Automatic Control
Dr. Anil Kumar
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture - 38
Design via Root Locus, Compensation – I

So, welcome to the lecture on application of MATLAB in automatic control, in this lecture we will discuss about design via root locus and compensation techniques. So, in this lecture we will discuss about problem related to PD controller design.

We have already discussed the theory behind the PD control that is ideal derivative controller. And we know that this is compensation techniques that helps us to design the root locus to pass through the desired dominant poles. And when we do the a this PD control we add a 0 in the existing system so, that the root locus passes through the desired dominant poles. So, we will take a problem.

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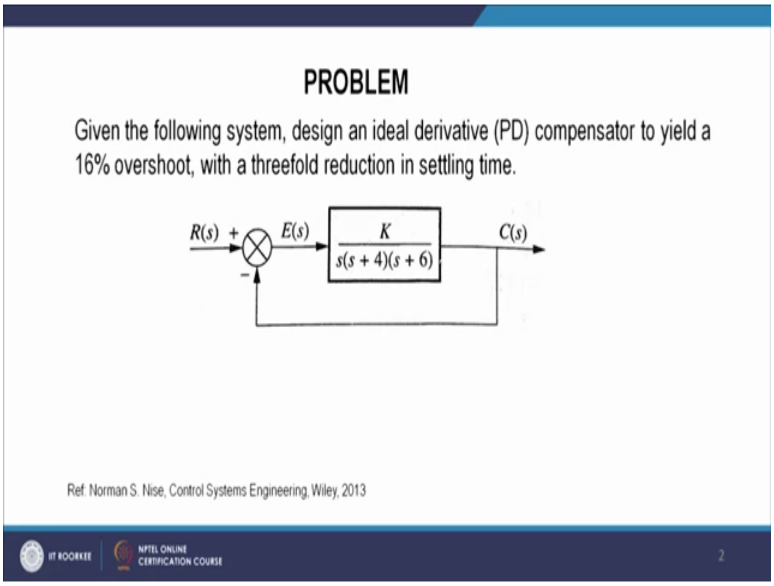

So, here we will have this problem. So, we are given this system, that is design and ideal derivative compensator to yield a 16 percent overshoot with the threefold reduction in settling time. So, this is our plant that is cage the gain and 1 by s s plus 4 s plus 6 is Gs and here Hs is unity that is 1 so, if we have this plant.

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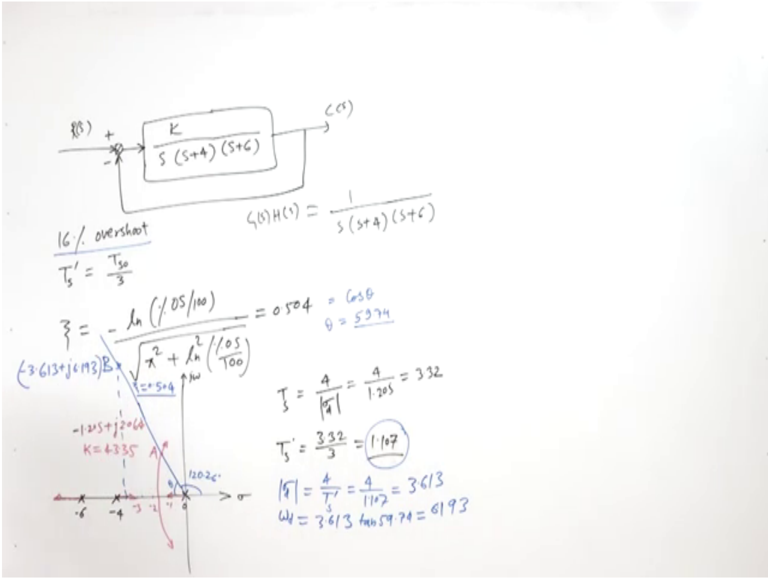

So, this is the system we have unity feedback here, this is Rs. So, we have to design ideal derivative compensator to yield 16 percent overshoot here and with a threefold reduction in settling time. So, 3 fold reduction means the Ts should be equal to Ts by 3. So, the this system has some settling time and we have to design it so, that the settling time is reduced by 3 fold.

So, now we first calculate the damping corresponding to this overshoot and we know that damping equal to minus ln percent overshoot by 100 root pi square plus ln square percent to overshoot by 100. So, this is the formula and when we put this percent overshoot equal to 16 so, we will get this 0.504 this damping.

