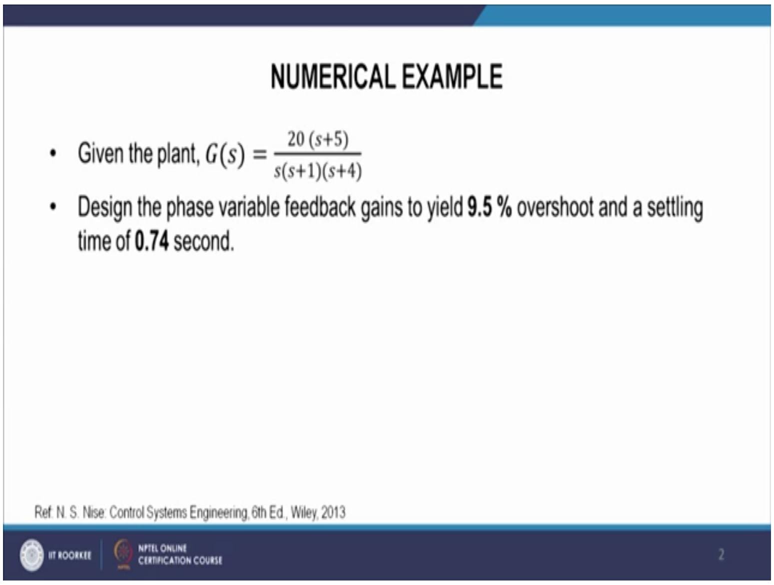
**Automatic Control**
**Dr. Anil Kumar**
**Department of Mechanical & Industrial Engineering**
**Indian Institute of Technology, Roorkee**

**Lecture – 35**
**Controller Design and Controllability**

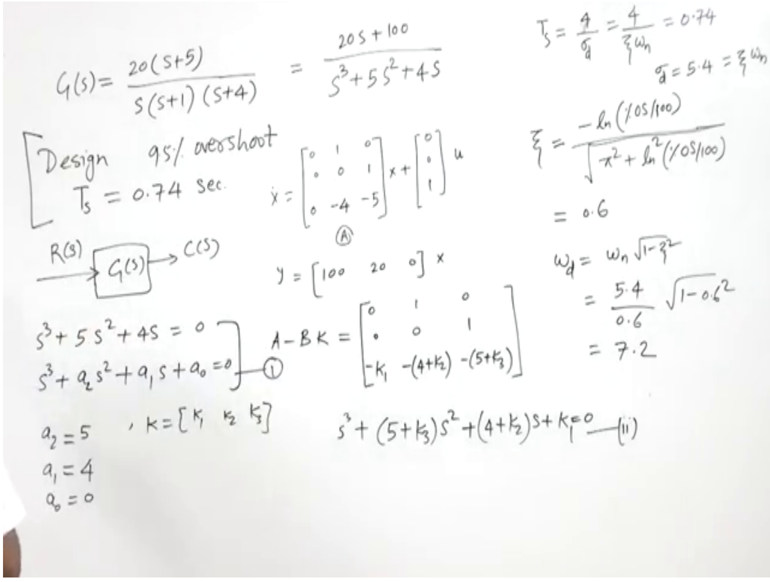
So, welcome to the lecture on State Space Method. In this lecture we will discuss a problem related to controller design and about controllability.

(Refer Slide Time: 00:43)




So, we discussed this problem that we are given a plant that is G s equal to 20 s plus 5 upon s plus 1 s plus 4 and we design the phase variable feedback gains to yield 9.5 percent overshoot and a settling time of 0.74 second.

(Refer Slide Time: 00:58)




So, we were discussing that we have this transfer function G s and we have this is the characteristic equation. So, we have this equation that is s cube plus 5 s square plus 4 s equal to 0. Now, we can write this as s q we can compare plus a 2 s square plus a 1 s plus a 0 equal to 0. So, if we compare this we will find the coefficients of this a 1, a 2, a 0.

So, here a 2 equal to 5, a 1 equal to 4 and a 0 equal to 0; so we have the system if we represent this system in the state space we can write x dot equal to 0 1 0 0 1 0 sorry 0 0 1 and here we can write reverse order with negative signs which is 0 and this is minus a 1. So, minus 4 and this is minus a 2. So, minus 5, this is matrix a. So, a x plus b. So, b matrix we have 0 0 1 and u and here y equal to so, y matrix come from here. So, 100 20 and 0 and here x. So, we know that c matrix we can write these by these coefficients.

