Automatic Control
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Lecture - 39
Design Via Root Locus, Compensation – II

So, welcome to the lecture on application of MATLAB in automatic control, in this lecture we will discuss about design via root locus and compensation techniques. So, we will more specifically we will discuss about lead compensators. So, we have already discussed about the lead compensators. So, lead compensators are equivalent passive systems for the PD controllers.

So, in PD controller we put 0 at desired location to obtain the desired poles locations, in case of lead compensations we put a desired 0. So, we put a 0 at desired location and then we put the another pole compensator pole at some locations to satisfy the condition of angle so that the root locus passes through the dominant or desired pole points.

So, here we have one problem and that we will discuss on I theory we will discuss the theory and then we will discuss the MATLAB coats. So, this problem is taken from the Norman S Nice Control Systems Engineering.

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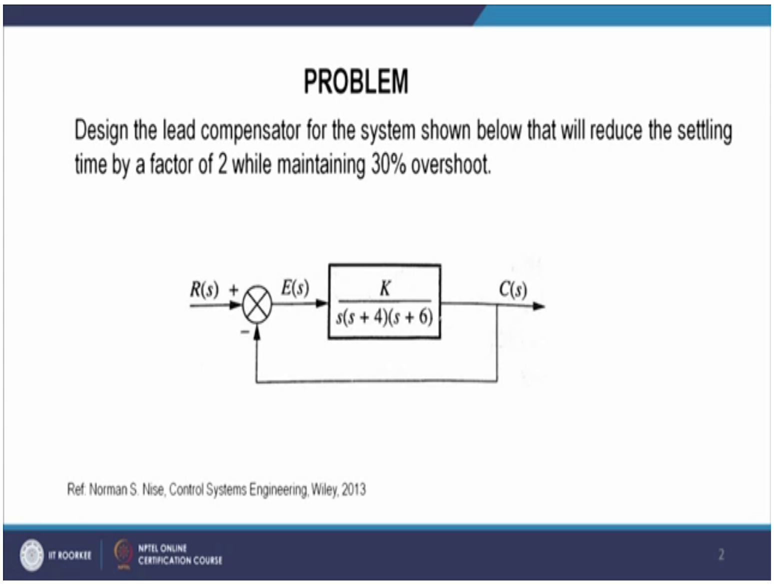

So, the problem is that we have to design the lead compensator for the system shown below that will reduce the settling time by a factor of 2 while maintaining 30 percent overshoot. So, this is the system and so, we can.

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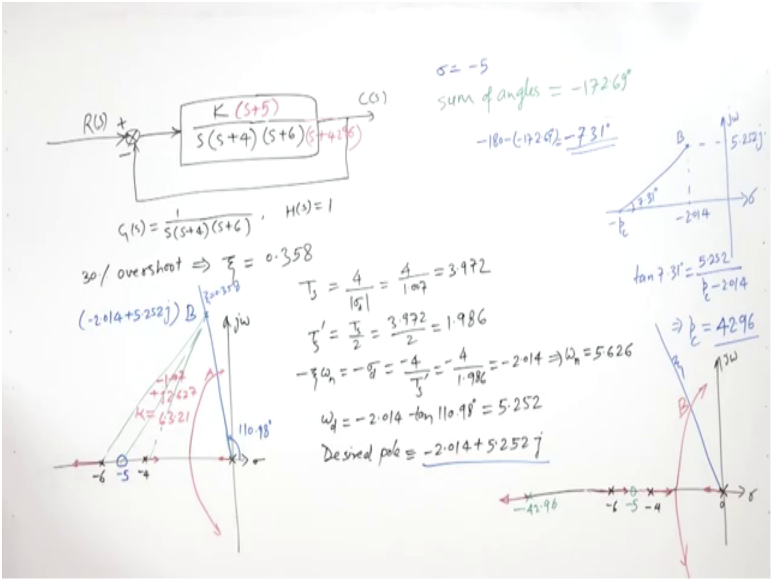

Have this system we can have the input Rs and this is K by s s plus 4 s plus 6 and we have unity feedback system plus. So, here we have Gs equal to 1 by s s plus 4 s plus 6 and Hs equal to 1 unity feedback.

So, first we have to find this 30 percent overshoot. So, if we convert this to damping with the formula so, we find this damping equal to 0.358. So, we can calculate the damping as we discussed in the previous lecture and then the settling time that is that is we can calculate. So, to get the settling time first we have to make the root locus. So, here sigma, this is j omega so, we have to plot this line and this is this line is we are getting. So, we can get this line that is zeta equal to 0.358 and this angle is 110.98 degree and this angle is 180 minus this angle.

So, now we have to plot the root locus so, root locus here we have s equal to 0 minus 4 and then minus 6. So, here the root locus will start and this will go to infinite. So, here they will separate and they will cut this line at some point, let us say point. So, this point A is minus 1.007 plus j 2.627, 2.627 so, this is the point A and at k equal to 63.21.

