Automatic Control

Dr. Anil Kumar
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

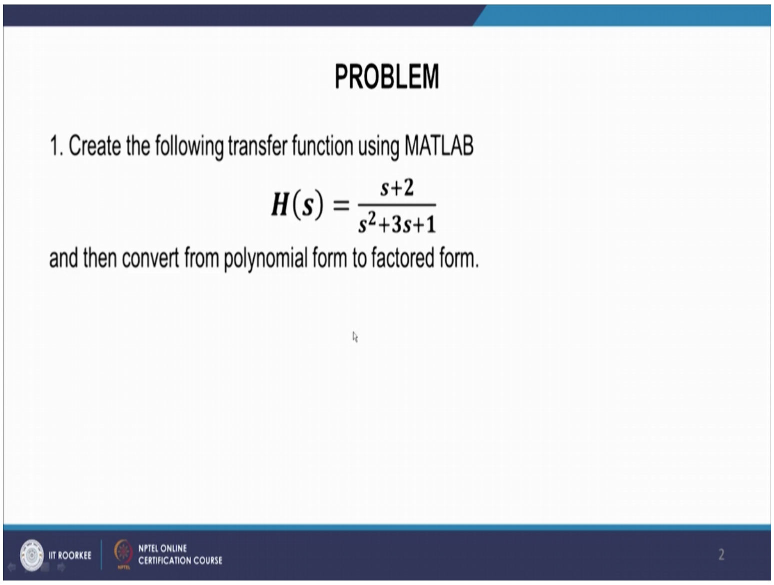
Lecture – 36
Transfer Function, Poles, Zeros, Response

So, welcome to the lecture on application of MATLAB in Automatic Control. So, in this lecture, we will discuss; how to plot or how to find the transfer function, poles, zeros and responses of the system of any system using MATLAB. So, I think most of you will be familiar with the MATLAB; so MATLAB is a mathematical tool that is used to modeling, analysis and design in modeling analysis and design. So, we can use matrix algebra, we can use differential equations, we can plot any function. So, there is quite application. So, how we can use this in automatic control?

So, what the theories we have already discussed; in the last lectures, and we have discussed some numerical problems also in those sections. Now, when we want to solve those problems in MATLAB or similar problems, how can we do? So, MATLAB we discus about transfer; we will discuss about transfer function, so how we can obtain transfer function? How we can obtain poles? How we can obtain 0 and the response, if there is some mix step response, impulse response and so on for a system.

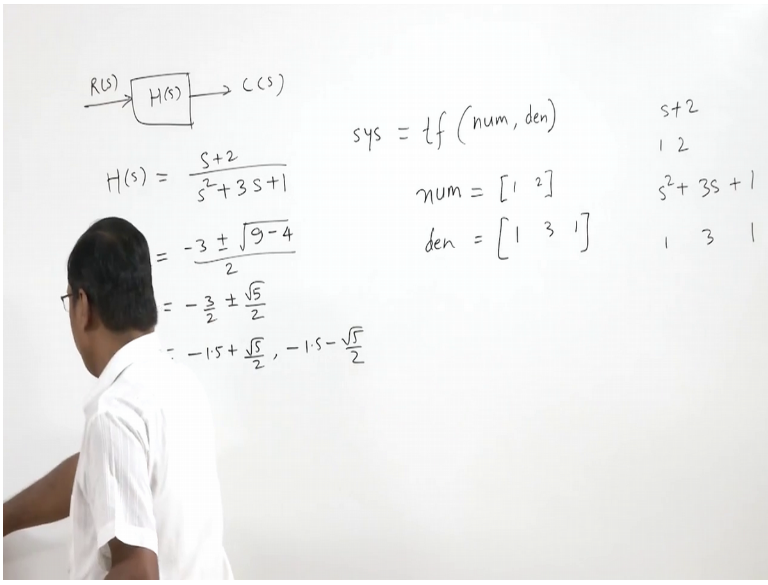
So, here let us take the first problem that we can see that create the following transfer function, using MATLAB and then convert from polynomial form to factored form.

(Refer Slide Time: 02:30)



So, here this is the transfer function, we knows this is the transfer function s plus 2 by s square plus 3 s plus 1; and factored form is if them there is some factored, because this is the square in the denominator, this ruminator is always factored. Here in denominator we; there are two routes we can factor into 2; so that is factor form. How can of course, we can do this manually, but for example, if you have manually we can have like this is our Hs and this our input and this is our output.

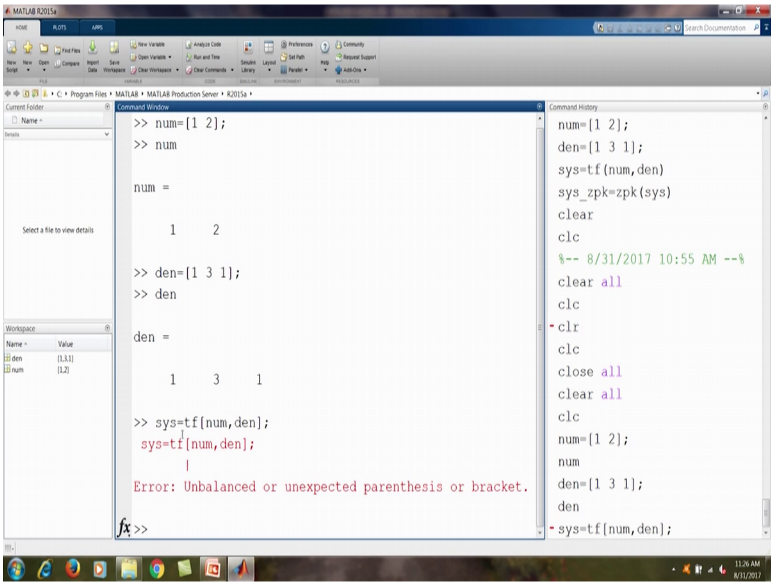
(Refer Slide Time: 03:13)



So, here Hs as given as s plus 2 upon s square plus 3 s plus 1. So, we can factored this by finding the roots; so s equal to minus 3 plus minus under root b square. So, 3 square minus 4 is upon 2. So, we can write minus 3 by 2 plus minus; so 9 minus 4; so this is root 5 by 2.

So, we will have the; if you want to; so we have this root minus 1.5 plus root 5 by 2 and minus 1.5 minus root 5 by 2. So, here if you want to use the MATLAB, so we have to open.

(Refer Slide Time: 04:47)

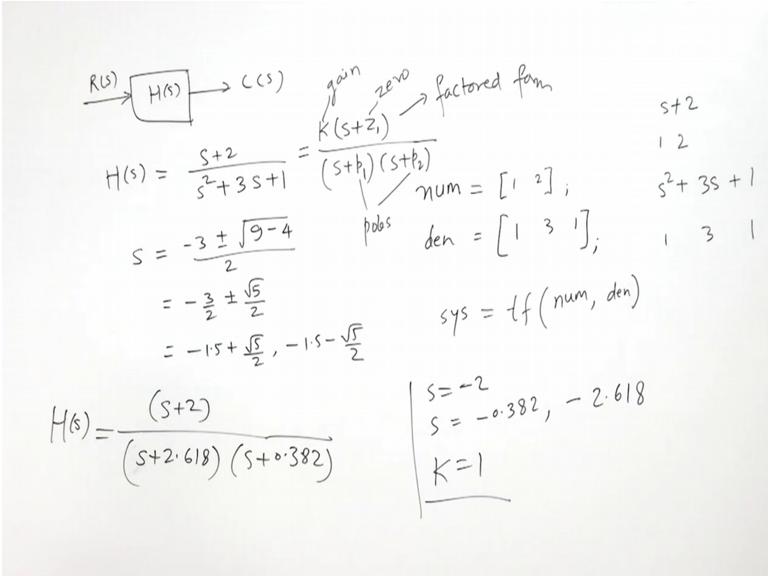


So, this is the MATLAB command window, we can write here the commands. So, here the if you want the transfer function, the we can use command at t f; t f command we can used and that is we can say t f numerate and denominator. So, this is the command.

