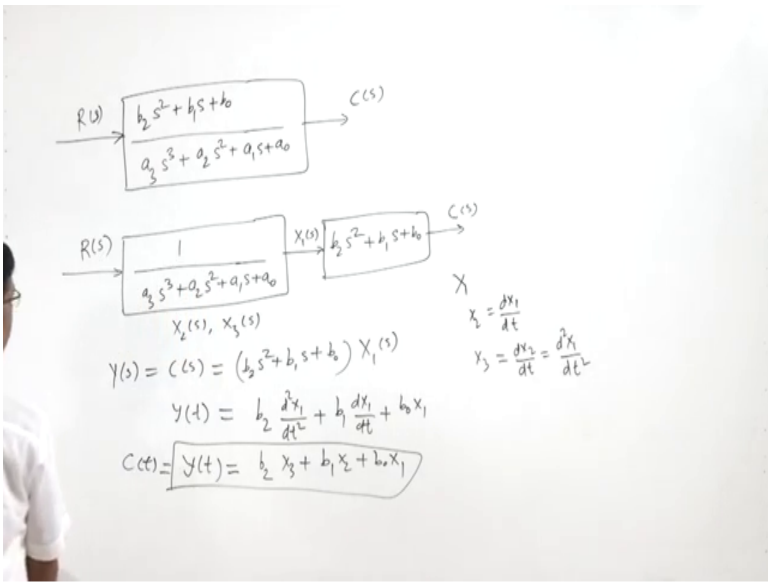
Automatic Control
Dr. Anil Kumar
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture – 33
Converting from State Space to Transfer Function

So, welcome to the lecture on the state space method. So, in this lecture we will continue with one example the conversion from transfer function to state space, as well as we will discuss also, how to convert from state space to transfer function. So, we were discussing that when we have a transfer function, we first convert this transfer function to a differential equation by taking the inverse Laplace transform.

And then we select the state variable as phase variables. So, we select one state variable the output, and then the next state variables will be the derivatives of the variables called phase variables. So, we can write the state space state equation and output equation for a transfer function. Now we take one case fine we have the numerator, there is a transfer function in the numerator as well. So, how to deal? So, if we have a the this case.

(Refer Slide Time: 01:40)



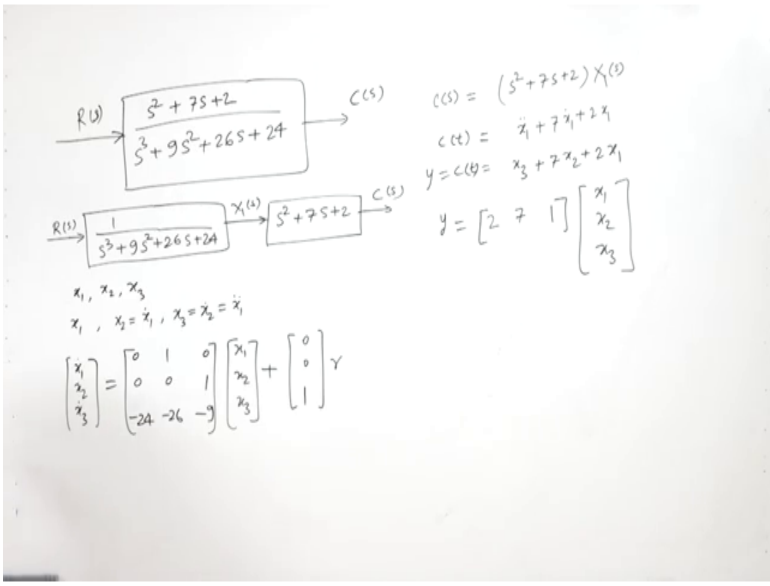
So, we have here, this is the numerator of the transfer function and this is. So, earlier we have only deal with there was no any function on the numerator, but it was taken with some constant parameter. But now it is function of S. So, when we have transfer function has a polynomial in numerator, that is of order less than the denominator, we can separate into 2 parts. So, if this is our output C S and this is R S, we can write this like we can separate it. So, first we can write as only denominator part, and the output of this part would be input to the other part. That is the numerator part b 2 S square plus b 1 S plus b 0.

Let us say this is x 1 S and this is R S. And it is a internal variable here is x 2 S and x 3 S, and x 1 S is the output of this part. So, here y S equal to C S, that is equal to b 2 S square plus b 1 S plus b 0 into x 1 S. And we take the Laplace transform with the initial conditions 0 of this we will get b 2 into d square x 1 by dt square plus b 1 d x 1 by dt plus b 0 x 1.