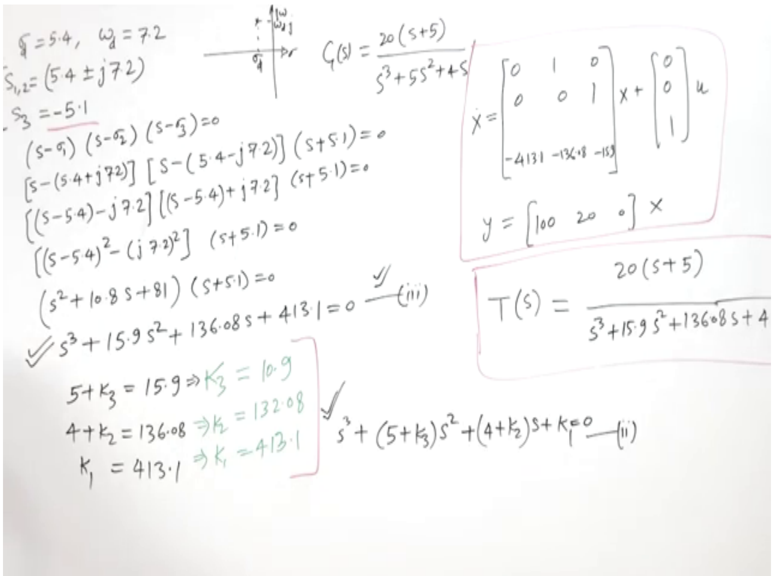
Now, if we write A minus BK matrix. So, A minus BK because we assume that there are. So, we there are 3 coefficients a 1, a 2 and a 0. So, we need 3 K value. So, K equal to K 1 K 2 and K 3. So, here we have K. So, here we have K that is equal to K 1, K 2, K 3. Now, this matrix we can write. So, A minus BK. So, we can write this as equal to. So, this is 3 by 3 matrix. So, 0 1 0 0 0 1 and this will be minus. So, here we will have K 3 sorry K 1 minus K 1 because here it is minus a 0 plus K 1. So, a 0 is 0. So, it is minus K 1 minus K 1 plus K 2, so, 4 plus K 2 and minus 5 plus K 3. So, this is A minus BK.

Now, the characteristic equation that is this will be changed. So, it is like s cube plus now here we will have 5 plus K 3 s square plus 4 plus K 2 s plus K 1 that is equal to 0, this is the new characteristic equation. Now, what is the design requirement? This is the design requirement. So, we have got two equations this is the first equation and this is the second equation.

So, this is the initial system and this is system we want. So, we want the values of K 1, K 2, K 3, ok. So, we have to select K 1, K 2, K 3 to obtain what objective these objective. So, here we have T s equal to 0.74. So, we know that here T s equal to pi sorry 4 upon sigma d or zeta omega n sigma d or 4 upon zeta omega n. So, that is 0.74. So, we can find sigma d equal to 5.4 or zeta omega n equal to 5.4.

Now, we have percentage overshoot we have to find the damping. So, damping is equal to. So, we have if we are given percent overshoot we can find a damping from here and we have percent overshoot 9.5. So, we put 9.5 and we will get the value of damping equal to 0.6 and here this is zeta omega n. So, here we can also find omega n and omega d equal to omega n root 1 minus zeta square and omega n equal to sigma d upon damping that is 0.6 into root 1 minus 0.6 square. So, we get omega d equal to 7.2.

(Refer Slide Time: 09:02)



So, now, we obtain that that we have this sigma d equal to 5.4 and omega d equal to 7.2 it means we to obtain 9.5 percent overshoot and damping equal to. So, that equivalent to damping equal to 0.6 and settling time equal to 0.7 for we need to put the poles at this real part and this imaginary part. So, we want to put this poles here somewhere. So, that we have this is omega d and this is sigma d omega d into j. So, we have 5.4 plus minus j

7.2. So, this these are the two poles, where they the two poles because this system has s cube. So, they are total three poles out of three the two poles complex poles would be this.

So, we have here let us say s 1 and 2 equal to this now what about s 3. So, the third pole where should I put. So, we know that here we have G s equal to s 20 s plus 5 upon this system that is. So, this 0 is at 5. So, if we put the third pole near to this 0 the pole 0 cancellation will take place and therefore, the higher order pole will not affect the response of the dominant poles.