So, tf num, den; now numerate we have to write; so here numerator is s plus 2. So, highest power of s is 1, it is coefficient is 1 and then the next powered coefficient that is 2. So, here we have to write 1, 2 that is numerator and denominator. So, we have denominator s is square plus 3 s plus 1. So, highest power of s is 2, it is coefficient is 1, and then next lower power is s that is coefficient is 3 and then s 0. So, it is 1.

So, were to write 1 3 1. So, here let us say this is system. So, of course, this command, when write in MATLAB; MATLAB we have to first define this variables. So, num den here we can write den, then we can use let say sys is equal to t f num comma den.

(Refer Slide Time: 06:58)



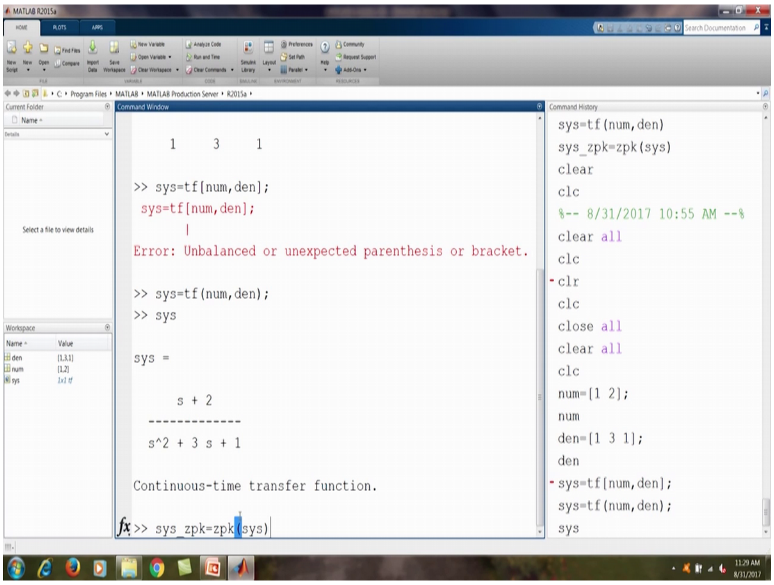
So, this is our; we will get the sys is the transfer function that have the numerate this and denominator 1 3 1.

So, we see if we write this here, so let us write num equal to 1 space 2 then; so we already defined we can see here we have defined num here in workspace, we have defined num 1 2. So, here if you want look see here what is num? So, we see num is 1 2. So, now den we defined den equal to 1 3 1. So, this is den, and we can see here in the work is space, there is the den variable created that has elements 1 3 and 1. So, if we write den and we put enter we will find this the elements of the den, so it is we defined the vector or matrix, so this is den; now, we want find transfer function.

So, these are we have to find sys; so sys means we are giving some name to the system. System equal to t f and we use this command num, den. So, here; so they should not be; we have done some mistake here; so here we have unbalanced of unexpected parenthesis; so we should not use this we have to use the small bracket. So, sys equal to t f. So, this one, because here we see we are getting t f small parenthesis num, den. So, here num, den; so we have know this and den we can use this icon.

So, now we obtain the sys, you can see we obtain sys that is 1 by 1; 1 by 1 matrix you can see a vector. So, here we have 1 by 1 sys. Now, if you want to know; what is this sys?

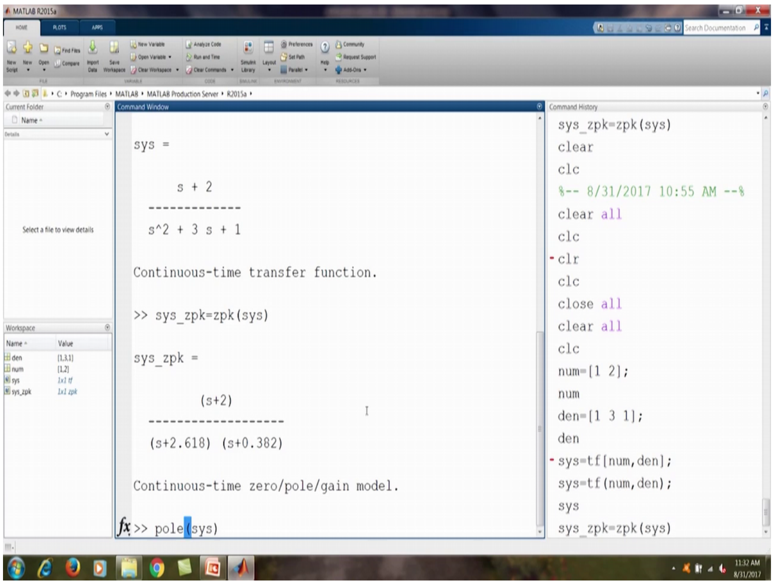
(Refer Slide Time: 10:24)



So, we enter and you can say we can find this transfer function s plus 2 s is square plus 3 s plus 1. So, we have found continuous time transfer function. So, this is the continuous transfer function. Now, we want to obtain the factored form. So, we want to obtain this factor form has I describe. So, we can write, so that is called zpk. So, z means 0, p means pole and k means gain. So, if this transfer function can be retained in a form like; let us say s plus s plus z 1 by s plus p 1 and s plus p 2, and if there is some gain; if it is 1 if there is some gain 5, 10 under; so that should be here.

So, this is the factored form. So, now, we say 0, that is z poles and gain. So, we call it zpk. So, here we have to find this sys underscore zpk equal to. So, now, this sys is the now in the transfer function; this is the transfer function form we want to convert this in this form factored form. So, we use this sys for the command zpk, sys means this zpk command will convert this sys that is the transfer function to 0 pole and gain form that is the show on here.

(Refer Slide Time: 12:41)

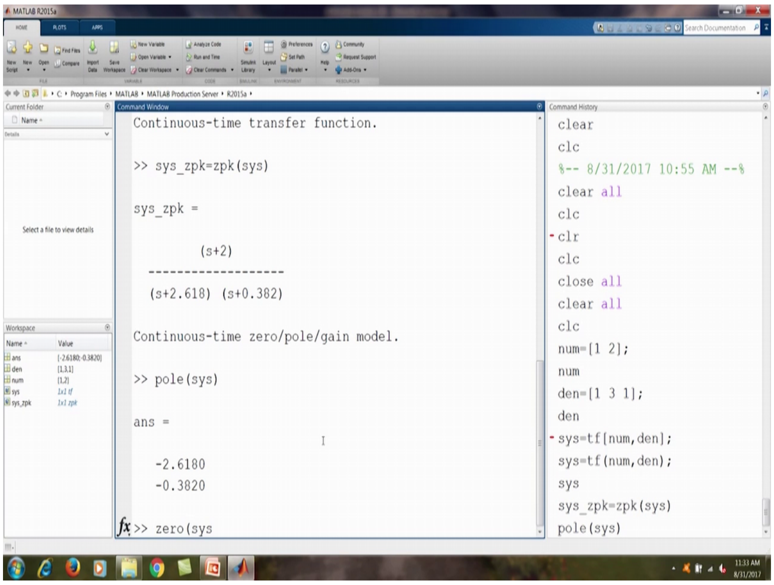


So, if we put this, so we are getting here you can see here s plus 2 upon s plus 2.618 and s plus 0.382.

