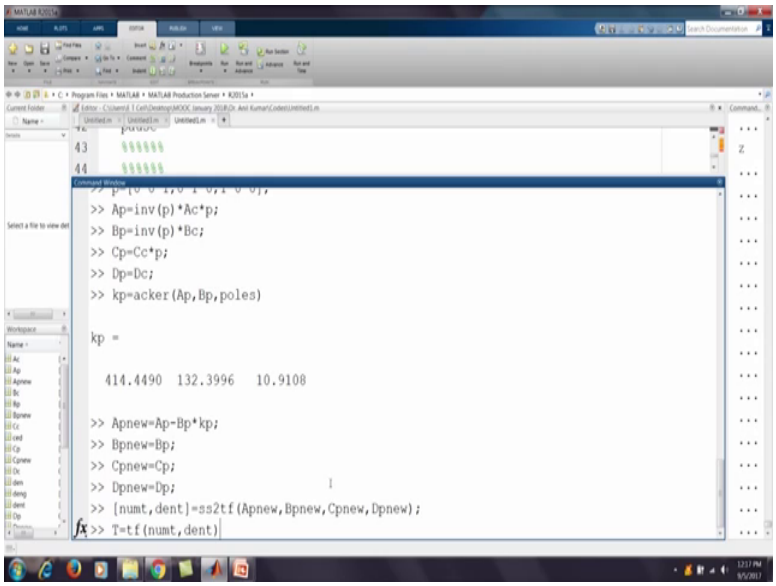

So, these are the gain parameters 414, 132 and 10.9.

So, now we have with these gain parameters we have the new is Ap minus Bp into kp. So, we know that the new A matrix is A minus pk for this feedback with this feedback gains and then we have. So, other matrices are the same so, Bp new equal to Bp, Cp new equal to Cp and Dp new equal to Dp. Now so, we want this to convert into transfer function so, numt comma dent equal to ss2tf Ap new comma Bp new comma Cp new comma Dp new.

So, these are the this and so, t equal to the transfer function tf with this numt and dent so, we can see the transfer function.

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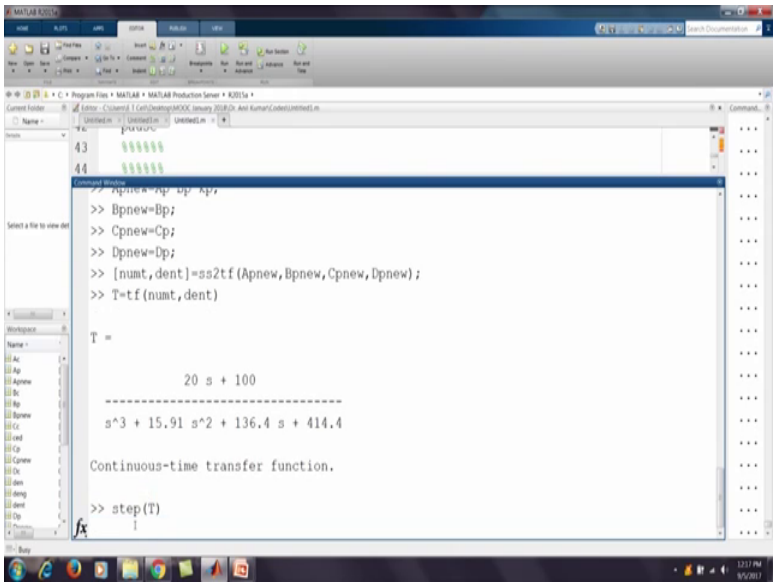

With this feedback gains so, these are the changed coefficients of this denominator characteristic equation and so, if we see the step response.

