# Automatic Control

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## Lecture – 34
## Controller Design

So, welcome to the lecture on state space method. In this lecture we will discuss about controller design using state space method. So, if we recall the design in frequency domain using root locus techniques. So, in case of root locus technique, we were trying to change the gain so that the root locus could pass through the dominant poles that will give us the required tangent responses.

And we used the compensation techniques so that if the root locus is not passing through the dominant poles just by varying the gains, we can put some compensation poles and 0s that can force the root locus to pass through the dominant poles. Then we are more focusing on the complex pole poles that is the 2 poles, but for higher poles we had no any control. We were not able to control the system pole that the that has the higher than 2.

So, the third pole if it was very far, it resulted to be very far, then we could approximate the second order assumption. If it is close to the pole, we were expecting that there are some 0 that will cancel that pole. And so, the effect of higher order police reduced. In case of state space method, we get the possibility to control each pole of the system by designing some parameters that can one parameter for each pole.

For example, if we have a transfer function G S.

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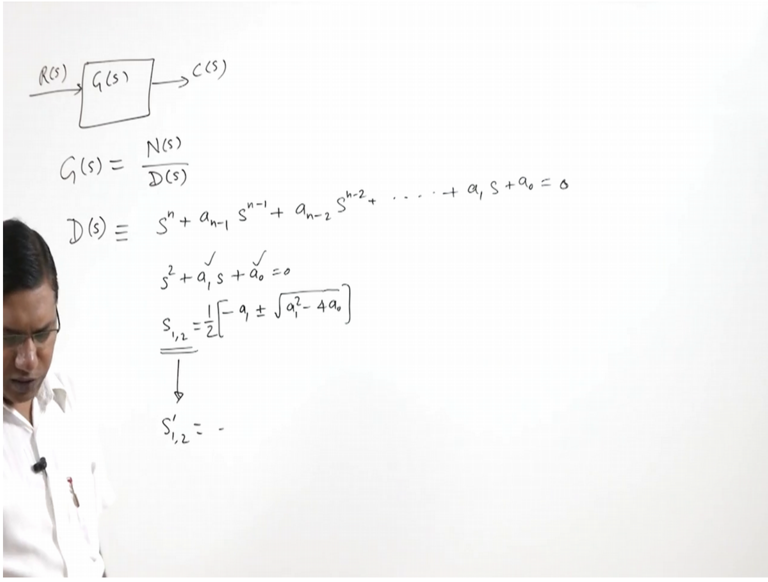


So, G S could be some numerator and some denominator. So, this D S could be a characteristic, it is a characteristic equation that is function of S. So, S power n plus a n minus 1, S n minus 1 plus a n minus 2, S n minus 2 plus plus a 1 S plus a 0; that is so, so, equal to 0, this will give us the n number of poles of the system.

So, this is a equivalent transfer function of a feedback system. So, it is equivalent closed loop transfer function G S. And so, if we take for example, S square plus a 1 S plus a 0 equal to 0. So, if you want to calculate these poles; so, S 1 2 equal to so these are the 2 poles of the system. So, we can see that, in these poles, we have the coefficients of S coefficient of S bar 0 that is constant term. And they govern the value of the poles.

So, if we want to get these poles to reach some another location, some other values. So, we have to change this a 1 and a 0. It means that, we have to change these coefficients of these characteristic equations. So, if we can, we want to change the n poles of the system or of the characteristic equation, we need n parameters one parameter to adjust each coefficient.

So, if we have n parameters for one for each of these. So, here if we have 2 parameters one can change a 1 other can change a 0. So, we will be able to change these poles. So, that we can reach to the desired pole location, in case of root locus techniques or design using root locus, we had this limitation that we were only able to design for 2 poles, that is complex poles, but for highest poles we were not having any method to design or to place those poles at desired location.

In case of state space method, we have this capability that we can put or we can place each pole at the desired location. So, we will discuss today and state dispensed method involves most mostly the matrix algebra, and that is supported by several software's like MATLAB and others so, it is more useful using the software's, and we will discuss about MATLAB in the last week of this course.

So, let us talk how to design this controller for the system so that we can keep each pole to the required location. So, we have this is the nth order feedback systems and there are n coefficients.