So, this is root 5; that is above something 2 point something by 2. So, it is about 1 point something, and if you did use this one it is about minus point so less then minus 0.5. So, that is minus here we can see 0.382, we are getting and the other is we are adding here one point something. So, it is minus 2.6. So, here we have s plus 2.618. So, when we will be do the factors form, now we obtain this factor form s plus 2 upon s plus 2.6.

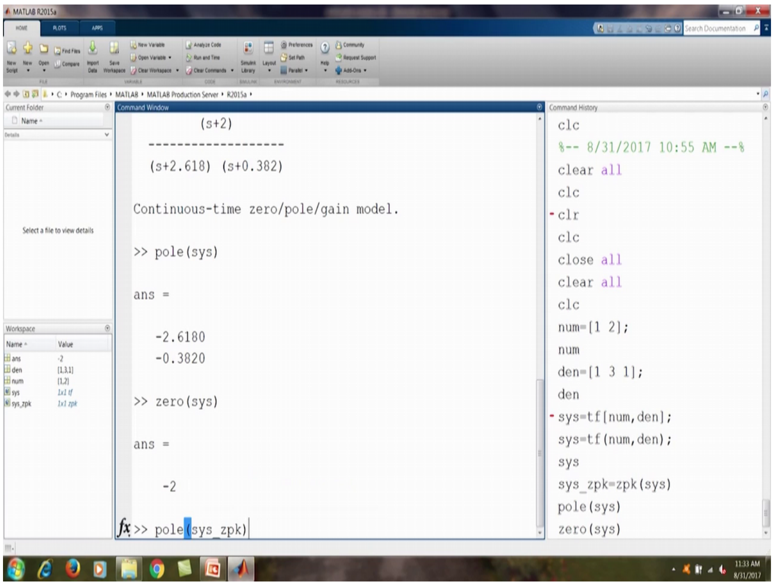
So, these Hs we represented in the factored form. So, now, form seeing the factored form we can tell that what is 0? So, 0 is at s equal to minus 2, pole poles are at s equal to minus 0.382 and minus 2.618 and then we known that gain here is 1. So, gain K equal to 1. So, by seeing the factored form, we can know these information.

(Refer Slide Time: 14:51)



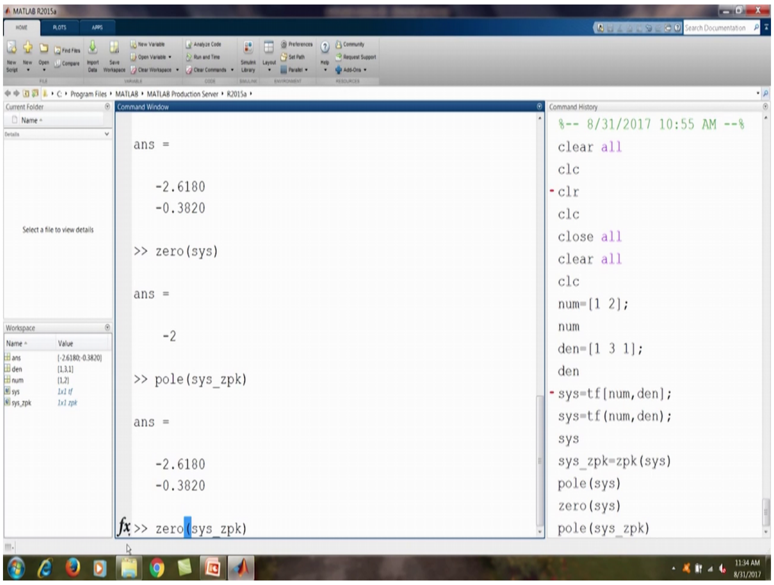
Now, here we can also known about the poles if he write poles sys, so you can see that by giving command pole we can pole sys. So, sys is the transfer function and if we give the pole command we can directly gets the poles; that is minus 2.618 and minus 0.382. So, we can get the poles, either we go in the factored form and we can find the poles or we can gives this command pole sys we can get; we can also give the command, that is called 0 sys; we get here minus 2.

(Refer Slide Time: 15:36)



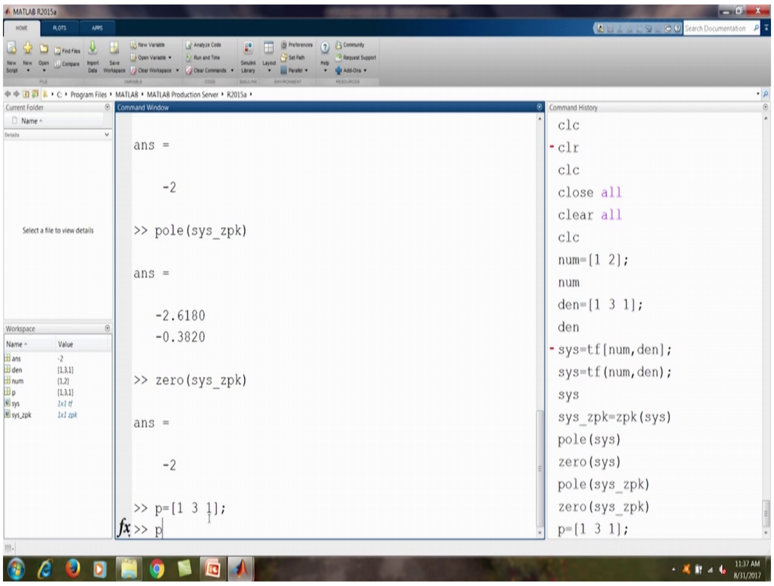
So, here the 0 is minus 2. Second thing is that, we can apply this same thing on the zpk system also. So, if he say pole zpk sys under scored zpk we can again get the pole.

(Refer Slide Time: 16:09)



So, we can either any form either the transfer function form or factored form, if he keep the pole command, we will get poles and if we give the 0 command; you can get the 0s. So, that is minus 2.