So, because we want to maintain the 30 percent overshoot so, this we have to the operating point is point a because, this point is on the 30 percent overshoot line and. So, the settling time for this point Ts we can compute the Ts. So, here Ts is 4 upon mod sigma d so, 4 upon here 1.007.

And we can get 3.972 is the settling time, now we have to maintain the settling time Ts dash we have to reduce by factor of 2 so, Ts by 2 so, that is 3.972 by 2 so, 1.986 so, this is the. So, we want to get the for this settling time we need some reduced settling sign time. So, we need some other points pole points because the root locus should pass through this point and so, the is new pole point is real part is minus 4 upon Ts dash that is 1.986 and we are getting this as minus 2.014, 2.014. So, this is the real part of the new pole and we can get the imaginary part omega d so, minus 2.014 sigma d into tan this angle.

So, this angle or we can get so, we are going to get at 5.252, 52. So, this is the omega d so, the desired pole is and we can also get here omega n from here because we know is zeta omega n is 2.014 and zeta is 0.358, we can get also omega n and omega n is 5.626. So, now, the desired pole is minus 2.014 plus 5.252 j so, this is the desired pole. Now, in the lead compensator we are going to put first 0.

So, we assume some location of some 0 and then we put the compensator pole to to satisfy the angle condition. So, let us we put a 0 at minus 5 so, we put sigma equal to minus 5. A 0 here between these points so, let us we put here at minus 5, a compensate is 0. And we calculate the angle so, we our desired point is here because at this point let us say this is our point B and that is minus 2.014 plus 5.252 j this is the desired point and we compute the angle of this point with respect to all these.

So, this angle so, the angle made by desired pole whether so, here the poles angle are added and 0 angle sorry poles angle are subtracted or in negative sign and 0 angle or positive sign. So, when we put the pole at sigma equal to minus a and we find the sum of angles so, sum of angles we obtain equal to minus 172.69 degrees.

So, when we are getting this angle 172.69 degree, now we have to add a pole so that it will complete this degree to minus 180 degree. So, we have how much angle we have to add by the pole is so, that is 7.31 degree. So, this is like minus 180 minus minus 172.69 degree so, that is equal to.

So, this is the angle because a pole will add a negative angle. So, this is the angle that would be made by the add by the pole. So, let us say this is the condition so, we have sigma j omega and we have this is our point B and this is 5.252 j and this is minus 2.014 and we are going to add some smaller angle so, our poles should be very far here. So, that it is going to add so, this is minus pc and this is 7.31 degree. So, this angle and so, we know that if we use this time 7.31 degree that is equal to this part, that is 5.25 to the imaginary part by this difference that is we say pc minus 2.014 and this will give us pc.

And we get 42.9, this pc we are getting 42.96 so, pc is here so, now if so, if we put this pole pc here the root locus will change now. So, if we have now this root locus and now our system is changed and the system is now first here a 0, that is s plus 5 and here is another pole that is s plus 42.96. So, now, this is our system that add 0 and a pole here. So, we have damping line zeta equal to 0.358 so, this is zeta line.

Now, we have the poles. So, first is s equal to 0 minus 4 minus 6 so, minus 4 minus 6 and this is 0, then we have compensator 0 that is here minus 5 and compensator pole here minus 42.96. So, compensator 0 and compensator 4 and this is the original systems poles. Now, we have to make this root locus and we have already our point B that should be satisfied.

So, here this root locus will pass so, these root locus will start here and they will separate break away, this will start and ends to this pole and this will go to infinity and this will go to. So, here will be the this will end here.

So, the root locus branch will be odd left of the odd poles so, here in this section then this section and th in this section. So, here it will start, from here it will separate, this will start and ends to this 0, this will start and go to infinity so, now these will pass through the point B here. So, this is all this design of the lead compensator, now we will see how in the code. So, now let us understand so, this is the uncompensated system with numerator 1 and denominator of polynomials with roots 0, minus 4 and minus 6.

