Automatic Control
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Lecture – 31
State Space Representation

So, welcome to the lecture on state space method. In this lecture we will discuss about state space representation of a system. So, till the the lectures we have discussed, we followed the transfer function approach of mathematical modeling and the root locus finding poles 0's. So, all these lectures were subjected to the complex frequency domain technique that is transfer function approach.

Now, we will discuss about the time domain approach, specifically the state space approach. So, here the state space, space approach is a modern time domain approach and a unified method for modeling analyzing and designing a wide range of systems. We remember in case of transfer functions, when we define the transfer function as the ratio of output and input, when the initial conditions are 0, and it is only valid for linear and time invariant systems. That was not valid for non-linear systems.

We took one case of non-linear system where we did the approximation of a non-linear systems to linear system, and then we applied the transfer function approach. So, there is limitation of the frequency domain approach, because they are only applicable to non-linear systems, only to time invariant systems. And single input single output systems; however, when we follow the state space approach that is a time domain approach. This is applicable to non-linear systems.

So, the systems that have the saturation be heavier backlash or inertia. So, we can use the non-linear systems to model using the state space approach; time varying systems; the systems for example, missiles where the systems parameters are changing, the mass is changing. So, we can model the systems using stage space approach. And systems with non0 initial conditions; because the transfer function approach we more we defined it as the 0 initial conditions.

Multiple input out or and multiple output systems status based approach can deal with the multiple input systems and multiple output system. It is more suitable to digital computers and digital simulations. So, more involvement of computers digital computers and simulations. We know that the frequency domain approach was more intuitive, because we were able to represent the graphical lead the the root locus the movement of poles.

So, we were able to more visualize graphically the things, but this approach is not as intuitive as the classical approach, that is frequency domain approach. So, here we have to perform several corporations, and then we have to physically interpret is interpret the physical interpretation of the model we have to make.

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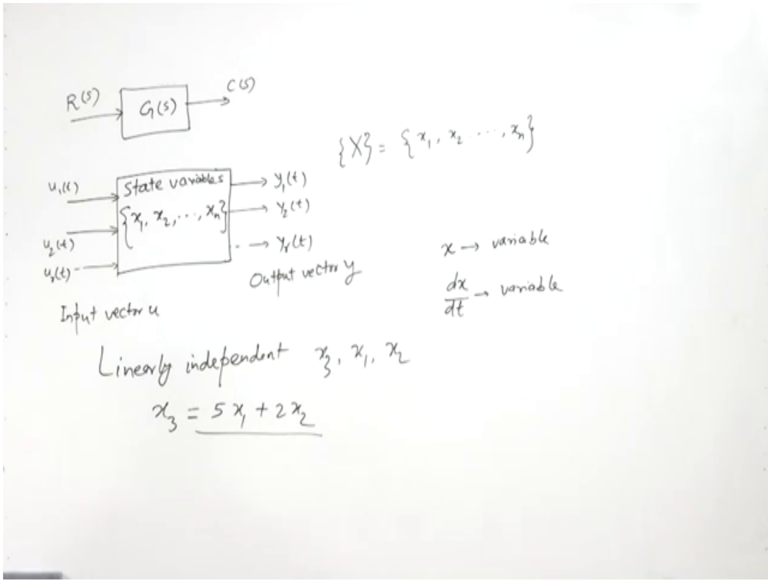

So, here we have a system if we compare here the transfer function approach, there was some input and output, and there was the transfer function. Now we come to here the time domain state space approach. So, here we have several inputs are possible. So, here let us say e 1 t, u t these are the input vectors u r t. So, these are input vectors here is the output y 1 t y 2 t y r t. So, the output vector y so, here is the input vector u, and this is output vector y.

Now, what is inside? Because here it was transfer function, and we say that transfer function is it represents the characteristics of the system that is the internal properties of the system. So, here we have the internal properties of the system defined in terms of the state variables so, here we have state variables. So, system is described by state variables x 1, x 2 and x n.