(Refer Slide Time: 16:40)



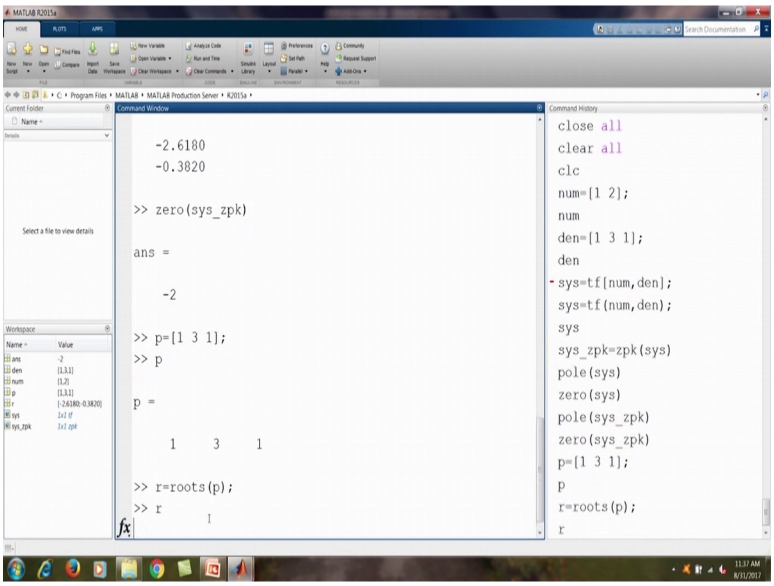
Now, one more thing that, we have this transfer function and we know that we are here to get the poles we are calculating the root of this polynomial.

So, we can; also get this using the roots of the polynomial. So, here we have a polynomial s is square plus 3 s plus 1 so if; so if we do not know that how to find we have to see for the help, so here is the help and in the help we can go to the documentation. So, in the documentation, if I want to find the roots we can search for the roots and enter. Here, is the roots; so polynomial roots we can go in the command how to use this? So, it is retain all this helps that polynomial roots the root concerns all's polynomial equation of the form P 1 s n plus P n s P 1 plus is equal to 0, that can polynomial equation contains a single variable with non negative exponents. To find the root of the types of equation is f 0. Now here the syntax r equal to roots p.

Now, roots p we can see here like example; that if he have this equation 3 x square minus 2 x minus 4 it is equal to 0; 3 x square minus 2 x minus 4 it is equal to 0; we have to write define the polynomial p equal to again the coefficients of the highest power that is 3, then the next lower power of x, that is minus 2, then the next lower power that is minus 4. Suppose if there is no x here this part is not here, we put here at in an place of minus 2 to be you put 0, you cannot leave this. So, 3 minus 2 minus 4, and then we say r equal to root of p, and we will find the root.

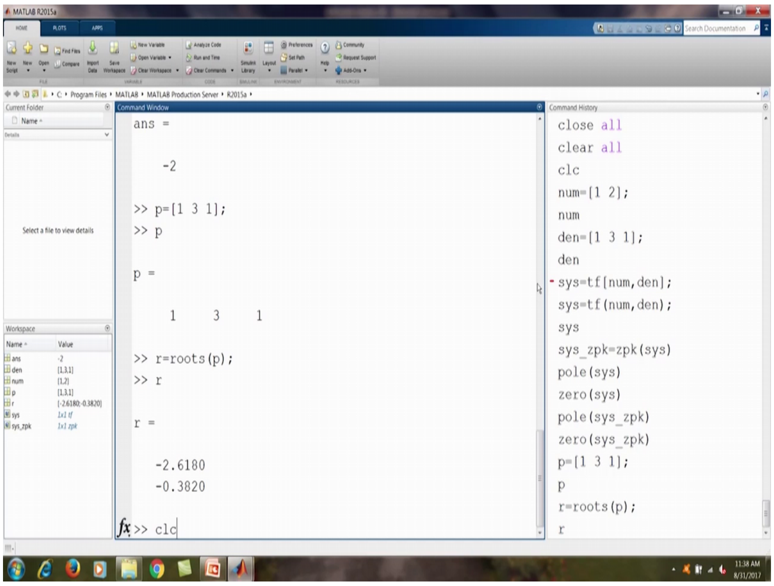
So, let us apply this here; so we have this is the polynomial s is square plus 3 s plus 1, and we have this polynomial 1 3 1, and we can find here; if he say r equal to root; so we first defined the polynomial p; p equal to you can define you can see how we can define; so p equal to 1 3 1; so we defined the polynomial so we can see here p equal to 1 3 1.

(Refer Slide Time: 20:15)



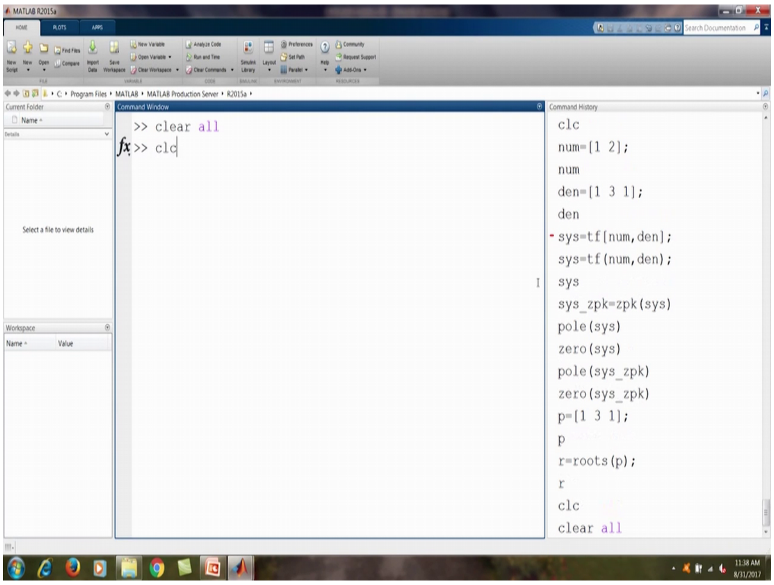
Now we say r equal to roots and p so; so we have got the roots and if he say r, you can see here we have got the roots minus 2.618 and minus 0.382.

(Refer Slide Time: 20:23)



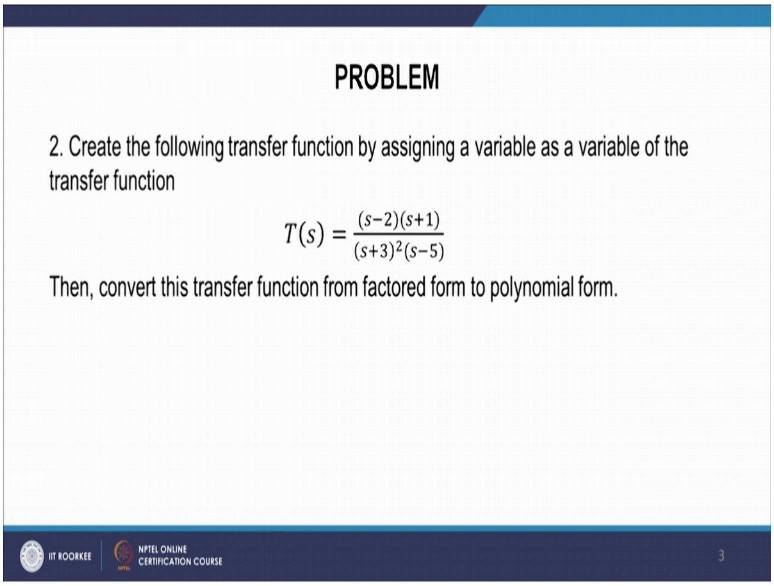
So, they are the poles of the system, because the poles are nothing, but the root‘s of the denominator or characteristics equation and the roots will give also the pole. So, we see how many ways we find the poles of the system of a transfer function. So, here now we want to go for the second problems and we want to clean this screen, because so we say clc; so we have cleaned in command window, if you also want to clean this workspace we can say clear all.