So, therefore, let us put this third pole at near 5. So, let us say 5.1. So, now, what should be the resulting equation we have s minus sigma 1 into s minus sigma 2 into s minus sigma 3 equal to 0. So, this should be the characteristic equation if we have three poles, so, s minus 5.4 plus j 7.2 s minus 5.4 minus j 7.2 and here s minus 5.1. So, this is the characteristic equation. This is the desired characteristic equation because we this is the other desired pole locations. So, now, if we solve this equation we will obtain this equation as. So, we can have s minus 5.4 minus j 7.2 and this is s minus 5.4 plus j plus j 7.2 and this is s minus 5.1 equal to 0.

So, this we can write s minus 5.4 square and minus j 7.2 square into s minus 5.1 that is equal to 0 and so, here we can obtain this as s square plus 10.8 s plus 81 into s. So, sorry this is plus because here this is minus 1, we are putting at negative side because here the 0 is at s equal to minus 5. So, this s 3 is minus 5.1. So, we have here s minus minus 5.1. So, it is plus. So, s plus 5.1 that is equal to 0. So, we finally, obtain this as s cube plus 15.9 s square plus 136.08 s plus 413.1 equal to 0.

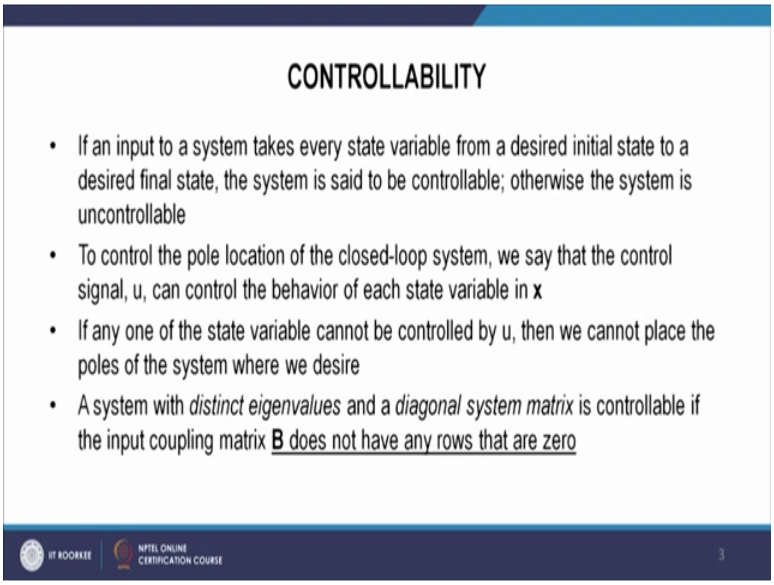
So, this is equation number third, 3 let us say now, this is the desired characteristic equation because you that will put the poles at these locations and this is also the desired characteristic equation that will set these values of K 1, K 2, K 3 to put the poles at the desired location. So, these two equation can be compared and we can find the values of K 1 required values of K 1, K 2, K 3 that will put these poles put the system at this locations.

So, we compare these coefficients. So, here if we compare s square so, 5 plus K 3 equal to 15.9 and 4 plus K 2 equal to 136.08 and K 1 equal to 413.1. So, from here we can obtain that K 3 equal to 10.9 from here we can obtain K 2 equal to 132.08 and the K 1 equal to 413.1 and here we have the 0 is the same.

So, now, what is the new system. So, here the new equation is X dot equal to a x. So, 0 1 0 0 0 1 and here we have. So, minus 413.1 minus 136.08 and minus 15.9 into x plus 0 0 1 u and y equal to c x and the transfer function because this is the characteristic equation we can write transfer function. So, transfer function is 20 s plus 5 upon here we have this is the characteristic equation. So, we can write s cube plus 15.9 s square plus 136.08 is plus 413.1. So, these are the systems. So, this is in the state space we have obtained this system and we have obtained the values of feedback gains that will be feedback and they will put all the 3 poles at desired location.

So, here we have taken this third pole at minus 5.1. We can also select this value may be very higher. So, we can put this pole at very far from the dominant real part of the dominant pole maybe at s 3 equal to 30 or 50 and we can find the value of gains that will corresponding this value. So, we can put the poles all these system poles at the desired locations. Here we applied the pole 0 cancellation, but here we could say that condition that the pole third pole is very far. So, it is not affecting much to the dominant poles response and this is the transfer function. So, the system is the same we have two different representation one is a transfer function that is the frequency domain approach and this is the time domain approach that is state space. So, now, we have discussed for these systems how to find this.