So, now if we plot the root locus so, this is the we want to plot the root locus for the system so, here the root locus can be plotted. So, we can we have here GsHs the open loop transfer function GsHs equal to 1 by s s plus 4 s plus 6. So, here we have 3 poles starting poles at s equal to 0, s equal to minus 4, and s equal to minus 6 and there are all the zeros at infinite. So, root locus starts at these poles and ends to infinite so, here s equal to 0, s equal to minus 4 and s equal to minus 6.

So, here minus 4 minus 6 and this is 0. So, here the root locus and we have damping line that is 0.504 so, we can find the cos theta so, because we know that damping equal to cos theta so, we can find theta equal to 59.74. So, this angle angle from a this axis this real axis from in clockwise. So, here we can plot at 60 about 60 degree this line. So, this is the theta that we have calculated that is 59.74 and this angle is 120.26 because that is 180 minus theta and this line is damping zeta equal to 0.504.

Now, the root locus will start here, here and it will break away between these points so, this is let us say minus 1, minus 2, minus 3. So, suppose it breaks here and it passes through and it cuts this point at A. And this point will go to infinite directly and these 2 will also lead to infinite because they will end to 0.

So, here we have we can find this point where it cuts this line. So, this line we point at this point we find minus 1.205 plus j 2.064 at gain K the value of K equal to 43.35 so, here it cuts this point. So, we can compute the settling time so, settling time here h Ts that is 4 upon sigma d that is the real part.

So, we can a find 4 upon 1.205 and we can compute the settling time that is 3.30 3.320. So, we find this settling time and we can find other peak time also a steady state error and other error constant and third pole is found at minus 7.591. Now, the objective is the settling time here should be Ts dash that is equal to one third so, 3.32 by 3 so, this is 1.1 06 or 07 we can write it. So, this is the desired settling time and to get this settling time we need to calculate the to obtain the settling time of course, we have we need to shift this pole. So, this real part will be now equal to 4 upon settling time Ts dash. So, that is 4 upon 1.107 and we want this pole at 3.613, 3.613.

And because, we have where the poles should be lie on the 16 percent overshoot or zeta equal to 5.104, we know that this theta is same if the pole is suppose here at B so, it has the same theta. So, we have this relation and we have found something here this 3. So, we have found here this point let us say because we found the sigma that is minus 3.613 here and this theta is same and we can find this omega d. So, omega d equal to so, tan theta equal to omega d by sigma d so, omega d equal to 3 sigma d into tan theta so, 3.613 tan 59.74.

So, we can find omega d that is equal to 6.193 so, the desired point here B that is equal to minus 3.613 plus G 6.193. So, this is the desired point B because, now we want that the root locus passes through this point. So, now we want that this point beyond the root locus so, the angle condition must be satisfied. So, that this point this is a desired point and it is angle with respect to these open loop poles and we will add a compensated 0 according to the theory of PD controller. So, with that 0 and these poles we consider this angle that should be odd multiple of 180 degree.

So, first we compute how much angle these points is making with respect to these poles. So, if we compute these angles we find that it is making angle of minus 275.6 degree.

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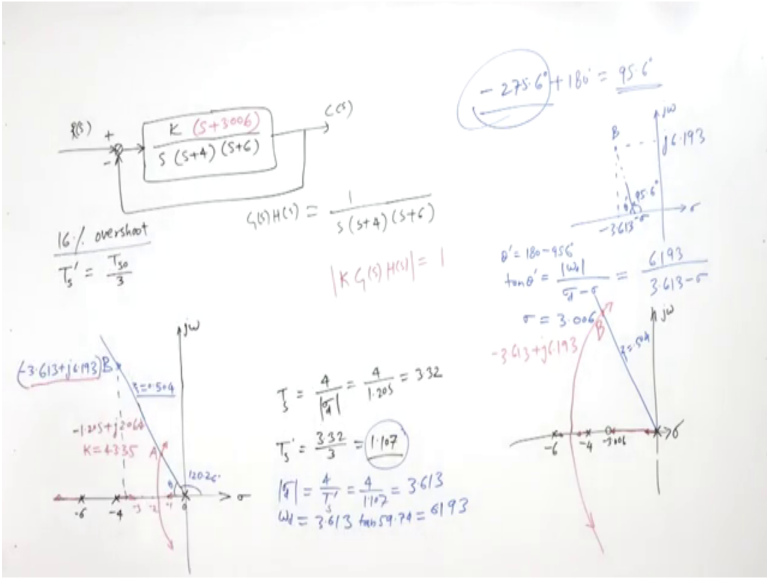

So, now if we want that it makes an angle of 180 degree because this angle should be either 180 or 3 times 180. So, it is better we are if we want 180 so, if we add 180 we will get 95.6 degree. So, we should add compensator to 0 at because we know that these poles makes negative angles.