(Refer Slide Time: 21:05)



So, it will clear this workspace and clc so it will clear the window. Now, we want to go for the next problem here.

(Refer Slide Time: 21:23)

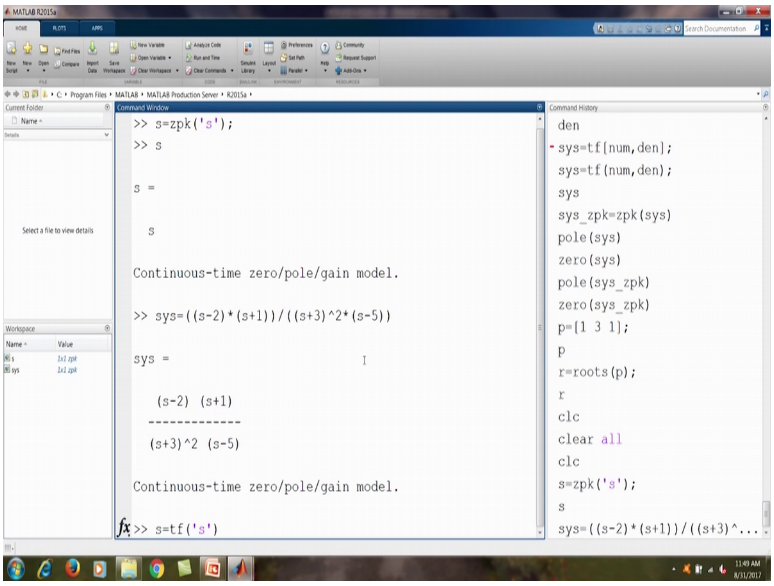


So, this problem 2, we want create the following function by assigning a variable as a variable of the transfer function.

So, here this is the transfer function factor form and then we want to convert the transfer function from the factored form to polynomial form. So, the transfer function we what;

this is the zpk form, first we have that is factored form we have to define a variable s, and then we have to write this in factored form and then convert in the polynomial form.

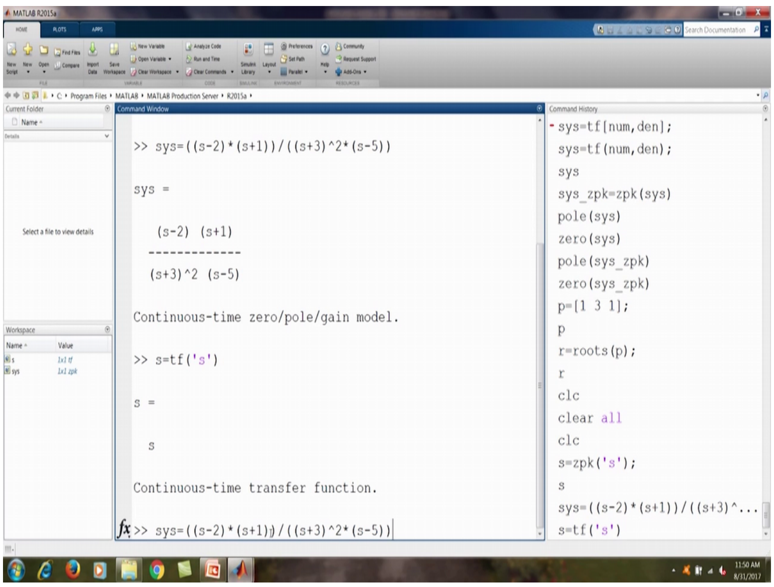
(Refer Slide Time: 22:10)



So, we can go in MATLAB so here we can define hear a variable s; that is zpk s. So, this is the; s is the variable of this modal get zpk and then we type the system sys equal to so s minus 2 into s plus 1 upon s plus 3 square into s minus 5 .

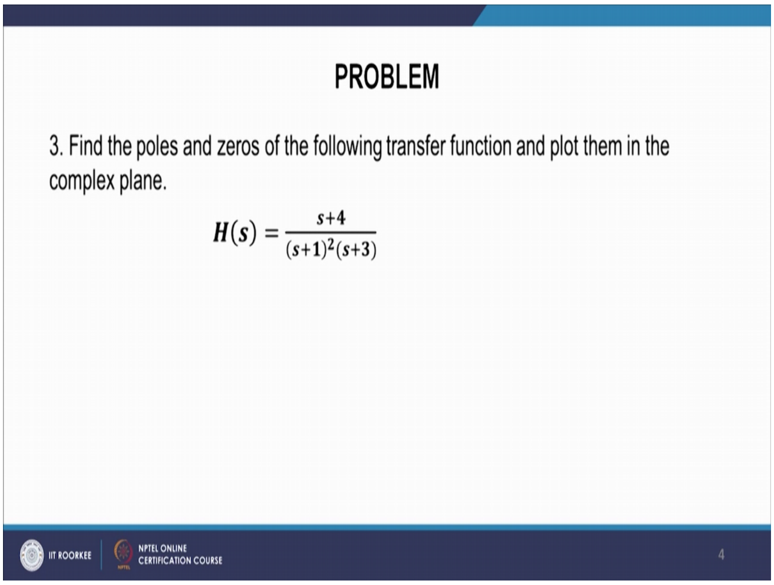
So, here we have defined sys that is s minus 2 s plus 1 s plus 3 square s minus 5. Now we defined the variable s variable of transfer function s is equal to t f, s .

(Refer Slide Time: 23:14)



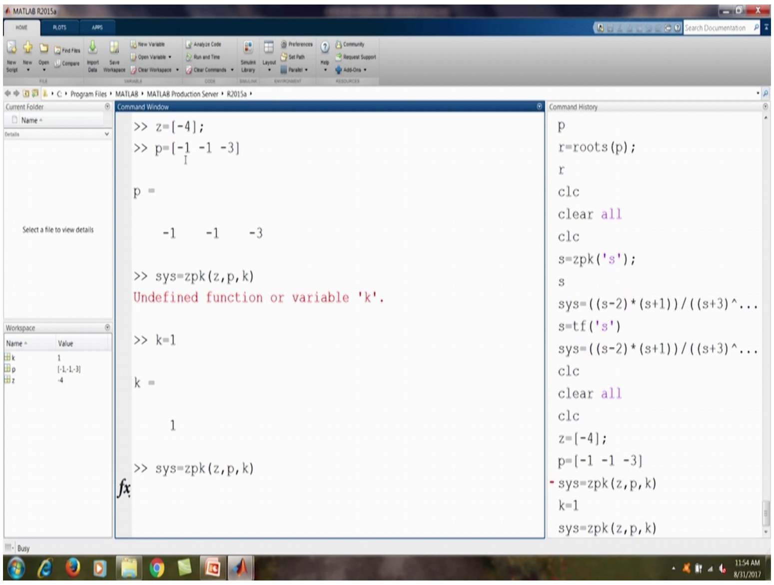
So, now, this is the s is the variable of transfer function earlier at write when variable of 0 pole of gain model. Now, we have to write sys 2 equal to; so we can also bring this. So, sys is equal to s minus 2 into s plus 1 so this. Now, if you do this we will get the transfer function. So, we have obtain the form factored form two polynomial form earlier form also factored form and here we have got the polynomial form of the transfer function. So, we now come to the ah; so, let us say clc command so be cleared this clean and clear all. So, we clear all.