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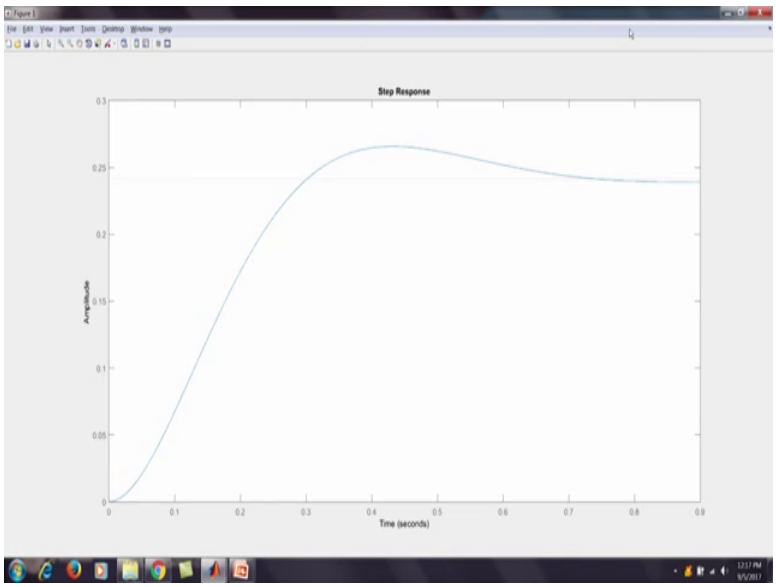

So, step t so, we can see this step response on this matrix ok.

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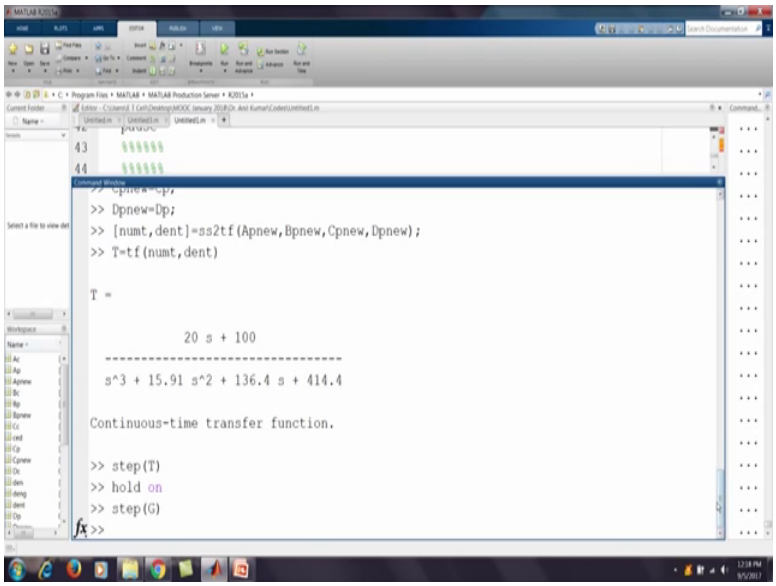

So, here I think we design the we design the state space feed by gains so that the system earlier system that is 20 s plus 100 by s cube plus 5 s square plus 4 s can give this percentage overshoot of 9.5 and settling time of 0.74. And so, here we first find the desired poles r1, r2 and we put the third pole at minus 5.1 so, that we can cancel the 0.

And then we found the state space canonical model matrix A B C D and then we converted to phase variable with using the transformation matrix. And then this phase variable form because in phase variable form we could apply this command to find A and B matrix for desired poles. So, the value of feedback gains that will give this desired poles location.

So, once we find the k the new matrix will be A minus B into k and other matrix will be same and then this is our new transfer function this that will give us the desired characteristics that it in the response. So, now we come to the last problem that is s.