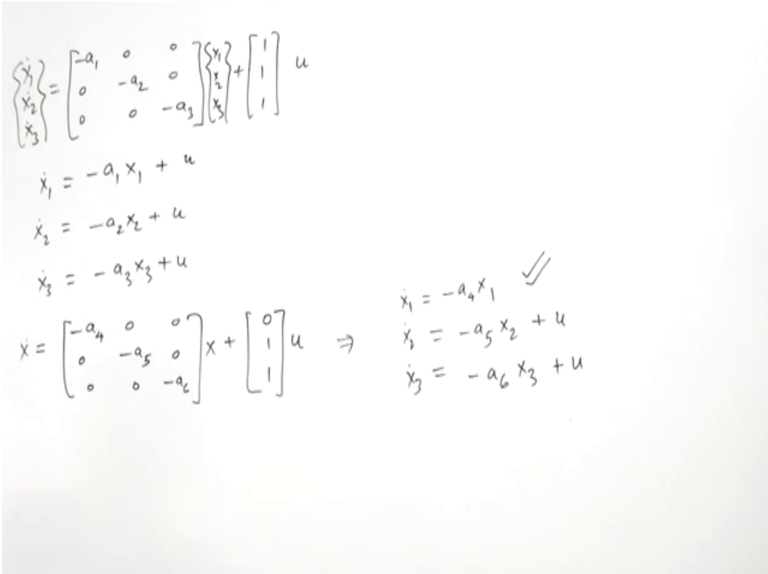
(Refer Slide Time: 19:42)




So, now, we will discuss the concept of controllability. So, when we define the state variables. So, we said that the state variable is takes the system from one state to other when we apply some input or initial conditions. So, if we input to a system takes every state variable from a desired initial state to some desired final state then the system is said to be controllable. If there is no effect of input on any state variables means if we are giving input, but the state variable is not affected or it is not changed. So, it means the system is uncontrollable. Therefore, to control the pole location because we were trying to place the poles at desired location.

So, if we want to do this we are controlling the pole location of the closed loop system by giving some input you are giving some feedback gain as an input. So, if we say that the control signal u we want to through this control signal if we want to control the pole location, so that we can control the behavior of each state variable in x. If any of the state variable cannot be controlled by u, then we cannot place the poles of the system where we desire.

(Refer Slide Time: 21:58)

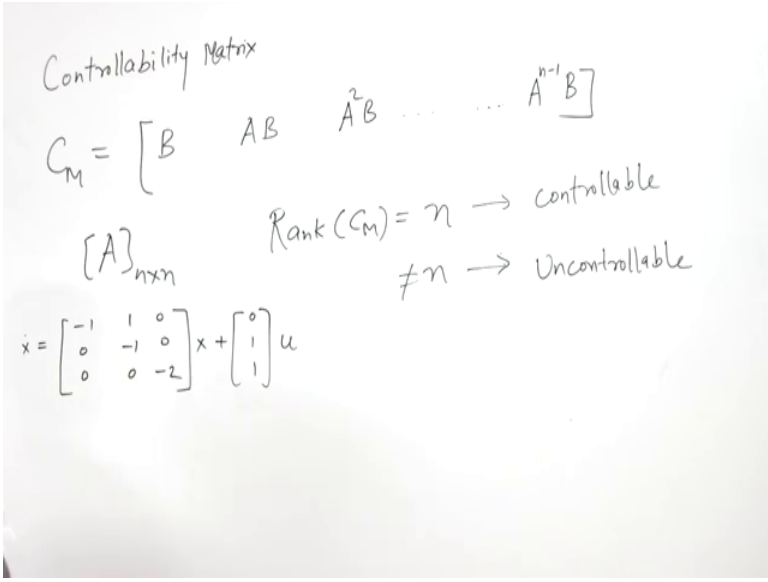


So, for example, we let us take some system like. So, which we have this system x dot equal to minus a 1 0 0, 0 minus a 2 0, 0 0 minus a 3 x plus 1 1 1 u. Now, what is this system? So, if let us say here we write this x 1, x 2, x 3. So, here we can write x 1 dot equal to minus a 1 x 1 plus u here we can write x 2 dot because here it is x 1 dot x 2 dot and x 3 dot. So, here x 2 dot we can write minus a 2 x 2 plus u, then x 3 dot equal to minus a 3 x 3 plus u.