(Refer Slide Time: 24:16)



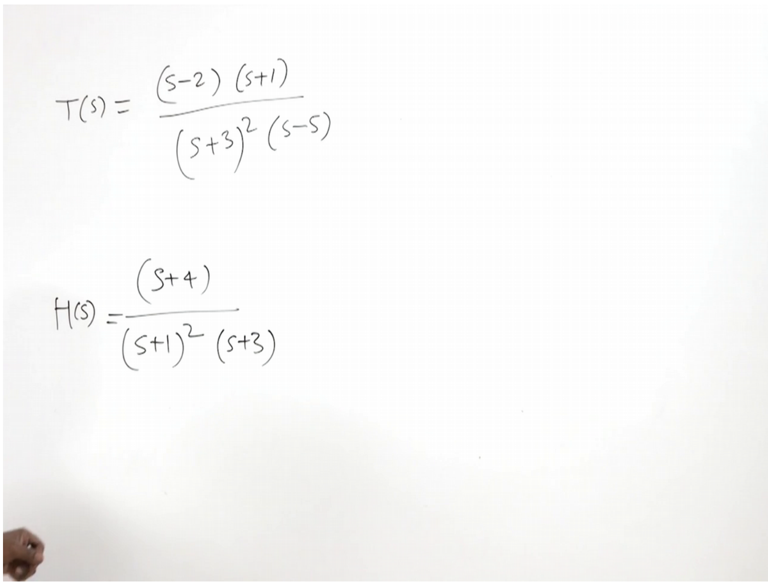
Now, we come to the next problem; that is problem number 3. Find the poles and zeros of the following transfer function and plot them in the complex plane. So, we have Hs equal to s plus 4 upon s plus 1 is square and s plus 3. So, we have to; find the poles and zeros. So, we go in the MATLAB.

(Refer Slide Time: 24:46)



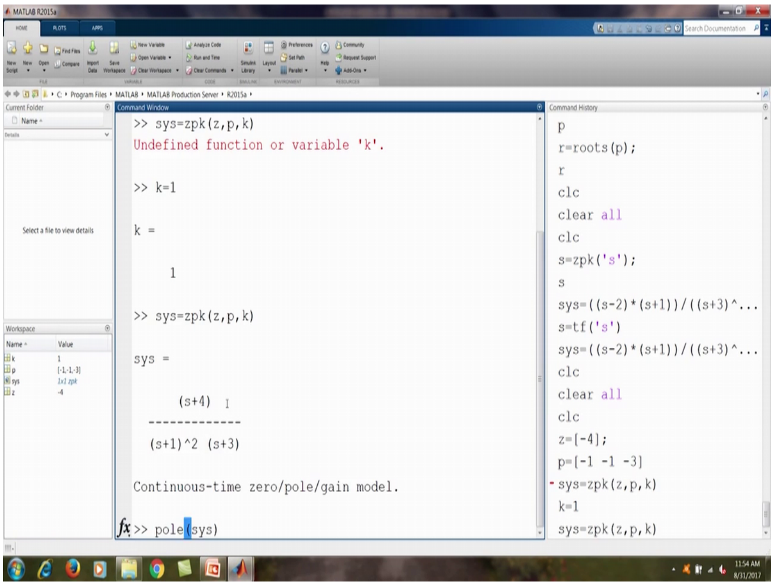
So, here we have; we can define the 0, z that is z variable; that is has 0 that is minus 4. So, this is the z is the variable that is 0 and then p is a variable that is. So, here we are we are writing that it is s plus 4 upon s plus 1 s square s plus 1 is square s plus 3.

(Refer Slide Time: 25:35)



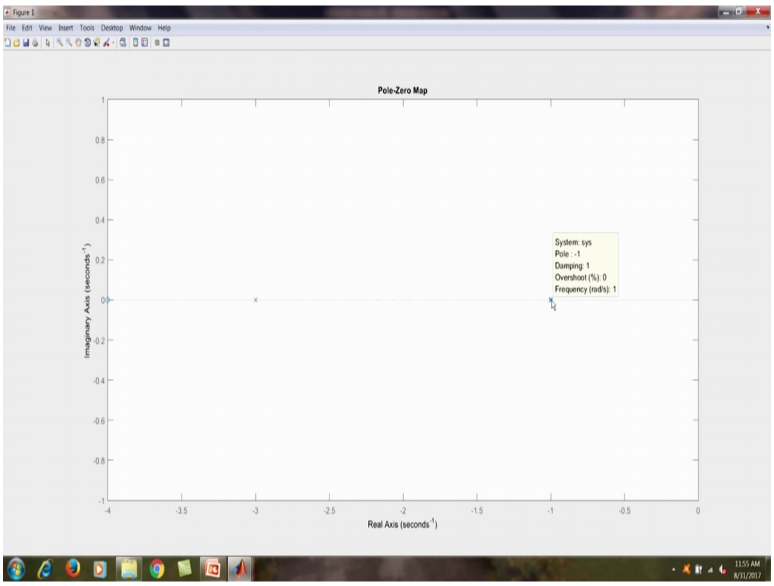
So, this is our Hs the transfer function. Now here we have zpk model. So, we write z as the polynomial. So, it contains here we are all ready writing this the 0s in minus 4; the p is minus 1 minus 1 minus 3. So, there are 3 poles at minus at minus 1 minus 1 minus 3. So, we can write sys equal to zpk z, p, k. So, we have new get zpk find the system. So, we are not defined k; k is the gain which is 1. Now we can again use this command and we will find this transfer function; that is in the zpk model or factored form.

(Refer Slide Time: 26:52)



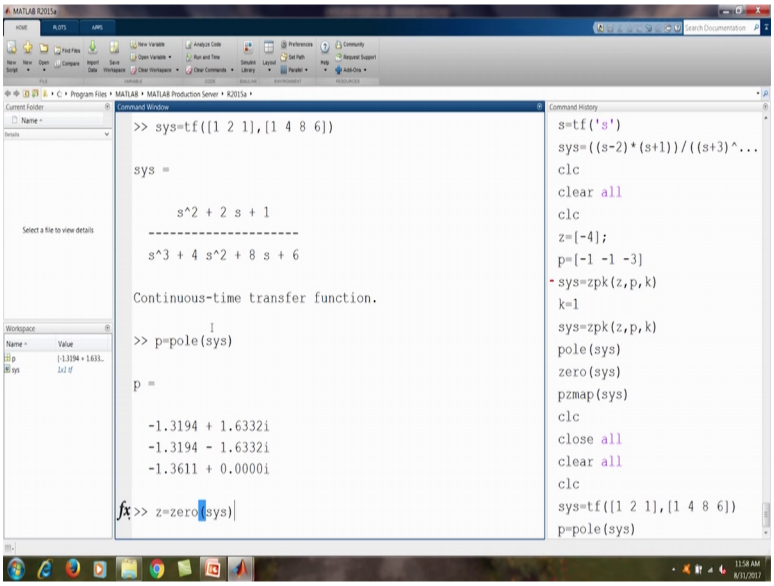
So, now, we can find pole. So, as we already know the poles and zeros, but ones we can create this in this form, we can find the pole of the system; that is minus 1 minus 3 minus 3 and 0 of the system. So, now, we want to plot this pzmap system. So, we want to plot this pole’s and zeros.