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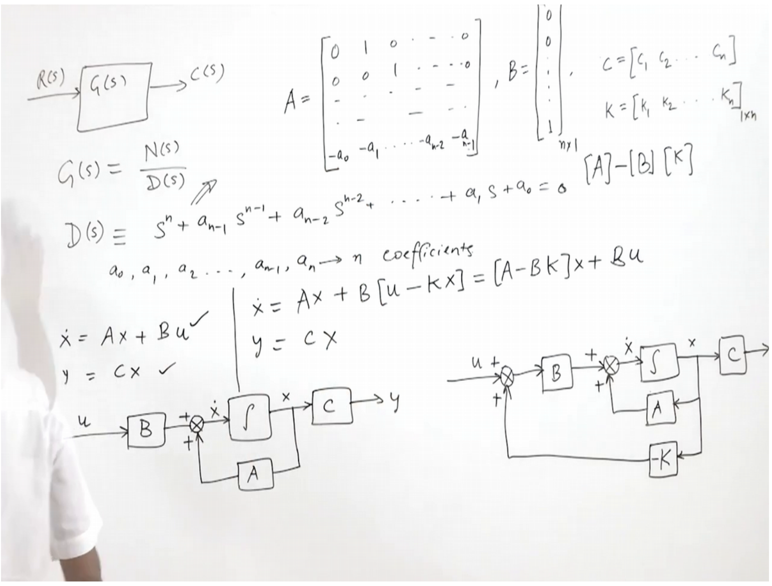


So, a 0, a 1, a 2, a n minus 1 and a n so, there are n coefficients of this characteristic equation.

Now, if we want this system to show in state space so, we know that the in-state space we represent X dot equal to Ax plus Bu, and y equal to CX. Here the Dv we assume that we are not giving any value to D. So, D is 0, we are not giving any feedback to output from the input so, we take this case.

So, what we do? We have this system, if we represent this system. So, we can represent as here we have B, this is u, there is X dot, there is x. So, here we have X dot equal to A into x plus B into u. So, this equation, and here y equal to C into X. So, this output equation. So, this is a straight equation this is output equation.

Now, in order to design the controller so that it can place it can control each of these coefficients. So, what we do? We give this state variable as a feedback to the input. So, we give this as a feedback to the input. So, here what we do? We have here this u B, now we are going to give this state to here at the input with some parameters gain parameters K, minus K K So, we are going to give this.

So now what will happen that; we are given these parameters, these case these values of K K we can adjust. So, we give this as input so that it will change these values of these coefficients and we can see how. So now, the state equation will be change. So, what will be the state equation? We can see X dot here will be A x plus now B and here is u plus minus K x. So, we can write minus K K x.

So, this we can write A minus B K So, A minus B K x plus B u. So, here this matrix now A is A minus B K and with changing the K K we can change this matrix. Now y equal to CX so, now, this system we can present in state space form. So, we can write this A as 0 1 0 0, then 0 0 1 0. And so, we can write this last row will collect these coefficients in negative and reverse order.

So, minus a 0 will be here, then minus a 1, then here minus a n minus 2, and minus a n minus 1 so, these terms will be here; so, this is matrix A. So, we can directly I have written directly because we already know how to write a transfer function to state space matrix. Now the matrix B; B we know that we had we have only this input u. So, we write B 0 0 here 1 so, into u. So, we have input here now let us see so, we assume that matrix C is their elements are C 1 C 2 and C n.

Now, we assume that we have matrix K So, K is having this K 1 K 2 K n. So, n parameters that can help to adjust this. So now, here if we could now for this system the matrix A minus B K, matrix A minus B K we can write if you want to write matrix A minus B K So, a minus B K is we have this matrix. So, there will be no any effect. So, B into K we will calculate first, matrix B into K.

So, we know that this is n into one, and this is one into n. So, we will get n into n matrix, and then we compute A minus B K So, this matrix minus this matrix. So, what will happen? We have we will have the coefficients K 1 K 2 K and they will be minus here.

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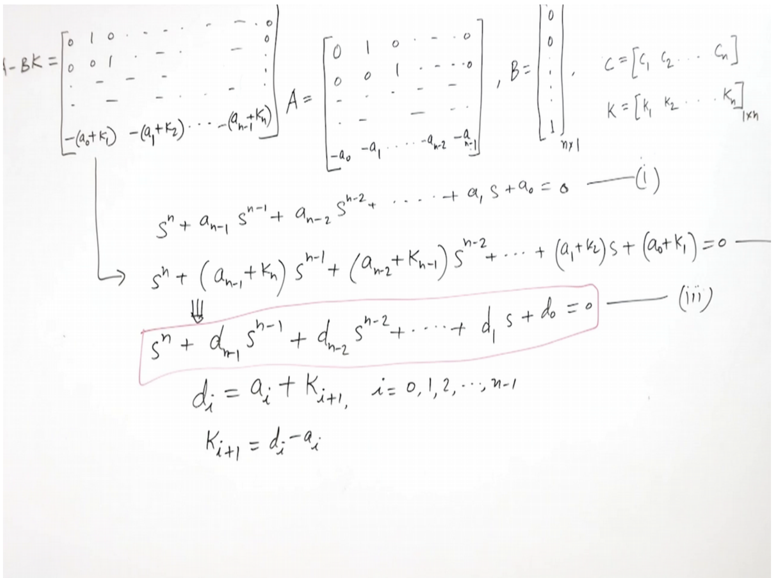