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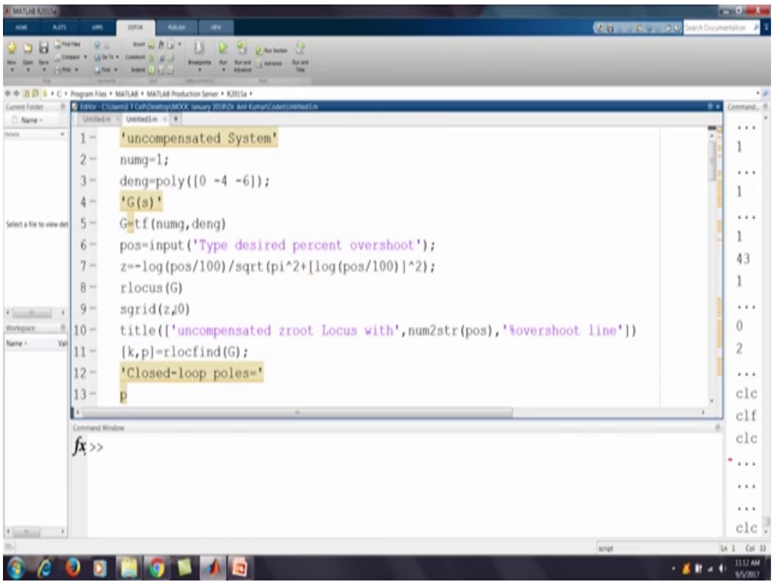

Then we get the transfer function by a numerator and denominator then the percent overshoot we input so; we asked to input the percent overshoot. So, here we will input 30 percent overshoot then we will calculate here the damping, then we will plot the root locus.

So, this original root locus without this compensator points and then here we will plot a line that is damping line; So, this line damping line.

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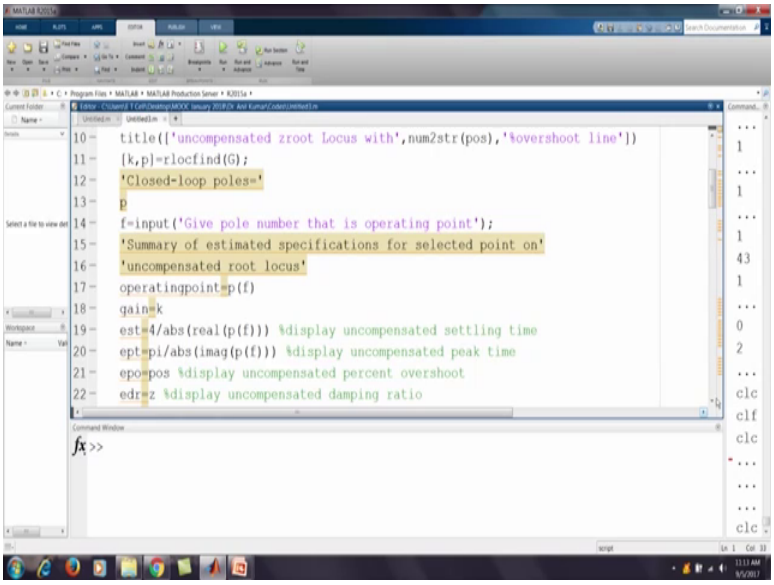

Then we will select, here we will rloc, loc find is find on the root locus some points. So, we will select this point A on the screen, the graphical mode and we select that point A and we store the gain value and the poles. So, there will be 3 pole set for that gain because there are 3 starting poles are open loop poles. So, this pole we will get this pole also minus 1.007 plus j 2.627.

So, once we select that pole give the pole number that is operating point so, we will give that pole. And for that operating pole we will calculate the gain so, gain we already obtain k, then we calculate the estimated settling time or for this system. So, we will calculate this Ts for this point A. So, 4 upon so, this formula 4 upon real part of the pole and then peak time, then percent over shoot be already entered damping we calculated. Then we find this natural frequency of the uncompensated system, then this static error constant and steady state error.

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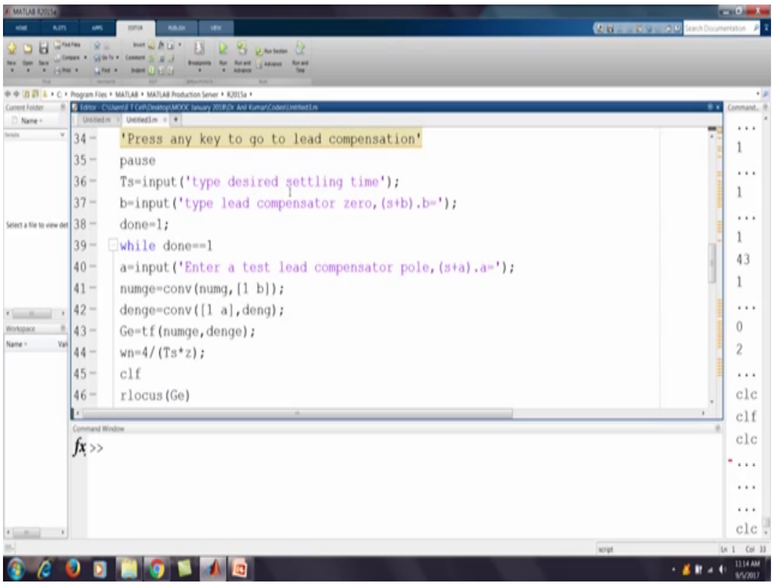

Then we get this is our closed loop transfer function with feedback of unity and k into G, we can see the step response for this system.