(Refer Slide Time: 27:37)



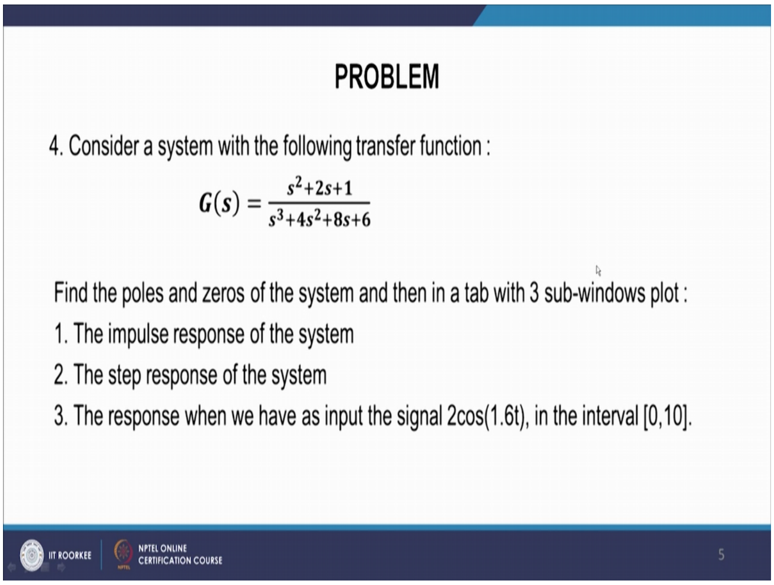
So, we can see here we have obtain this poles, so here we can see here this is the imaginary axis, this is real axles and there is pole; there are 2 pole at minus 1. So, they are over lapped here and here is 1 pole at minus 3 and there is 0 at minus 4. So, this is 10 we can see here a 0 at minus 4; this is the pole at minus 3 and here we have pole's two poles at this location; so this.

(Refer Slide Time: 28:25)



Now, we come to the next problem. Now, we have transfer function of a system, you to find the poles and zeros of the system and then we to find the impulse response and step response and response with some arbitrary input signal that is 2 cos of 1.6 t in the interval of 0 to 10.

(Refer Slide Time: 28:30)

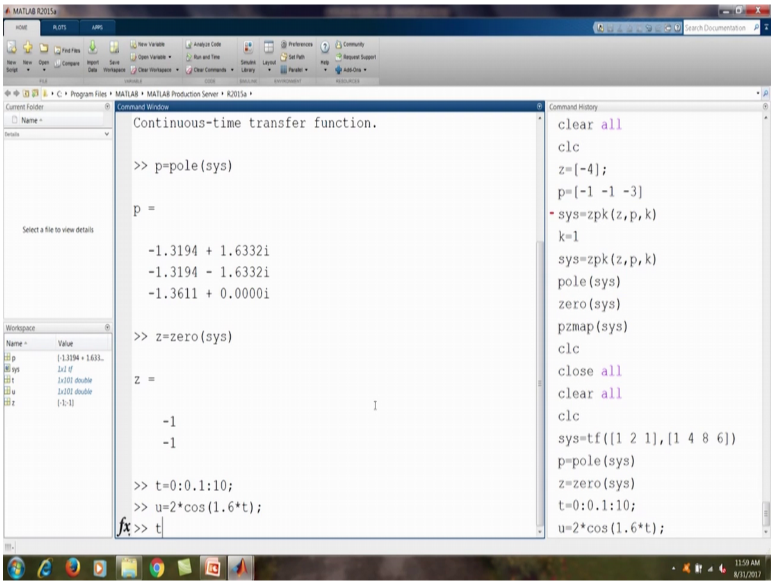


So, we have this system; that is transfer function Gs is equal to s square plus 2 s plus 1 upon s cube plus 4 s square plus 8 s plus 6. So, this is our system and here we have to find the response of impulse and step response of the system. So, here let us going the MAT LAB .

So, now we have to first find the system we can directly write system is equal to t f, and we know that numerator comma denominator. So, numerator here is; we have s square plus 2 s plus 1 so s is square coefficient is 1, then 2 s and then 1; so this is numerator comma denominator. So, denominator is s cube so 1 then 4 s square so it is 4, then 8 so 8 then 6 so 6 and so this is the system and we enter we will get sys s square plus 2 s plus 1 s cube plus 4 s square plus 8 s plus 6.

Now, we want find the pole's. So, we know that the p equal to pole sys. So, we have got the pole's. Now we want to find the zeros. So, z equal to 0 sys; we obtained the 0 that is minus 1 minus 1. So, there are 2 0s at the same location.

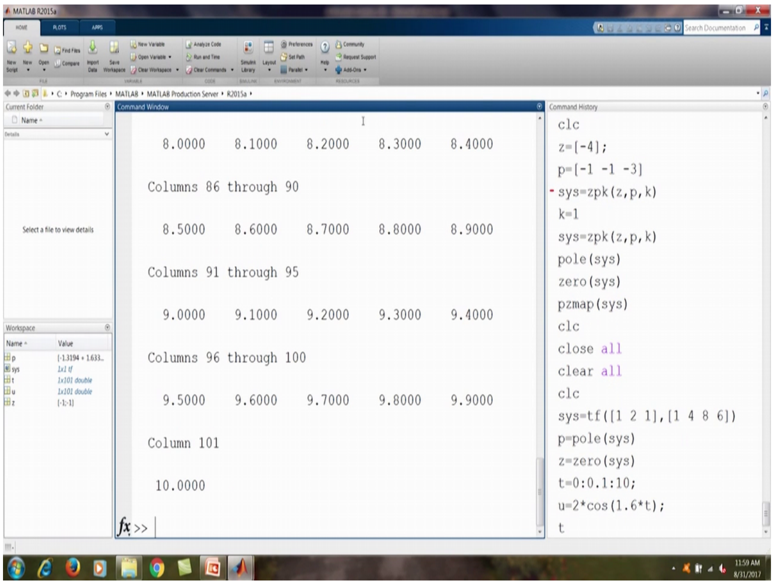
(Refer Slide Time: 30:47)



Now we defined the time "between" 0 to 10, so MAT LAB we have to give some time interval and this tell the time. So, we time is equal to 0 to 10, but we have to give the increment interval that is 0.1, so 0 to 0.1 to 10. So, this is the time we have given.