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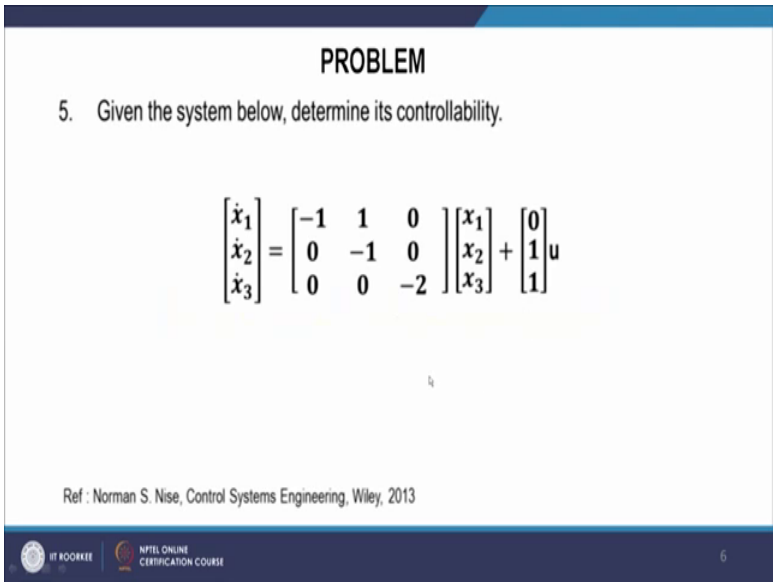

Given the system below determine that it is controllability because we discussed about controllability. So, this is the system minus 1 1 0 is 0 minus 1 0 0 0 minus 2 and 0 1 1 so, how to find this?

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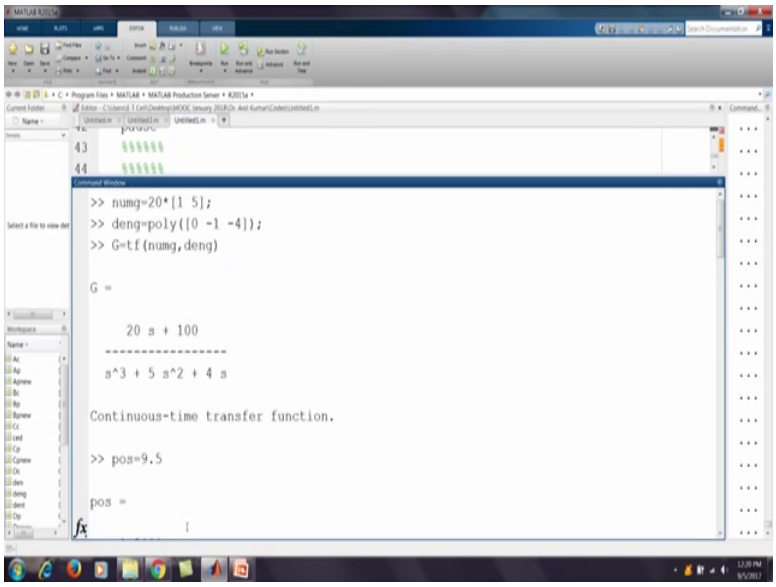

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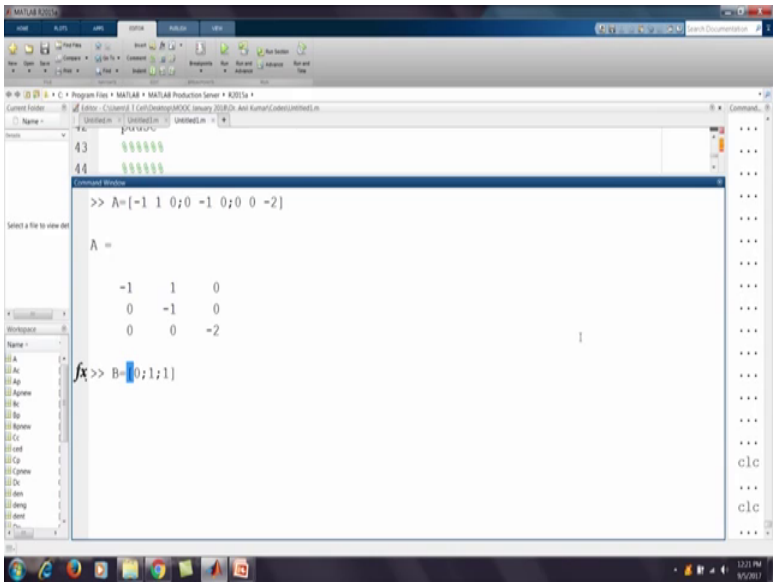

Let us try to find so, we so, we define this matrix A equal to minus 1 1 0, 0 minus 1 0, 0 0 minus 2.