Now So, if we put some 0 such that it adds some positive angle that is equal to 95.6 degree then the this pole desired pole at B will make 180 degree angle with respect to the compensator 0 and the system poles. And therefore, the condition of angle will be satisfied and that point we will be e on the new root locus because, new root and the root locus will be changed to pass through the point B. So, we want to compute this 95 point so, if we have this sigma, this is j omega and this is our desired point B so, we know this is j 6.193 and this is here minus 3.613.

Now, to make angle that is 95 so, 95 angle it should be here because if it is beyond this the angle is less than 90 so, it should be before this. So, if we have this here is zc and let us say minus let us say minus sigma and this angle should be 95.6 degree. So, this is the angle 95.6 degree and this angle is pi minus 95.6 degree. So, let us say this angle is theta dash so, theta dash equal to 180 minus 95.6 degree so, tan theta dash equal to mod so, mod omega d upon.

So, here mod sigma d minus sorry sigma d minus sigma; So, here we will have 6.193 upon here we will have. So, 3.61 minus this plus sigma so, we will have here let us do like 3.613 minus sigma so, we just take the values because we want this difference. So, we take this 3.613 minus sigma so, we just take the values. Now, we will get so, we will get this sigma that is 3.006 so, we know that we will put this pole at here 3.0 minus 3.006.

So, now the root locus will change and let us re-plot this root locus. So, now we have the system like this wise the original system we have added a 0. So, here the system will be compensated system and that will add a 0 at 3.006. So, here it will be into s plus 3.006 so, this is added this compensated 0 and this is our new system. So, we will make the root locus of this system and so, we have this line at damping equal to 0. 3 0.504 and we have these poles for this system now the compensated system that is at 0, at minus 4, at minus 6 and there is a 0 at 3.006 so, about here so, minus 3.006.

Now, the root locus will we can draw the root locus so, here the root locus will start and end to this point. Now, the root locus will start here and it will break away to reach to these points. Because, now we have one pole at finite and one 0 at finite and two 0 is at infinite so, these points will lead to infinite and they will intersect here this will be point B and this point B will be the point of desired pole location, that is minus 3.613 plus j 6.193. And we can find the gain corresponding to this point by using this rule that k Gs Hs equal to 1 the modernist so, we can find the gain for this pole if we put s equal to this we can find the gain for which gain this value is occurred.

So, now this thing we have discussed here how to do in MATLAB this problem so, we can see. So, the because the code is long I will run I have already written and I will run this code so, so here we can see.

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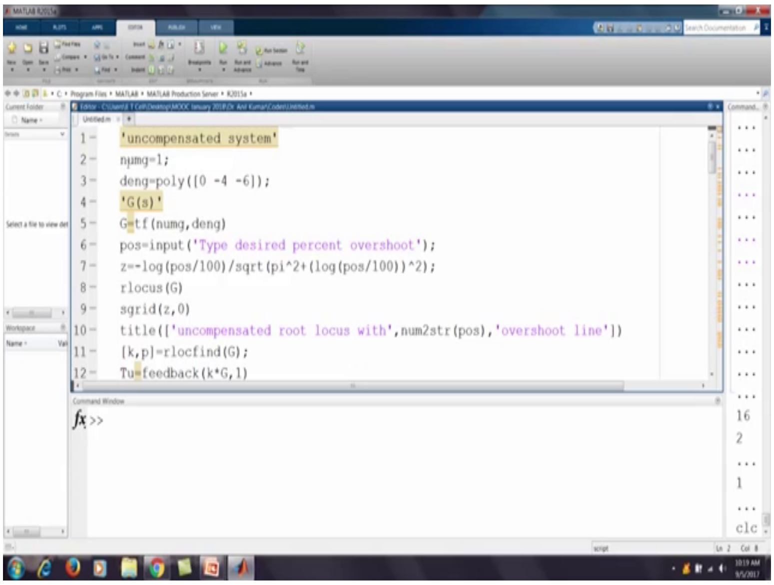

The uncompensated system we know that here the uncompensated system was a system without this compensated pole that is K by s s plus 4 s plus 6. So, we have numerator 1 because this gain we consider as a separate element K gain so, 1 by s s plus 4 s plus c is our uncompensated system.

So, here we are talking about Gs, Gs is 1 by s s plus 4 s plus 6. So, numerator is 1 and denominator here we say num numg and deng so, here a polynomial having this poles or roots 0 minus 4 minus 6 so, we get a polynomial in the denominator. And we can find the transfer function that is num from this numerator and denominator.