So, so, here a minus B K, this matrix we can write. So, 0 1 0 0 then 0 0 1 the last row will be minus a 0-minus K 1. So, minus a 0 plus K 1. Then second term will be minus a 0 plus K 2 minus a 1 plus K 2 sorry, here it is a 1 the second term.

And then the last term will be minus a n minus 1 plus K n so, this is the matrix. So, this is equation 1. And now from this matrix we can write the characteristic equation that is a power n plus, now a n minus 1 is a n minus 1 plus K in S n minus 1 plus a n minus 2 plus K n minus 1, Sn minus 2 plus plus a 1 plus K 2 S plus a 0 plus K 1. That is equal to 0 this is equation number 2.

So, this is the new characteristic equation when we have these parameters K So, we can rewrite this equation as Sn plus d d n minus 1 Sn minus 1 plus d and minus 2 Sn minus 2 plus d and d 1 S plus t 0, that is equal to 0. Where d i equal to a i plus K i plus 1 for I equal to 0 to n minus 1. And so, here Ki plus 1 equal to d i minus ai.

So, we have found this equation, so now we have n values of K and each K can change these coefficients these earlier coefficient. So, we can select the values of K such that these coefficient will be changed and each pole can be placed at the desired location. So, we will take one example and we can discuss about this application.

So, let us take one example. So, before that so, here we follow this. So, design via state space we understand the differences between root locus technique.

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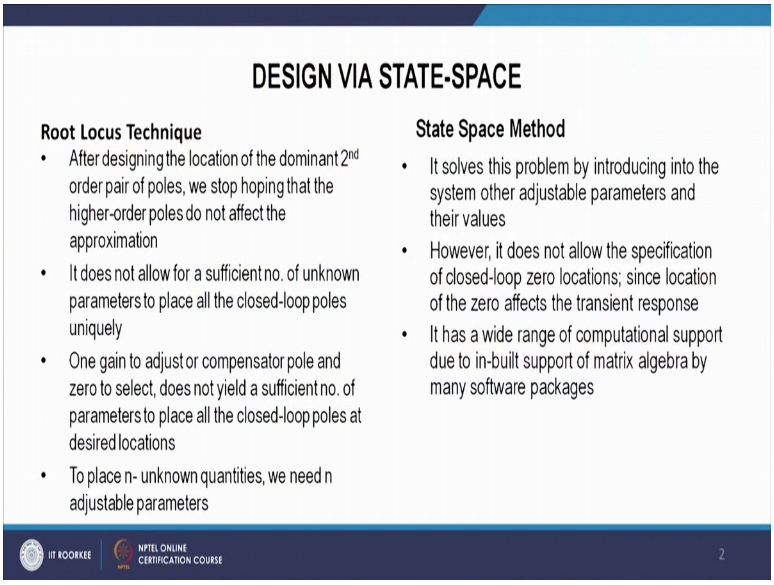


And state space method so, here after designing the location of the dominant second order pair of poles we stopped hoping that the higher order poles do not affect the approximation. The root locus technique does not allow for a sufficient number of unknown parameters to place all the closed loop poles uniquely. One gain to adjust or compensator pole and 0 to select does not yield a sufficient number of parameters to place all the second order poles at desired locations.

So, to place an unknown country we need n adjustable parameters. So, we have found the n adjustable parameters using the state space. But here we see that, the state space method does not allow the specification of closed loop 0 locations. So, here we cannot put the local 0's, but we are able to put only the poles.

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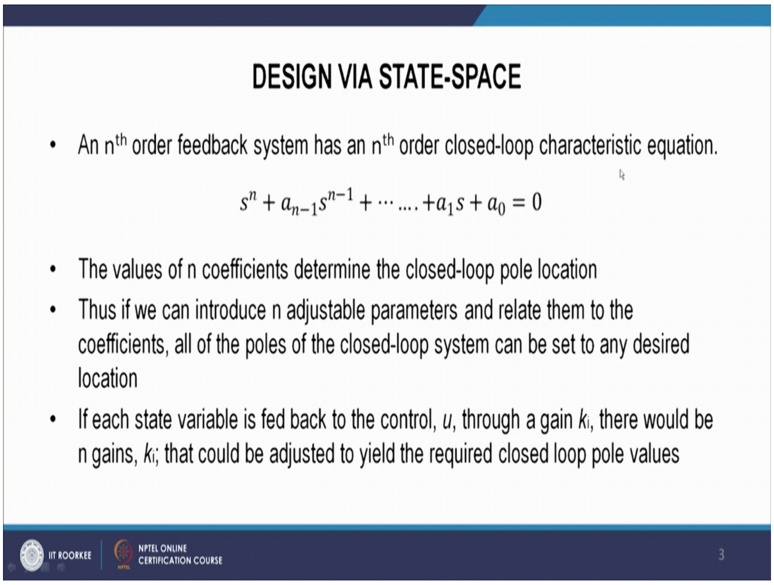