So, here we know that, we can write because this if we have x 1 as state variable we have selected. So, x 2 equal to dx 1 by dt, the next state variable. And x 3 equal to dx 2 by dt that is d square x 1 by dt square. So, we can write here as b 2 into x 3 plus b 1 into x 2 plus b 0 into x 1.

So, we can represent this output in terms of the state variables that we already defined in this first part. So, let us take one example. So, we can see that the final output so, this is csr. So, we here this is equal to ct we can say also. So, so we see that here this is only collecting the state variables that were already defined in this first part. So, here b 2 x 3 plus b 1 x 2 plus b 0 x 1 now, let us take one example.

(Refer Slide Time: 06:38)



So, we have an example. So, we have S square plus 7 S plus 2 upon S cube plus 9 S square plus 26 S plus 24. So, this is the transfer function having some numerator polynomial in the numerator.

So, we can represent this as R S 1 upon S q plus 9 S square plus 26 S plus 24. And this is x 1 S and there is second block, that is S square plus 7 S plus 2, and this is C S. Now we know that this part, we have already learned how to write the state equation.

So, if we have x 1, x 2 and x 3, the state variables where x 1 equal to the output of this and x 2 equal to x 1 dot and x 3 equal to x 2 dot, and that is equal to x 1 double dot. So, we can write this as x 1 dot x 2 dot and x 3 dot. These are the state equations we are writing.

So, we had earlier we solved for the same system except here was 24, and this 24 was coming here to the input. But here now we have input R and 0 is 0 and 1, because now here is 1, so, we will have here one. And we know that here we have x 1 dot equal to x 2.

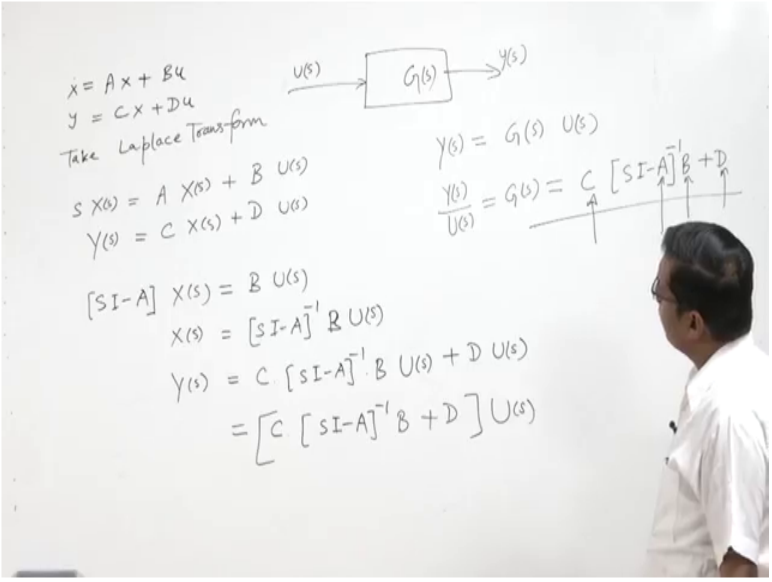
So, 0 1 0 and x 2 dot equal to x 3, 0 0 1, and this last row is these elements are reversed with negative sign. So, minus 24 minus 26 and minus 9 so, this is the state space form for this part. Now we can write x 1 this C S equal to S square plus 7 S plus 2 into x 1 S. And therefore, we can write ct equal to here x 1 double dot.

Because we take the inverse Laplace transform plus 7 x 1 dot plus 2 x 1. So, here x 1 double dot is equal to x 3 plus 7 x 1 dot is x 2 plus 2 x 1. So, we are representing in terms of state variable. So, we can write this is y equal to ct. So, output so, this is the output equation we can write y equal to x 1 x 2 x 3; so, 2, 7 and 1.

So, y equal to 2 7 1 so, we can see that this block, what they simply collect collected this out these variables to give the output y. So, in output we have 2 7 1 this matrix. So, that is how we will go for writing the state space equations for a transfer function having polynomial in numerator of less order than the denominator. So now, we will discuss how to convert from a state space to a transfer function.

So, we have to decide to discuss how to convert how to get the transfer function if we are we have the state space form.

(Refer Slide Time: 12:39)



So, if we have this equation x dot equal to A x plus B u, and y equal to C x plus D u. So, if we have these 2 equations so, we have the state space form. And we have to convert it into the transfer function. So, we take the Laplace transform.