Now, we can take pauses the percent overshoot so, we want it as input because here it is 16 percent so, we take it as input, there will be a command that type desired percent overshoot and we will enter the value of percent overshoot 16.

Now, we will calculate the damping with the formula that relates the percent overshoot with the damping so, that is minus ln percent overshoot upon under root pi square plus ln percent overshoot. So, this is a square root so, command so, now we find the root locus of the this G so, root locus of this G we can find.

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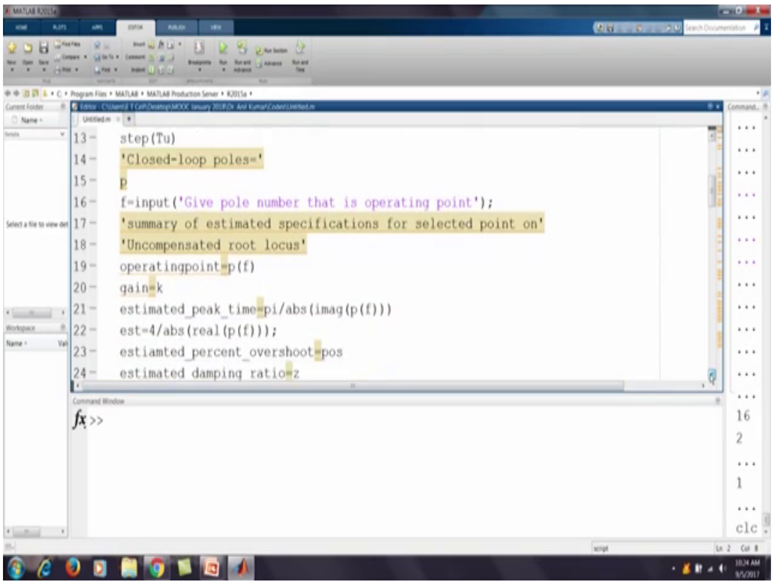

Then the sgrid command gives this a line that is of the damping value z that is calculated here and natural frequency is 0. So, it will plot a line and that for damping equal to 0.504, then we have a title uncompensated root locus with this percent overshoot.

So, we will calculate the a point so, we are here going to we will see the root locus on this screen and we will select the intersection point between this root locus and the damping line. Once we select that line, we will get here gain k is the gain and p is the poles. So, poles for this gain so, there will be 3 poles because these are the 3 poles so, 2 poles here 1 pole, second pole and third pole as I said it will it is going to occur here for that gain. So, here we will find these 3 poles and Tu equal to feedback k into G1.

So, now we want the transfer function of the equivalent closed loop transfer function we want. So, that is k into G is the open loop transfer function because, G here Hs is 1 so, k into G and 1 is the unity feedback here we are saying feedback is unity Hs is 1.

So, we get the uncompensated closed loop transfer function and once we give the step input we can see what is the response. Now, here then we are going to get the p will give the poles because, these poles already we have obtained here k into p, k comma p so, we have already got the poles of interest that is point A. So, where they intersect the root locus intersects the damping line so, we get the poles. Now, we want input give pole number that is operating point. So, our pole that is operating point is this A because, this A is passing at this overshoot or damping line.

So, we are get operating point at p f is if we because, p is a vector that contains the 3 poles. So, 1, 2, 3 and we will give if we give 2 we will get the second pole or if we give 1 so, our desired pole we can see at which number it is and we can give it f. So, we will enter f, then gain k gain equal to k we will get the gain here k because we already selected that point and that will give the gain, gain is stored here. Now we will calculate the estimated peak time.

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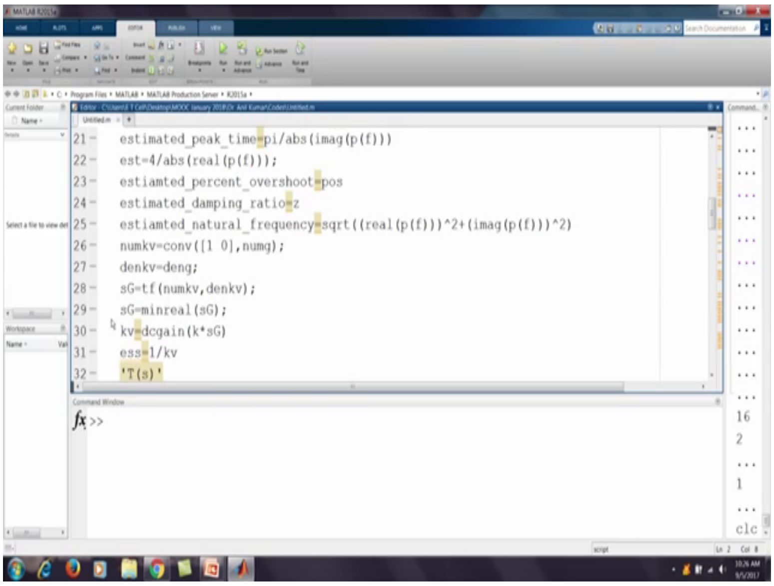