Then we go for the lead composition because we will here press the key to for lead compensation. So, we find input we input the settling time so, here type desired settling time. So, we can either put the settling time what we calculated or we just put estimated settling time by 2 because we want to make it half. So, we put the desired settling time then, we put the type lead compensator 0. So, we have to select the position of the compensator 0 as we selected here at minus 5. So, we have to enter this 5 because here it is given that s plus b and. So, we just enter b so, we have to enter plus 5.

Then we have variable done that is equal to 1. Now, we have to enter lead compensator pole. So, we have to this while loop when we entered in this while loop we are trying to find the value of compensator pole that will pass through this point b. So, that will make this to pass through point b.

So, here what we are doing? We are numerator we are doing the convolution or multiplication with the original funks system with the compensator 0 that is 1b is the compensator 0. Then denominator we are multiplying the original this denominator with the compensator pole that is a we are entering here. So, 1a is s plus a are here 1v is s plus b so, we are multiplying so, we are changing the system to this system s plus a and s plus b and we are trying with b.

Once we get the equivalent transfer of this transfer function with these compensated poles and 0, then we again calculate this natural frequency and.

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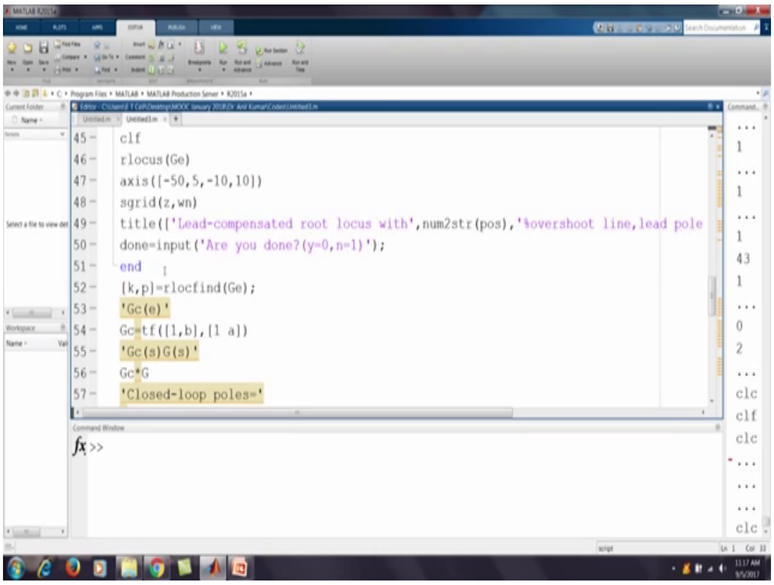

We plot the root locus for this change system and we see that whether s grid line z omega n so, it is passing through this compensator this overshoot or damping line. The point where it will cut is the point p and we see that we are getting this the proper point on the of the whether the root locus is cutting this point for this omega n. Once we are done with the point we say 0 and we will come out of this otherwise, if we put 1 we will be trying this.

And here k p equal to rloc find once we are we have.

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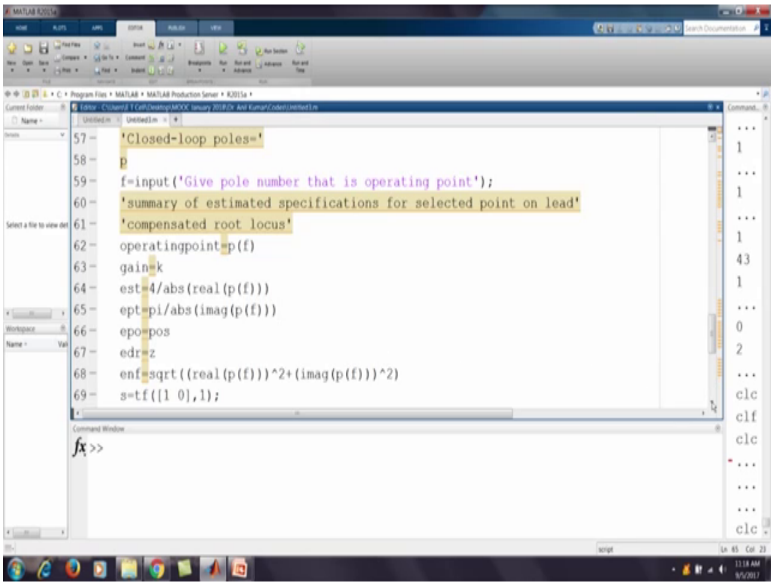

We have finalized the compensator poles and zeros, we will command this to select that point b and we will find the gain of this for this point and the poles. So, once we are so, this is our compensator 0 and poles and this is original system G and we multiply with compensator and we are getting this poles so, give the pole number that is operating point.