So, here we see that if we are giving the input we are getting the state variable is affected by this input, because they are function of u. So, here we can say that the system is controllable because each state variable is getting affected and by the input u. Now, we take another system let us say x dot equal to minus a 4 0 0 0 minus a 5 0 0 0 minus a 6 and here we have 0 1 1 and here is u. Now, if we want to write this equation we can write x 1 dot equal to minus a 4 x 1 x 2 dot equal to minus a 5 x 2 plus u and x 3 dot equal to minus a 6 x 3 plus u. So, here we see that this state variable is not affected from the input. So, x 1 is not controlled by input u and therefore, this state variable is not affected by u and so, the system is uncontrollable.

So, here we see that the system is have distinct Eigenvalues because there that is a diagonal element. So, the Eigenvalues are distinct and so, we can say that a system with distinct Eigenvalues and a diagonal system matrix is controllable if the input coupling matrix b does not have any rows that are 0. So, if we have this kind of system diagonal system we have if any row of this b matrix because this is b. So, b matrix is 0 then the system is uncontrollable. However, if we have the repetitive poles then if the system has multiple poles it does not have distinct poles, then how to find the controllability of a system. So, there the controllability is defined.

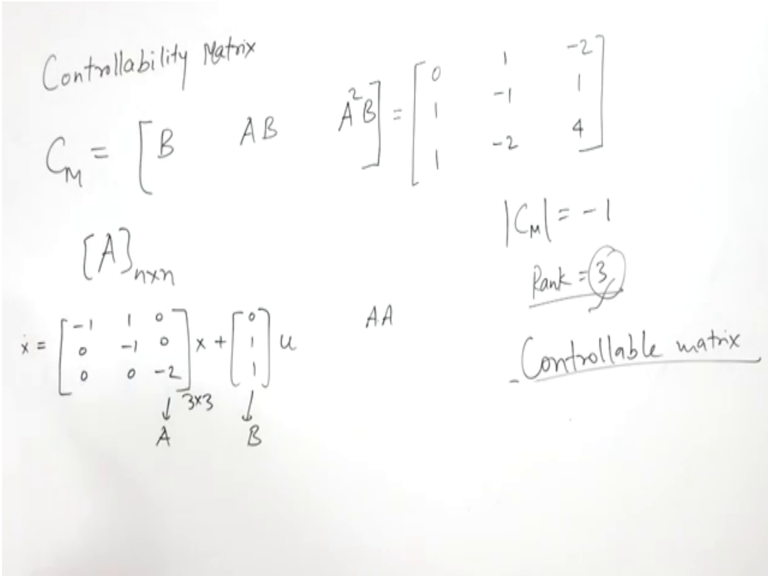
(Refer Slide Time: 26:17)




So, to find controllability of such a system we have to find the controllability matrix. So, we call it a controllability matrix C m and C m equal to B AB A square B and here it is A n minus 1 B and if the system is of nth order, so, if our matrix A is n by n matrix. So, our system is of nth order so, the rank of C m equal to n. So, it is controllable otherwise it is not equal to n then it is uncontrollable. So, for any system we can use this method to find the controllability. So, first we calculate the controllability matrix. So, let us take one example. So, if we have a system like x dot equal to minus 1 1 0 0 minus 1 0 0 0 minus 2 x plus 0 1 1 u.

Now, we can see that the here at multiple poles at minus 1. So, now, we calculate the C m.

(Refer Slide Time: 28:49)




So, we have to calculate the C m. So, this we have to calculate. So, here B is now we can complete this. So, B is this. So, this is A that is 3 by 3. So, A and this is B. So, here we have B 0 1 1 now we calculate A B. So, A into B we can calculate. So, we multiply this A matrix to B matrix and we will get 1 minus 1 minus 2 and when we calculate A square B we have to first calculate A into A and then we calculate this into B. So, we can get A square B. So, A square B we calculate we will get minus 2 1 4. So, so, determinant the C m equal to minus 1 and this rank is 3 because this is it is rank is 3 and that is equal to the order of the system that is n. So, this is controllable matrix. So, here just by saying this row we cannot say because here the first row of the B matrix was 0, but the session is controllable.

So, the examples discussed in this lecture were taken from the book of Norman S. Nise, Control Systems Engineering. So, I thank you for attending this lecture and let us see in the next lecture.

Thank you.