So, we calculate here because we are now doing this calculation settling time, peak time etcetera. So, at for these points so, we will use the peak time we can know by using pi upon absolute imaginary part because, we know that Tp is pi upon omega d and omega d is the imaginary part of the pole. Then estimated here est estimated settling time, that is 4 upon absolute real of the pole so, 4 upon sigma d, sigma d is the real part.

So, here we are using this sigma d sigma is 1.205 that is a here real part 1.205 modulus. So, we get the estimated settling time, then estimated percent overshoot equal to pause that is already we entered pos, then damping is z, natural frequencies under root of real part sigma d square plus omega d square, that is omega n we get. Then here we are getting the a steady state error constant, static error constants and a steady state error and.

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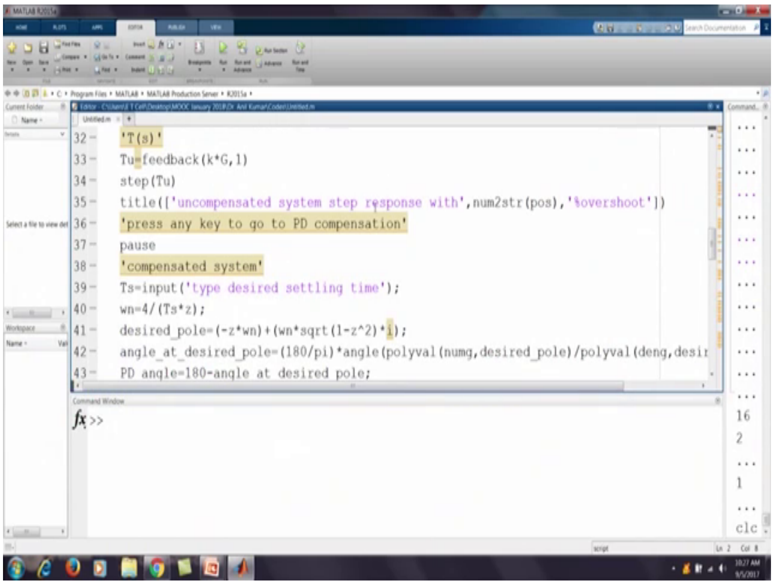

We already discussed how to get the static error constant. So, the here the it is type one system because s is there is one pole at origin so, we will calculate kv.

So, we got here the kv and the steady state error is 1 by kv, then we will take here the feedback k into G1. So, here we have we will get the step response of the uncompensated system for this pole A. So, we can see what is the response of this system at this pole.

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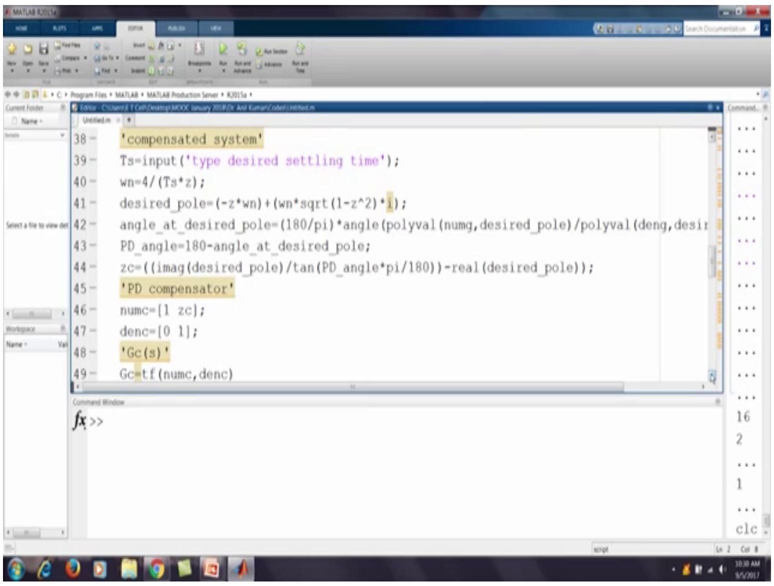

Now, we have we now want to go for the PD compensation and for PD compensation we have here Ts is the input. So, now, we want to input the settling time and here settling time is Ts by 3. So, earlier settling time, estimated setting time by 3 so, we will put this value either we put 1.107 or we put est estimated settling time that we calculated by 3.