So, we have to enter the point b here and so, operating point is p and the pole number f and then we calculate other parameters like settling time, peak time for this.

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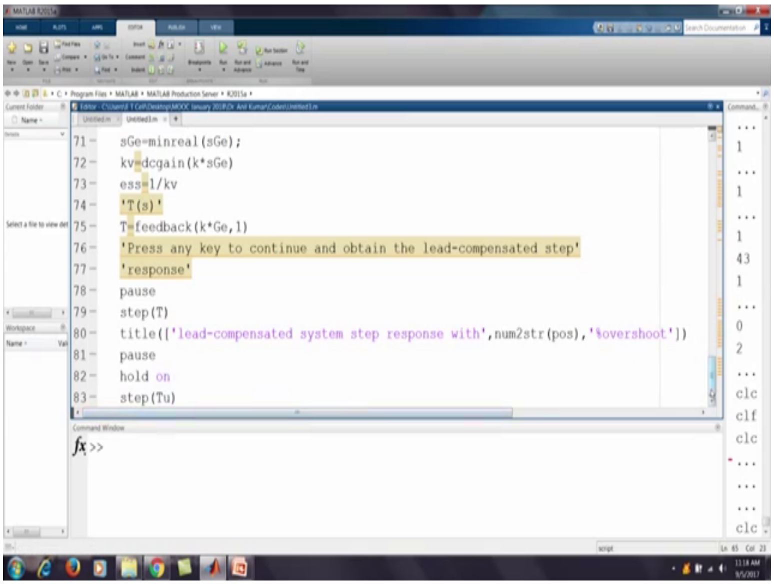

And the steady state error, then we plot this T equal to feedback k into G. Now, our transfer function is compensated 1, that is G k into G and feedback 1.

Now, we find the step response of this and we compare this step response with the step response of the uncompensated system T u that we defined here T u. So, here we can run this command. So, we can see here that it is asking type the desired percent overshoot so, we entered 30 percent because in the problem there is 30 percent overshoot we entered so.

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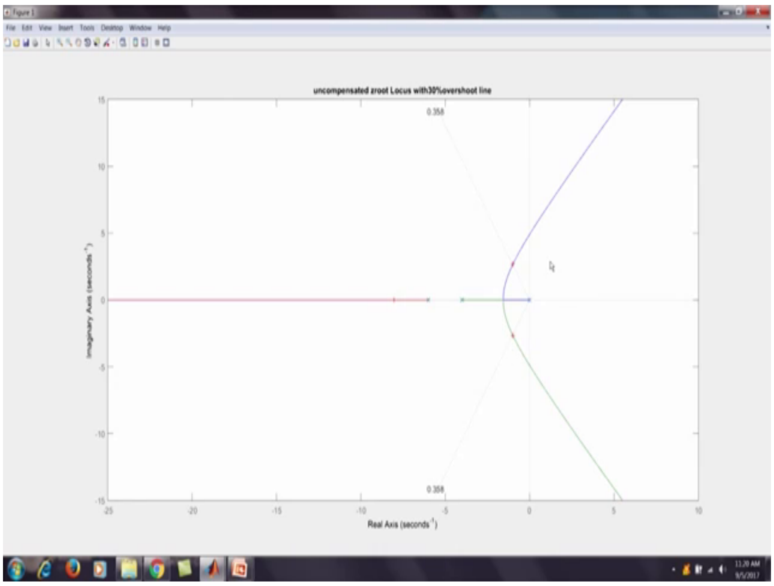

We are getting the root locus. So, you can see here the same root locus that we enter we described earlier. So, poles at 0, at minus 4, at minus 6 and this is the root locus and entering intersecting this line of 0.358 damping line. So, we select this point because this point of the root locus is on the this damping line so, we select this pole so, we all selected these poles and.

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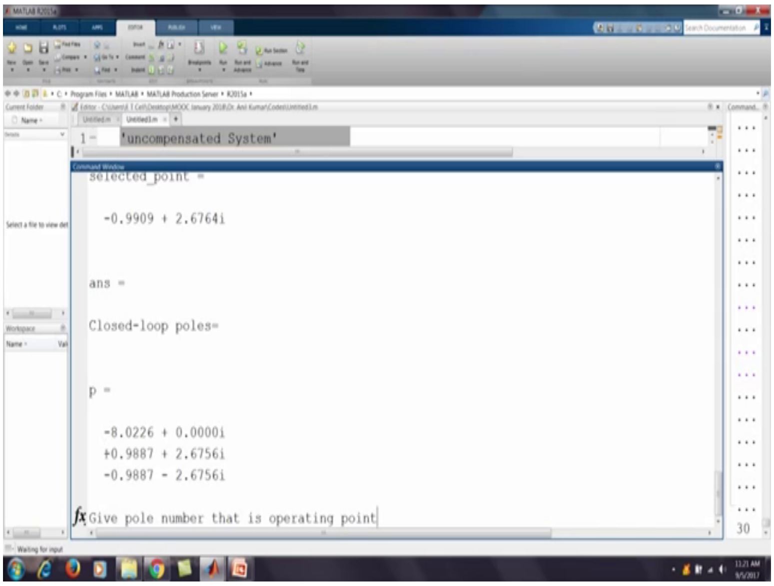

We can see here we got these p equal to this so, these are the 3 poles we got; So, here the imaginary pole minus 0.98 plus 2.67.

