Automatic Control
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Lecture – 32
Converting a Transfer Function to State Space

So, welcome to the state space method. In this lecture we will discuss about how to convert a transfer function to state space. So, we have defined the transfer function; that is in frequency domain and the state space approach belongs to the time domain. Now if we are given a transfer function, then how can we convert these to state space?

So, here to convert a transfer function to state space we first convert the transfer function to a differential equation by cross multiplying and taking the inverse Laplace transform assuming 0 initial conditions, then we select a set of state variables called the phase variables. So, phase variables we select the output and n minus 1 derivatives as the state variables.

So, in order to select the phase variable, we select one variable that is output and it is derivative, subsequent derivatives as the state variables. So, let us see if we have a different differential transfer function Ds or Gs. So, we have some transfer function.

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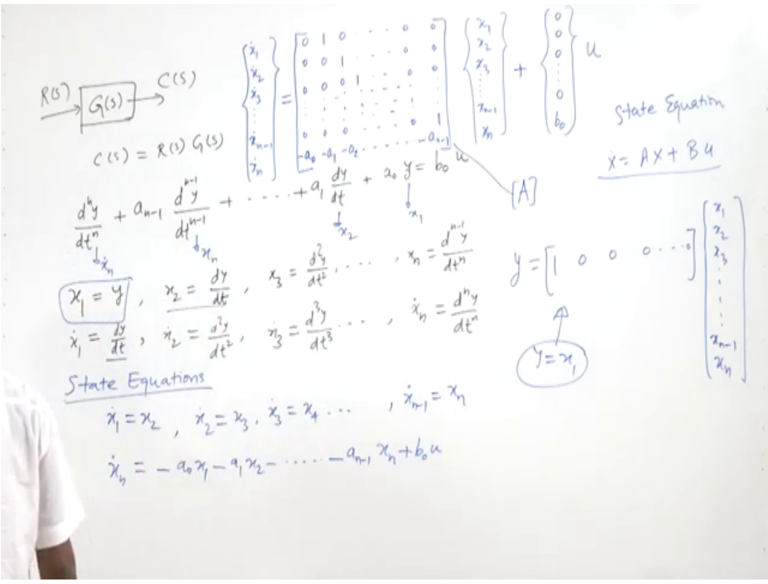

This transfer function we can represent as in terms of differential equation. So, here C s equal to (Refer Time: 02:11) s into G s and we take the inverse Laplace transform, and we will find the differential equation and we cross multiply these terms in numerator denominator and we get the nth order differential equation.

So, we get d and y by d t n plus a n minus 1 d n minus 1 y by d t n minus 1 plus plus a 1 d y by dt plus a 0 y. So, here we can have input so b 0 u. So, this is a differential equation that we obtain from this transfer function and inverse Laplace. Now we have to select the phase variables, state variables, because in state space we have to represent the state equations, and so the state variables and state variable here we select as phase variables.

So, phase variables are that, we first select the output as one state variable and the subsequent state variables like x 2 equal to dy by dt. So, here the subsequent variable is the derivative of the previous variable, and so this is called phase variable. So, x 3 equal to d square y by dt square and so on. So, xn equal to d n minus 1 y by dt n.

So, the x 1 x 2 x 3 xn, these are the state variables or phase variables. So, now, we have x 1 dot that is dy by dt and x 2 dot is t square y by dt square. So, here x 3 dot equal to dq y by dt q and here xn dot equal to d and y by dt n.

So, now we have to write the state equations. So, to write the state equations we have, so here x 1 dot. So, we have x 1 x 2 x 3 x and these are the phase variables and x 1 x 1 dot x 2 x 2 dot, all these are the state variables. So, x 1 x 2 x 3 xn and x 1 dot x 2 dot x 3 dot x n dot, these are the state variables. So, these state variables, so x 1 dot equal to dy by dt and that is x 2, because here x 2 is dy by dt.

So, we can write here x 1 dot equal to x 2 and x 2 dot equal to x 3 x 3 dot equal to here, it will be x 4 and x n minus 1 dot will be x n now xn dot. So, xn dot xn dot will come from this equation, because here now dn y by dt n is xn dot. So, this is xn dot dn minus 1 y by dt n minus 1 this is xn dy by dt it is x 2 and here this is x 1.

So, we can write xn dot equal to minus a 0 x 1 minus a 0 x 1 minus a 1 x 2 minus a n minus 1 xn plus b 0 u this term. So, we have got the derivative of these variables x 1 x 2 and xn. Now we can write this in a vector form, so here. So, if this is o we can write x 1 dot x 2 dot x 3 dot x n minus 1 dot and x n dot that is equal to.

So, this is a matrix and here we have the state vector. So, x 1 x 2 x 3 x n minus 1 and x n plus another matrix, because we have this u. So, now, we have to complete this matrix, because we have to write state equations, we have x dot equal to ax plus bu. So, we are trying to write these equations into this form. So, here we have state vectors x 1 x 2 x 3 xn minus 1 xn and this is the derivative of these state vectors.