So, we have found these values.

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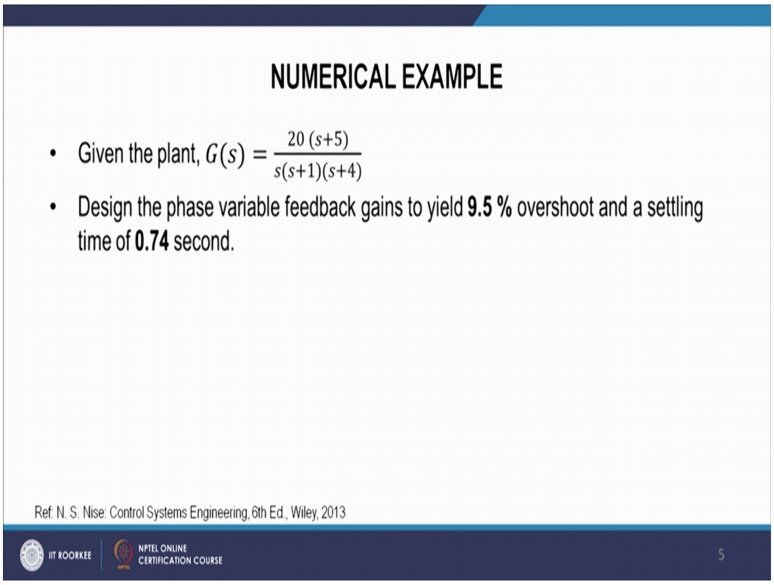


Now we come to this numerical example. So, here the numerical example is that a plant is given, that is G S equal to so, we are given G S equal to 2 s plus 5 upon s s plus 1 s plus 4.

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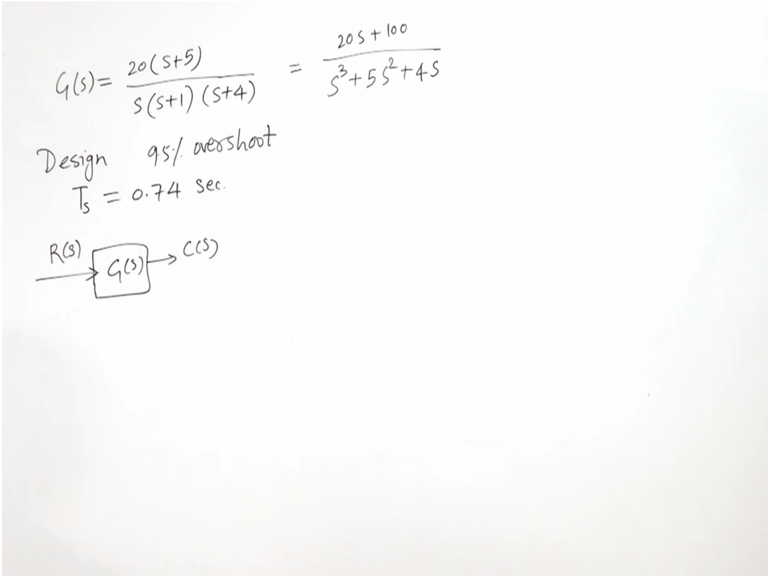


Now we have to design the phase variable feed by gains. So, that within the 9.5 percent overshoot design. So, that we get 9.5 percent overshoot. And the settling time equal to 0.74 seconds. So, we get a settling time T S 0.74 second.

So, we have to find those gains K value of K so that we can; so now, here we know that here we have the system like G S R S CS. So, we are given here the G s, and what is the characteristic equation? So, we can see here, we have this we can write this equal to 20 s plus 100 upon.

So, we solve this s s plus 1 s plus 4. So, we will find s cube plus so, we find this equation. So, this is the characteristic equation, and from on this problem we will apply the the theory that we developed in this lecture. And we will continue this problem in the next lecture. So, see you in the next lecture.

Thank you.