So, here we are getting the value of minus 1 plus j 2.6, here we are getting minus 0.99 plus 2.67. So, there is a little deviation due to the selection on the graphics mode. So, here give pole number that is operating point so, this is the pole number 2 because it is minus 0.99 plus 2.6i so, this point a.

So, we give the point this is at the number 2 so, give 2 and enter so, we get the already we got this uncompensated system with already we got the gain k and this system uncompensated system. So, this is s cube so, this is the original system has the third order because there are 2-3 s terms. So, s cube and in the denominator we only have k and value of k is 65.28. So, at k equal to 65.28 we are getting this uncompensated system that will pass through this that will give this 30 percent overshoot.

Now, press any key to go to lead compensation.

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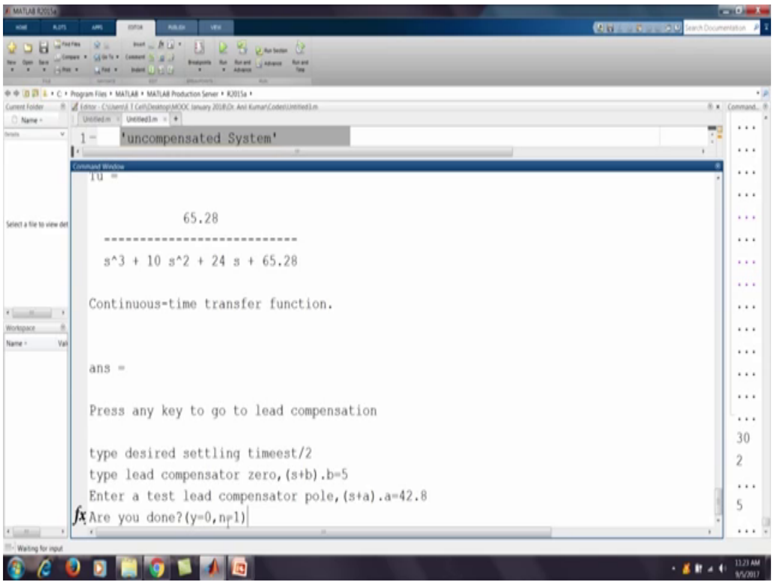

So, we press some key let us enter so, type desired settling time so, settling time we want to make half. So, we want to give tst by 2 because, tst was the variable we computed the settling time of the original system through ht by 2 we give. Now it is asking type desired settling time we enter type lead compensator 0 so, lead computer 0 v assume at minus 5. So, here it is in terms of this s plus b and so, b is 5 then only s plus b equal to 0 so, s equal to minus 5 so, b we have to enter 5 we enter. Now, enter a test lead compensator polls so, now, we have to test for the lead compensator pole. So, the value of a we have to enter.

So, here we used a different a methodology, here we are testing because we can see here easily graphically whether this point is cutting this damping line or not. So, we let us put a equal to so, 42.8. So, because we know that we are about 42.96. So, we enter so, we can see now here we are getting this.

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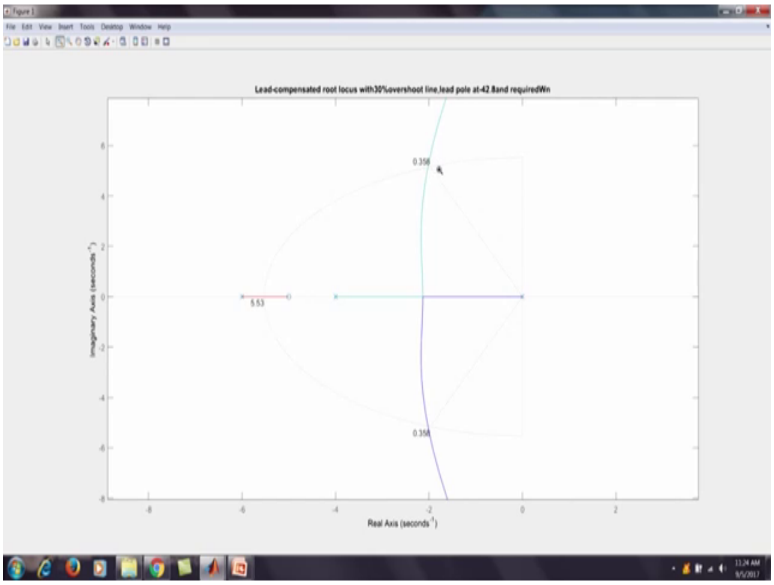

Here, but there is still there is some deviation here little deviation we can see here. So, this point is little bit away from this. So, it should shift to this side so, we have let us try 42.9. So, first we have to put 1 so that we are not done.