So, here state vectors are x 1 to xn and the derivative of these vectors are defined here. now we can write this matrix. So, when we have to write x 1 dot equal to x 2. So, x 2 here is second, so the second term. So, here 0, then one, then 0, then 0 and 0, only the second term x 1 dot equal to x 2 and here it should be 0, because this equation we will obtain.

Now, x 2 dot equal to x 3. So, x 2 dot equal to x 3. So, here at the third position these elements would be one, others should be 0, then x 3 dot equal to x 4. So, here 0 0 0 and 1 and then here 0 0, x 3 dot equal to x 4, now xn minus 1 dot. So, x n minus 1 dot equal to x n. So, here the last element should be one and rest will be 0

Now, come to the x n dot. So, here xn dot and of course, here also 0s, because there is no any component of u in these equations now come to xn dot, so here we have x n dot equal to minus a 0 x 1. So, these first elements would be minus a 0 then minus a 1 x 2. So, second elements would be minus a 1 and so on.

So, other elements like minus a 2 minus a 3 and so on. And then here minus a n minus 1 minus a n minus 1 x plus b u 0. So, here it should be b 0. So, what we see that, these matrix we can say this is a, these are the vectors and this is b. So, we have written this state equation. So, this is a state equation.

Now, the output equation. So, output; if we want the output. So, we can write, because output is y. So, this is the only output equation and with this we have to represent as y equal to 1 0 0 0 and here x 1 x 2 x 3 x n. So, this gives us in fact, y equal to x 1. So, the same equation we have written as output equation in this form. So, this is the output equation. So, this is how our transfer functions. If we have a transfer function we can write these as a state equation and output equation.

Now, we will take one example and we will try to find. Now here one thing we can note that how these coefficient are shifting to this matrix tell, because the other matrix, there is 0 1 and 1 this is shifting. Now here in this matrix we have the coefficients of x 1 is minus a 0. So, these coefficients are negative and reversed here. So, the coefficient that was here that is gone here.

And the coefficient here is coming here. So, coefficients are. So, this matrix can be retained by saying this differential equation, because this coefficient which negative they will be starting from this side minus a 0 minus a 1 minus a 2 and then minus a n minus one. So, let us take one example. So, we have a transfer function. So, this is input, this is the transfer function 24 by S cube plus 9 S square Plus 26 S plus 24.

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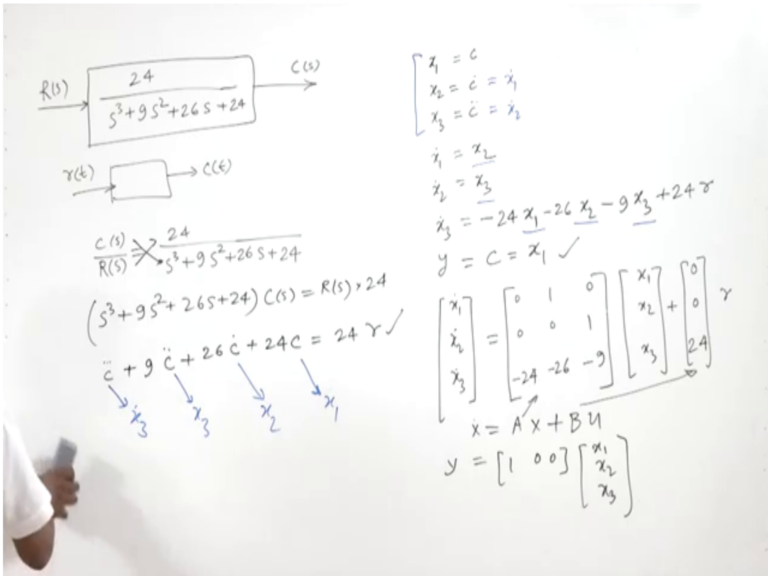

So, this is the transfer function and we have to find represent this in a state space. So, here first we have to find a differential equation. So, we have, so it is like we have r t here and here we are getting ct and this is our state space vector x. So, here let us first we find. So, let us say that we have Cs by Rs equal to 24 by S cube plus plus S square plus 26 S plus 24. So, this is we have to find the differential equation. So, we cross multiply these. So, S cube plus 9 Sc square plus 26 S plus 24 here at Cs equal to Rs.

So, now we take the inverse Laplace transform of this and put the initial conditions 0. So, here S cube, we have c triple dot, because if we have this output c t. So, c triple dot, because the Laplace transform of the nth order derivative is s power n. So, sq we take the inverse we will get c triple dot plus 9 c double dot plus 26 c.

So, here we have 26 c dot plus 24. Here c equal to 24, so here this is 24, so 24 r. So, we have obtained this. This is a differential equation we are obtained from the Laplace transform. Now we have to state variable as we have to select phase variables as a state variable. So, we assume that here x 1 equal to c is c is the output. So, this is our state variable and then subsequent.

So, we need third order differential equation, we need three state variables. So, x 1 equal to c and then x 2 equal to c dot and x 3 equal to c double dot. So, here we have x 1 equal to c. So, x 1 dot equal to. So, let us write x 1 equal to c and x 2 equal to c dot and x 3 equal to c double dot. So, here x 1 dot equal to x 1 dot, it is c dot and c dot equal to x 2. So, we can write x 2 x 2 dot. So, we are here writing derivatives of the state variable in terms of state variables. So, here x 1 dot equal to x 2 and x 2 dot equal to c double dot and that is x 3 and then x 3 dot.