So, here and we can get also omega n that is 4 by Ts into z and desired pole is minus that is sigma d plus omega di. So, minus damping is the same z into omega n we have calculated plus omega n under root 1 minus damping square, this is the desired pole. So, we are getting this desired pole here, what this is step sigma d we calculate 4 upon Ts dash or 4 upon Ts into that is zeta into omega n and omega d we are calculating.

So, omega d is omega n root 1 minus zeta square so, this is also this formula so, we use. So, angle at desired pole now we want to know the angle so, this angle that we calculated here that how much angle the pole at this point is making with respect to this system.

So, we are getting this angle so, we calculate this angle. Now, here PD angle that the compensator angle is 180 minus angle at desire poles. So, we are doing 180 minus this that will be the angle that the compensator 0 should add and so, this location zc sigma that calculated there the location of the 0.

So, this part sigma we are calculating so, we are calculating sigma using this formula and the code is written for this. So, here we have sigma equal to or zc equal to imaginary of desired pole by tan theta. So, this is tan theta PD angle into so, here we convert this into pi because this tan is going to take equivalent angle in pi and minus real of desire poles. So, we use that formula and we find this z at the location of the 0.

Now, here we are defining the numerator that is the 0 so, s plus zc so, one that is this point. So, this is the 0 that is here and here denc is compensator has compensator has only 0, no poles.

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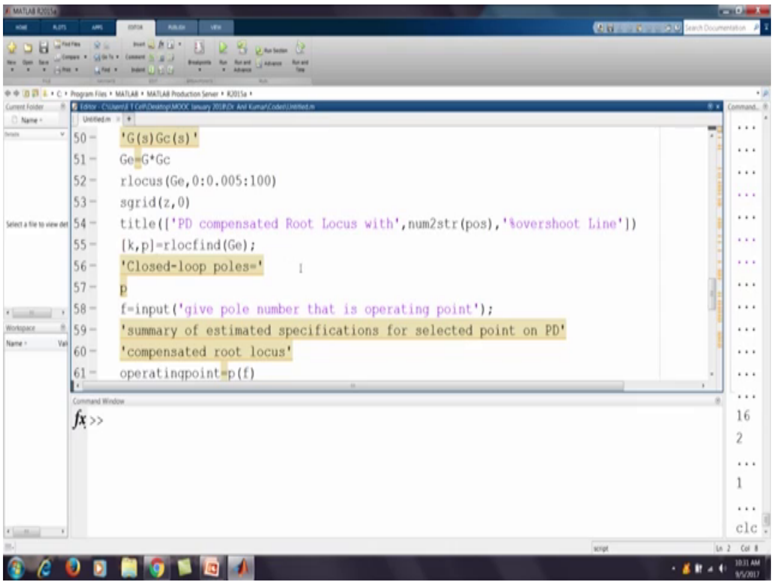

So, here we have 1 so, it is just unity and so, we find the compensator transfer function as numerator with this and denominator as 1. So, it is only the 0 and we can get here Ge like the transfer function that is G into Gc square. So, I initial transport function G1 by s s plus 4 s plus 6 into this s plus 3.006 and we plot this root locus and on this root locus again we select the point rloop find Ge.

So, from here we select the point k into p so, we select now on the root locus point B and this point B is our desired point and we get the gain k and the poles all the poles for this gain.

So, we give the pole number.

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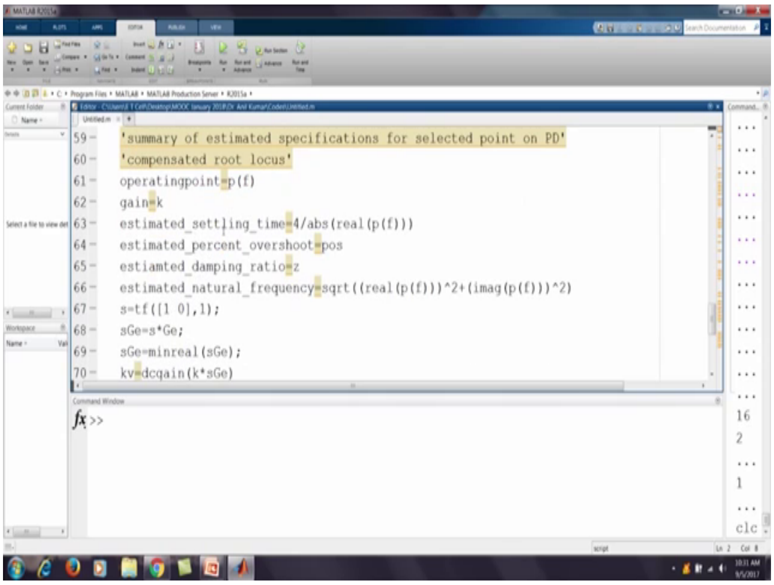

Here pole number and we get the all the other parameters, we can recalculate what is settling time, what is damping, what is and again we calculate here the steady state error and we find the closed loop transfer function T. And we calculate the step response of this and we can calc compare this with the step response of the Tu.