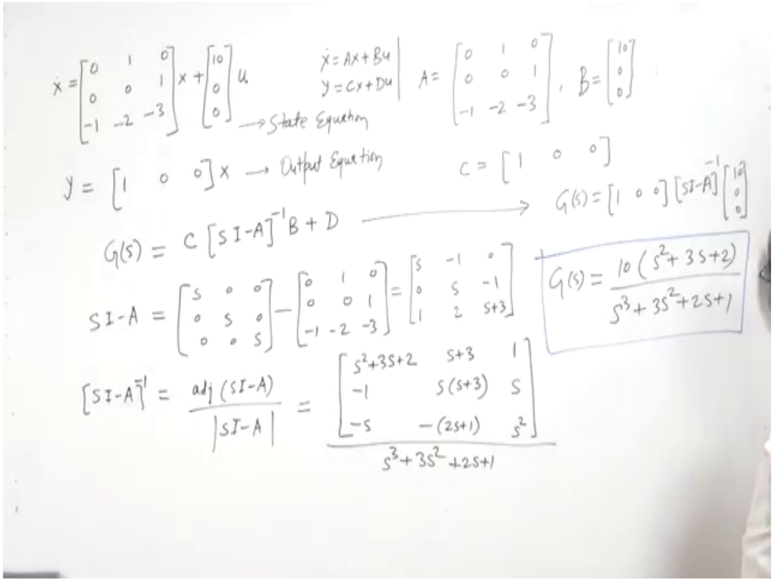
We take the Laplace transform so, this is x dot. So, we will have S X s so, one derivative. So, we get S plus so, equal to a into x S plus b into U S. Now y S we take the Laplace transform of this. So, y is equal to c into x S plus d into US. So, we solve for x s so, we find x s so, when we want to solve for x S. So, S I minus a x S. So, we take this they said. So, S I minus A equal to B U S.

So, from here we can get access equal to so, this is a matrix. So, we will get S I minus A inverse into B into U S. Now y S equal to c x S plus D U S so, here C in to X S we have this S I minus A inverse into B into U S plus D into US. So, we can write here this is as C into S I minus A inverse b plus D U S.

So, what is this? So, here u is u is the input so, here u are the input. So, us is the input and y, y the output so, y S is the output. And so, what we are getting here? If we say G S so, here this is we can write Y S equal to G S into US. So, our Y S by us equal to G S. And this G is equal to this quantity, that is C S I minus A inverse B plus D.

So, this is the transfer function so, we remember that these are the matrices. So now, we can take one example, and we can show how for a given state space we can write the transfer function.

(Refer Slide Time: 17:44)



So, we have this equation x dot equal to so, this is the we can see the this is the state equation, and this is the output equation. So, we are given the state space model for a system.

Now, we have to find the transfer function for this system. So, to find the transfer function v, we can find G S equal to C S I minus A inverse B plus D. So, this is the transfer function so, first let us calculate S I minus A. So, S I minus A equal to so, here the matrix A equal to 0 1 0 0 0 1 and minus 1 2 3. Because we can compare this with x dot equal to A x plus B u, and output equation we can compare with y equal to c x plus D u.

So, when we do this we get that A is this matrix, and B matrix equal to 10 0 0 and the C matrix here C x. So, c matrix equal to 1 0 0 and D matrix is null. So, we find S I minus A. So, S here S 0 0 0 S 0 0 0 S, this is S I, because we multiplied S in the unity matrix I matrix; so, this minus A. So, A is 0 1 0 0 0 1 and minus 1 2 3.

So, we will get S I minus a equal to S minus 1 0 0 S minus 1 1 plus 1. So, 0 minus 1, 0 minus 2, minus 2 and S minus 3, here this is minus 2 and minus 3. So, we have minus 2 minus 3 so, here this is 2, and this is S plus 3 so, there is a correction.

So now we find S I minus a inverse. So, to find the universe we can find A joint S I minus A by determinant S I minus A. So, we can find this ; so, this is the numerator part, then denominator it is S cube plus 3 S square plus 2 S plus 1.

Now, we can get the transfer function from here. So, we have G S equal to C C S 1 0, and we got S I minus A inverse and B, B is 10 0 0 plus D D we can have 0 0. So, from here we will get so, here we have obtained from the state equation, given state equation, we have obtained using this relationship. First, we calculated we had identified the matrix A B C and D.

Then we calculated the S I minus a then we calculated the inverse. So, adjoint S I minus A by determinant A S I minus A, and then we put these matrices. So, I think here it should be 0. So, we will obtain we will obtain this transfer function. So, we learned today about in this lecture about how to convert from state space to a transfer function end, from transfer function to by state space. So, I stop here, and we will continue in the next lecture.

Thank you.