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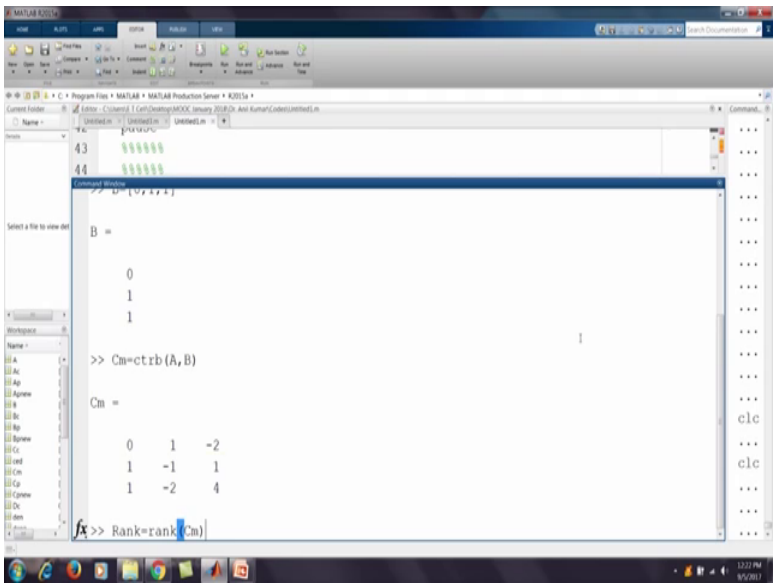

So, this is A matrix and the B matrix equal to 0 1 1 so, this is B matrix. Then we have to calculate the controllability matrix with in MATLAB with simple command that is Cm equal to ctrb, ctrb gives you the controllability matrix and we give only A and B here and we got this controllability matrix. Then we can find the Rank of this matrix Rank with Rank amount so, Rank Cm and.

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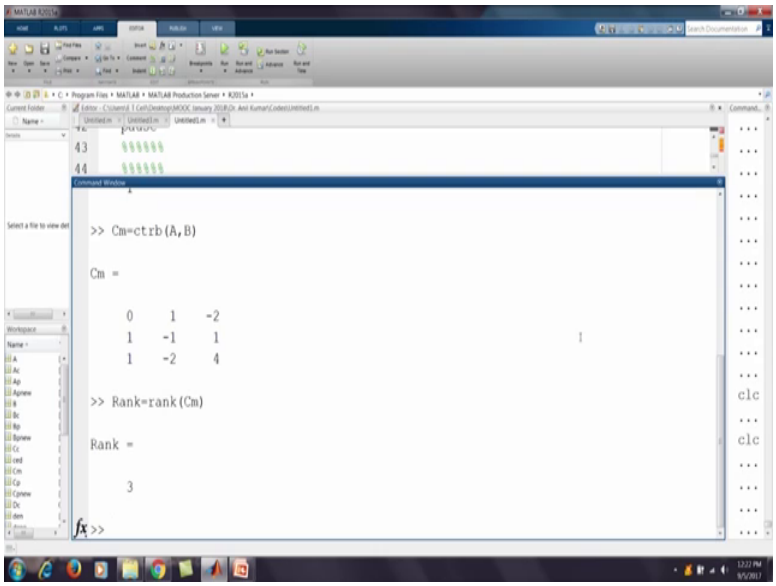

So, Rank we get 3 so, we know that the system matrix is of order 3A and Rank is also 3. So, this system has this is controllable because it is equal to the order of the system matrix.

So, here we discussed about state space models and how to use MATLAB to solve the problems in state space. So, I thank you and this is the last lecture of this course so, I thank you for this lecture and I hope that you will learn several things that will be useful to you in the control and design of control system.

So, I thank you very much.