So, here we will we can run this code so, so we are going to run this so, we just control v and. So, we run this code and it is asking type, type the desired percent overshoot. So, let us type the 16, we type the desired percent overshoot 16.

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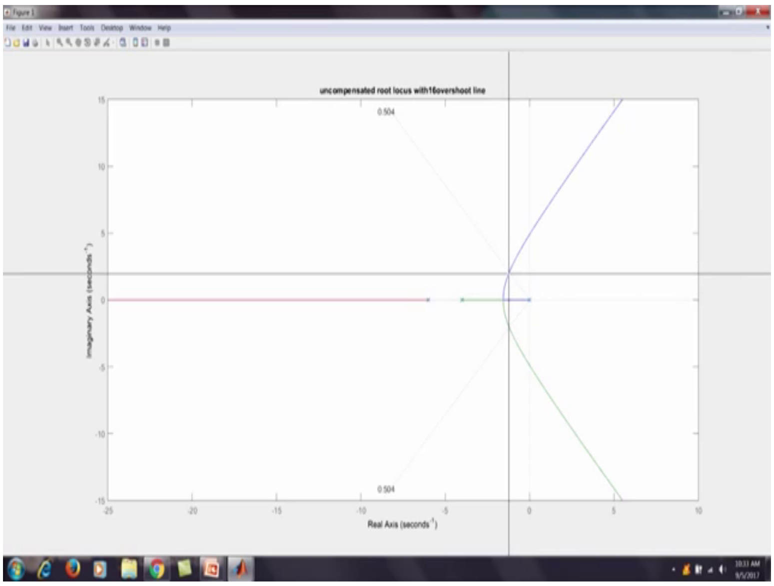

So, you see here we obtain this root locus and you can see here the root locus that we plotted here we are getting the same here. And we have to select the point he here we have the this cursor to select and we select the intersection point here, also it is a bit approximate but, we select here.

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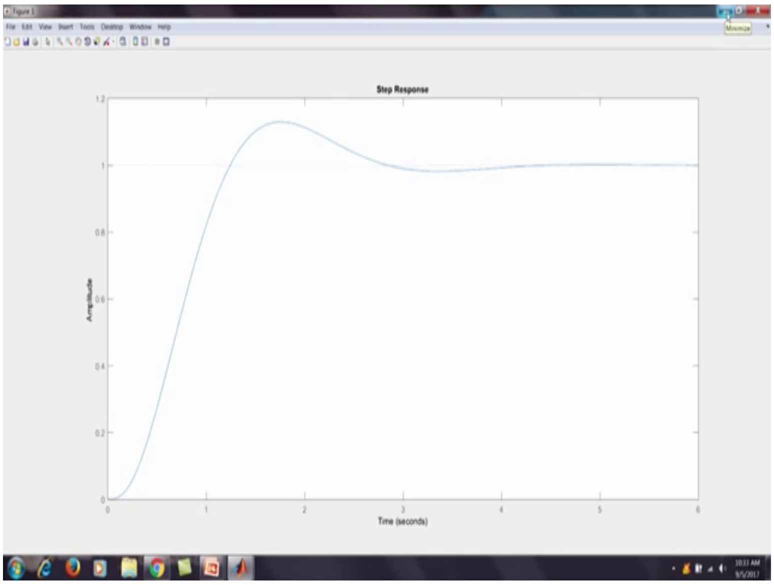

And we find the step response for this uncompensated system corresponding to this selection point A; this is the step response of this system. So, now we see this is the pole so, one pole is at minus 7.5 another is minus 1.2 and 1.96. So, here we are getting 1.2 and 2.064 so, there is little bit deviation due to the selection approximate selection.

So, here our desired point the point is operating point is this second pole because, this is the first pole, this is second pole, this is third. So, we have this is the second pole that is the desired point A. So, we put here 2 we enter so, we get the transfer function and we now want the and you see here we already find the k the gain and this transfer function. Now we press any key to go to PD compensation so, we in put some key type desired settling time. So, desired settling time here was est so, est by 3 because we want the one third of that settling time.