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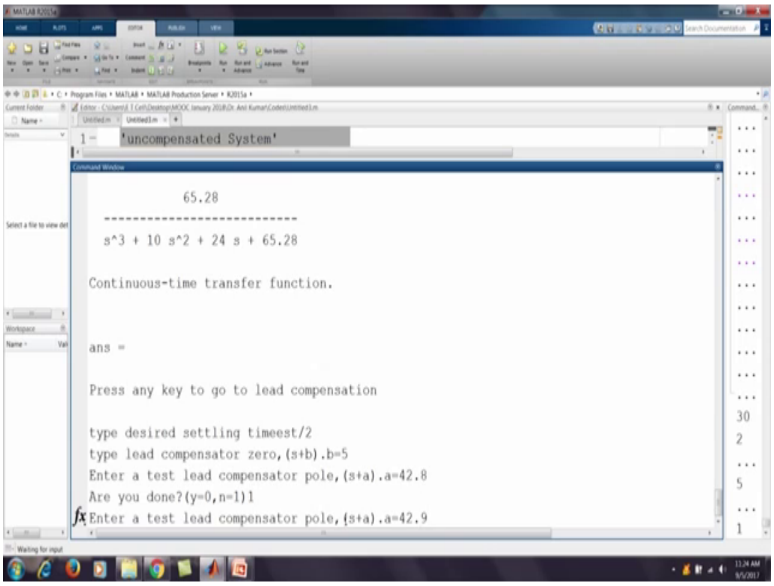

And it will go again in the while loop so, 1 and now we again put a equal to 42.9 so, we can again check.

So, here we are more close to this point so, we are more we are entering here cutting this point at z line and this z this curve. So, we are we can say this is our pole compensator pole location and we put 0 so, we will come out. So, once we come out we are getting the root locus. So, we have obtained this root locus and because the we have here at 0, at minus 4, minus 6 there is compensator 0 here and these 3 poles and one pole here this scale is just tail minus 10. So, it is beyond that so, we select this point on the root locus and we select at this point.

So, see we are getting this minus 2 plus 5.16; here we obtain the point b as a minus 2 plus 5.25. So, there is little deviation due to the selection on graphics mode so, here.

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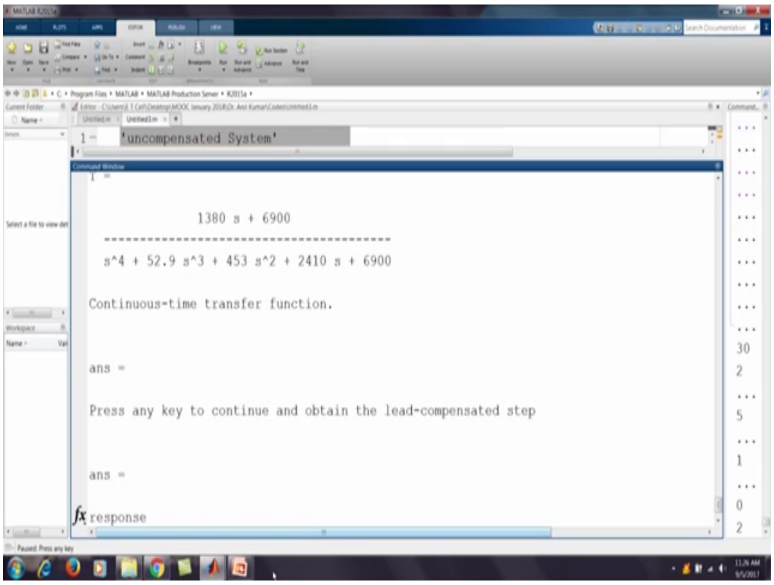

The pole is second pole minus 2 plus 5.16. So, second pole here enter so, we are getting the here this transfer function you can see here we have a 0 here and now it is power 4. So, we have already one compensated pole here so, this is now order of 4, now press any key to continue and obtain the lead compensated step. So, step response we can see of the compensated system.