So, x 3 dot we can find from here this equation. So, here x 3 dot a c triple dot. So, here this is x 3 dot. So, this is x 3 dot, because x 3 dot is c triple dot and this is x 3, this is x 2 and this is x 1 and this is r. So, we can write 2 x 3 dot equal to minus 24 x 1 minus 24 x one. So, this is x 1 minus 26 x 2 and minus 9 x 3 and plus 24 r.

So, now we have. So, y equal to c equal to x 1 this is output equation. So, because here y output is c and that is equal to x 1. So, now, let us write the state equation. So, here we have x 1 dot x 2 dot and x 3 dot; that is x dot and we have this matrix 3 by 3 and then the state vector x 1 x 2 x 3 plus r.

So, here we have x 1 dot equal to x 2. So, we have 0 1 0 and x 2 dot equal to x 3, so 0 0 1. So, x 2 dot equal to x 3 and here it is 0, because there is no any are terms here. Now x 3 dot equal to minus 24 x 1 and minus 26 x 2 and minus 9 x 3 and plus 24. So, here it will be 24 r.

So, here we can see that these elements are reversed. So, here 24 is minus 24. These 26 is minus 26, and here it is minus 9 with negative sign. So, now, this is like x dot equal to Ax plus Bu and this is the system matrix and this is the matrix B and this one

Now, the output equation can write y equal to 1 0 0 x 1 x 2 x 3, because here y equal to x 1 from this equation. So, this is the output equation. So, we saw that how we can convert a transfer function to a state space. So, we first take the, we write the transfer function with the input and output, we take with cross multiplying, we take this equation and we take the inverse Laplace transform with all the initial conditions 0.

And then we are, we get the differential equation, then we select the state variables. So, first state variable is the output, we select the output as the state variable. Here we selected the x 1 equal to cc while the output and then the subsequent state variables are the derivative of the first variable. So, here x 2 is the derivative of x 1; that is c dot. So, x 1 dot and this is the derivative of.

So, here this is the derivative of x 1 dot and this is the derivative of x 2 dot. So, we obtain this as state variables, and because in the state space equation we need the derivatives here, we represent derivatives in terms of the state variables x 1, x 2 and x 3 and here is this r is the input.

Now, if we want to represent this system as a block diagram. So, how can we do?

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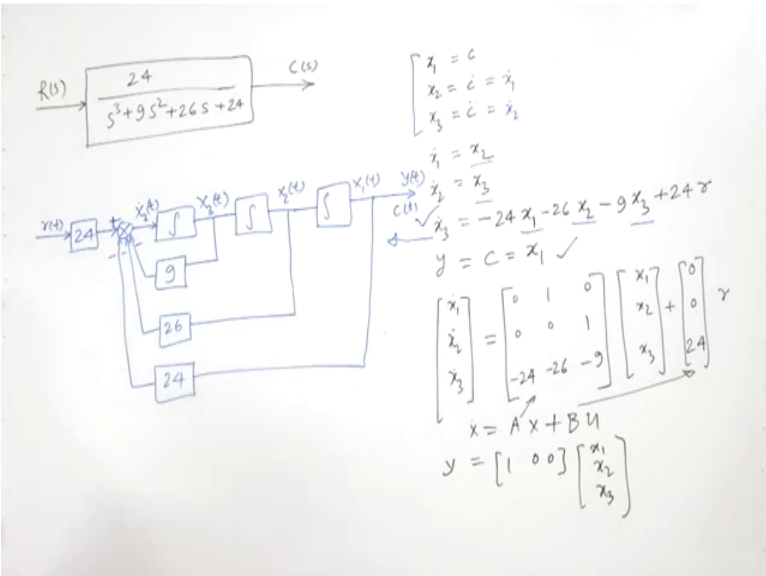

So, we have this 24. So, here into r t here we have summing junction, we have the integrator we get x. So, here we get x dot then here we get x, here we have for the integrator. So, here we have from x 3 we have to send minus this 9.

So, this is minus then x 2 we have to send minus 26. So, we have here with a minus sign and then x 1 we have 24. So, here we have minus. So, we can see here is the input rt with

24 rt. We have input here, we have x 3 dot t we integrate, so x 3 dot t. So, this is this equation, this is equal to we have integrator.

So, we get x 3 t and then we further integrate to get x 2 t, because here we can get, we integrate x 3 dot we get x 3 and we integrate we get x 2. So, we have, and then we integrate x 2 we will get x one. So, we integrate here this x 2 we will get x one. So, now, this equation we have arrived represented here, minus 24 x 1, here minus summing junction minus 26 x 2 and minus 9 x 3.

And this is plus 24 r here, it is plus and that is making x 3 dot t and y equal to x 1 t. So, output is x 1 t here yt or ct, whatever we can write ct also. Well so this is how we represent this system. So, this example we took from the book of Norman S Nise control systems engineering.

So, I thank you for attending this lecture and we will continue in the next lecture.