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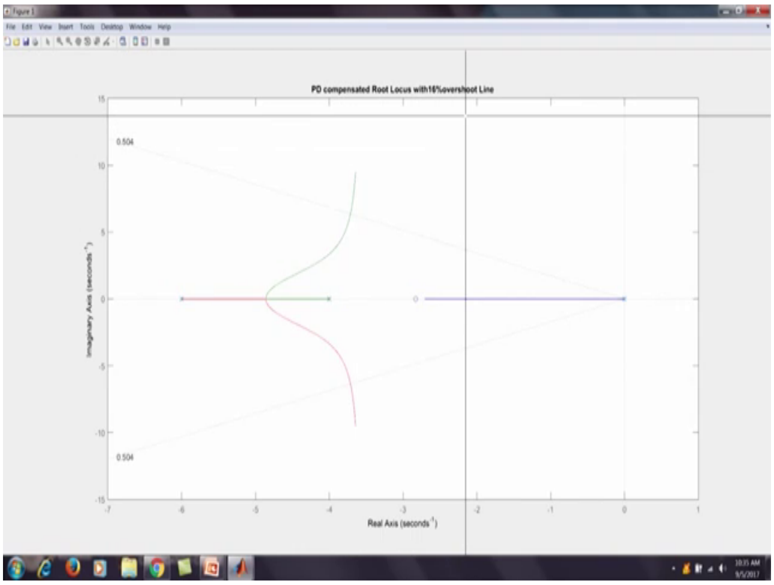

Now, we have obtained this root locus. So, when we put the desired settling time we have obtained the root locus and we want the desired so, this is our desired so, that the root locus should pass through this point. So, we have now this first point you can see first point is minus 3.7 plus 6.39. So, there is little deviation than what we calculated due to this selection of the point.

So, of course, we can also write the code so that we can get the intersection points and exact points that we are getting here. So, here this is for the design purpose initial design purpose and.

So, here we are using the graphical interface. So, here we are selecting this 0.1 because you can see here it is minus 3.6 plus j 6.19, here it is coming minus 3.7 plus j 6.39 so, little difference. Now, we select point one here this is the desire desired point. So, we are we already got designed this thing and everything calculated here you can see, we are already getting the steady state error everything and this is at final transfer function. And there is one 0, that is entered here, that is about s plus 3 because here if we take 50 outside so, it is s plus 3 about so, we have already.

Now, we press any key to see the step response so, we entered and we can see the step response. So, this is the step response of the PD compensated system. Now we want to compare the response with respect to the uncompensated system. So, if we enter so, we can see here.

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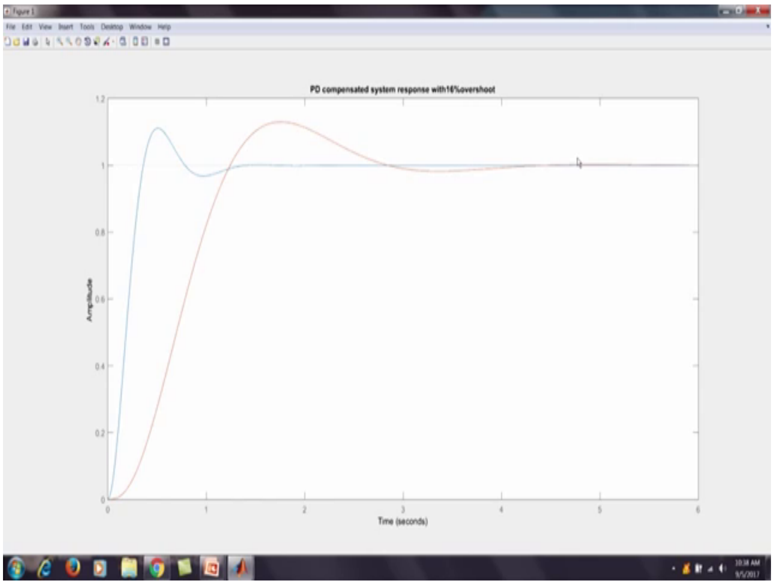

So, here the red one is the response of uncompensated system and you can see here the settling time it is settling here somewhere and we have a decreased the settling time here. So, and you can see also we are we have a better this, we have also decreased the peak time of this system because peak is also occurring earlier. So, we have designed the PD control system. So, this is the compensated system and this is the uncompensated system.

So, here so, we learned how to the we learned already the theory of PD controller. So, we saw in this example how to do the PD control theoretically and then how to use the MATLAB code to make this and these examples were taken from the book of Norman S Nise Control Systems Engineering.

So, I thank you for attending this lecture and see you in the next lecture.