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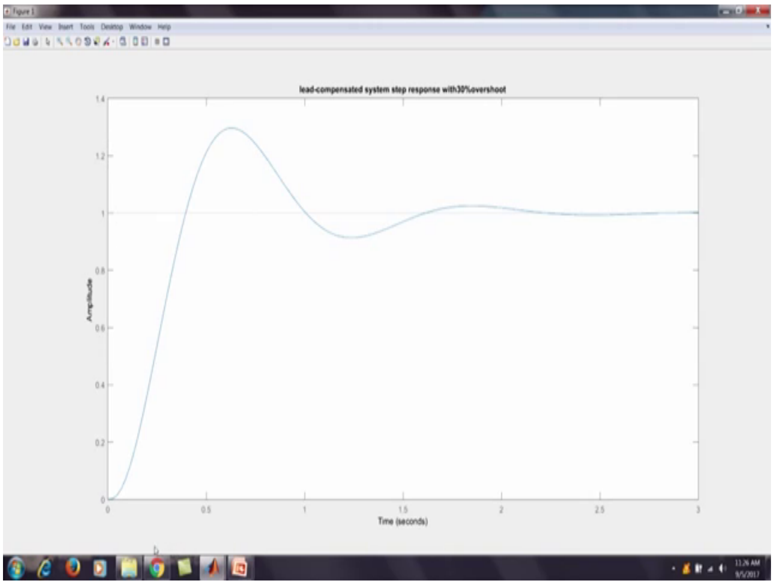

So, this is the lead compensated systems response for the step input. Now, we want to.

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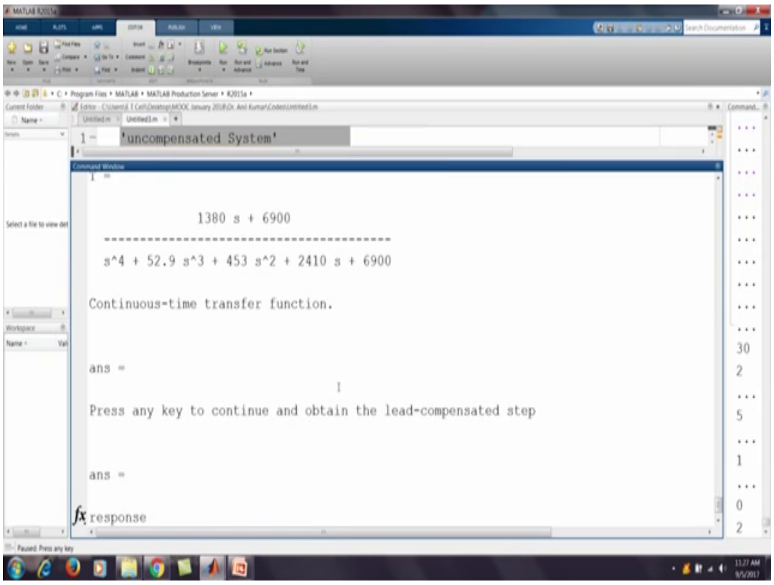

We want the step response of uncompensated system so, that we can compare. So, I put let us enter so, we are going to get the step response of.

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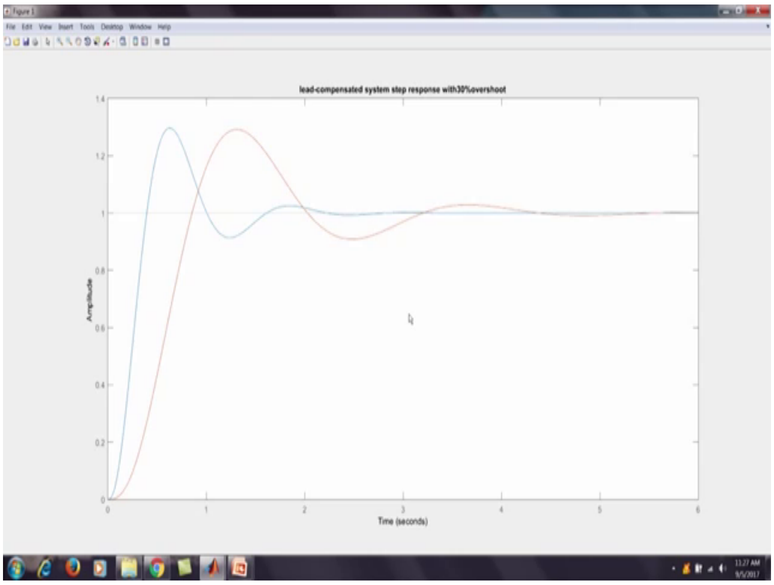

So, you can see here the red one is uncompensated systems response and the blue one is lead compensated systems response. So, we are here we are getting a peak earlier than the uncompensated and so, the see this settling time also we are getting reduced here for this compensated system.

So, here we saw how to design a lead compensator and how we can use in MATLAB. So, here we use the some graphical selection we can also use some other MATLAB functions we can write different codes so, that we can get exact point that we calculated here. So, I stop here and I thank you for attending this lecture.

Thank you.