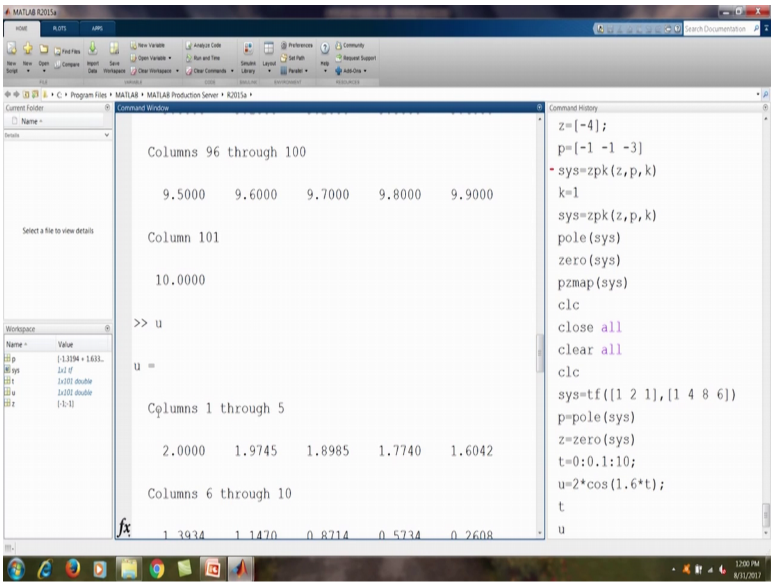
So, there is vector time t we have already created. Now, we defined because we have to find also the last part the arbitrary input u equal to 2 cos 1.6 t. So, we have 2 into cos 1.6 into t. So, now, we can see we have created input u. Now you can see what is t; so t is the time you can see here; we are starting time is 0, we have given the increment 0.1 till 10 seconds.

(Refer Slide Time: 32:09)



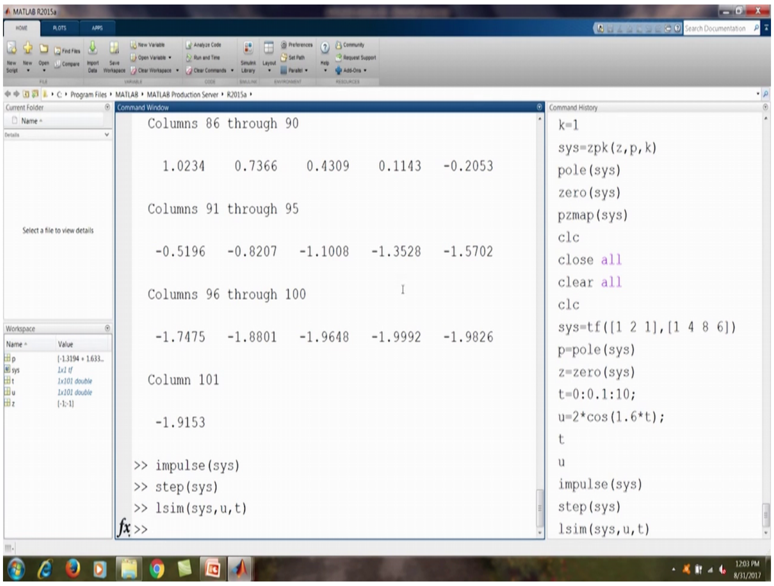
So, 0, 0.1 then increment a 0.1 will be make it .0.2. So, we are getting value of time 0, then 0.1, 0.2 and so on till 10 seconds. So, here we can say 9.9 and then 10. Now u is defined for each of these points with the function 2 cos 1.6 t, so u is the equal to again it is calculate; it has calculate the values of u.

(Refer Slide Time: 32:46)



So, you can see here, from t equal to 0, u is 2 and that you can see there is 2 cos 1.6 t and t equal to 0, so cos 0 is 1, so we will get maximum value 2 and then t will increase the value in value increase and vary and then we at 10 we are to going to get this minus 1 point; so we have got a input of like this.

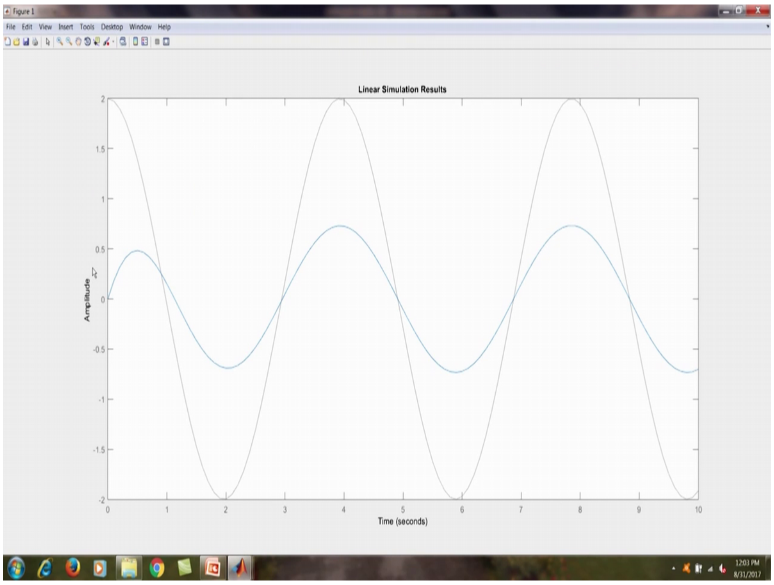
(Refer Slide Time: 33:07)



So, we have the time t and we have this cos and function. So, this is 2 and varying till here we have time t equal to 10 second and here t is equal to 0. So, this is our input. However, we have find the step in; impulse input first or a step input. So, let us first find impulse. So, we just write impulse and we say system sys. So, we have defined the system sys. So, impulse is we obtain here the impulse response.

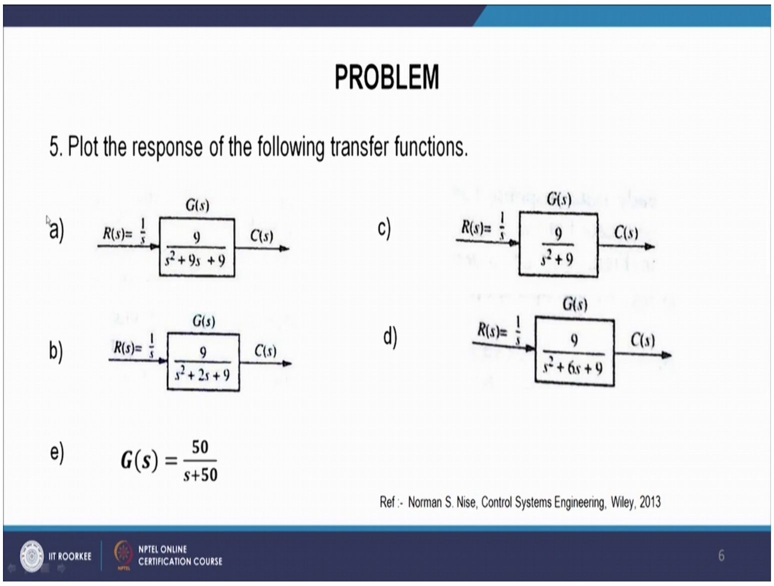
So, we are giving some impulse and we are the getting response. Now, we get by step by step and sys. So, we will get sys; so it is step response. So, we are giving the step input and we are giving the steps. So, here is some step response of the system, then we use the command lsim sys, u, t. So, this is lsim is doing the simulation under some arbitrary input u. So, this is the input and time and reactance and this is a system. So, we do lsim; so we will get the input .

(Refer Slide Time: 35:20)



So, you see we are getting here is the input that is 2 cos 1.6 t and this is the output we are getting here the output. So, here it shows that the system has some is out put that is below the input .

(Refer Slide Time: 35:45)



So, these are the some plot the response of the following transfer function. So, this are the transfer function 9 upon s is square plus 9 s; 9 s plus 9. So, let us continue this in the next lecture. So, let us stop and see you in the next lecture.

Thank you.