So, there are n state variables, and we define this system. So, here system variable is any variable that responds to an input or initial conditions.

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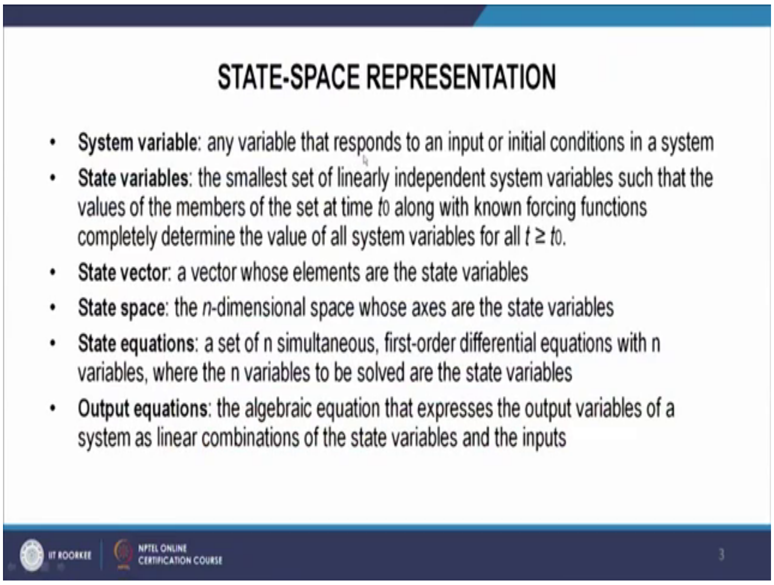

In a system so, system variable is defined as any variable, that is response to an input or initial conditions in a system. So, for example, if we have a spring mass system, and we apply someinput force f, and mass is moving with certain position, it is position is moving so, it is position is varying.

So, we can take it is responding to the input that is for. So, the position can be one of the system variable, the state variables the state variable is the smallest set of linearly independent system variables. So, state variables are those variable system variables, and this is the smallest set of linearly independent system variables.

So, system variables can be many, because any variable that responds to the input or initial conditions, that is a system variable. But it is possible that one system variable is varying because the other variables are wearing. So, and the particular variable system variable can be represented as the linear combination of other system variables. So, this variable cannot be a state variable. It must be linearly independent system variables and linearly independent means, when we say linearly independent.

So, if we could write. So, there are 3 variables x 3, x 1, x 2 and if I can write x 3 as 5 x 1 plus 2, x 2. So, here x 3 is represented in terms of the 2 variables x 1 and x 2. So, x 3 is linearly depending on these variables. So, x 3 is not linearly independent variable.

And therefore, x 3 cannot be a state variable. Although, it can be a system variable because, but it cannot be a state variable; whether x 1 and x 2 if they are linearly independent they could be a state variable. Derivative of a variable is linearly independent variable. Because if I say, x is a variable dx by dt is another variable, and this dx by dt cannot be represented as linear combination of the x, but it is a derivative.

So, therefore, these both could be state variables. So, the smallest set of linearly independent system variables such that the values of the members of the set at time t 0 along with known force functions completely determine the value of all system variables for all t greater or equal to p 0. So, if we have at any time t 0, we know these values of these state variables for a forcing functions, we can completely know the value of all the system variables.

So, only with we know the state variables at any particular time instant for under some input conditions, we can represent the other system variables in terms of state variables. Because other system variables are somehow they are expressed in terms of the state variables. Then state vectors, state vector is a vector whose elements are the state variables.

So, if these are the state variables x 1, x 2, x n, we can say x is if x is state vector, because it is a vector that is elements are state variables, now the state space. So, there are n dimensional space, the n dimensional space whose axis are the state variables. So, if there are n state variables. So, n dimensional space is the state space, where axis of one acts represent one state variable.

Then state equations so, each state equations are a set of n simultaneous first order differential equations with n variables; where the n variables to be solve are the state variables. So, here the state equations or if there are any state variables. So, there are n first order equations, differential equations. And these are called the state equations. Then the output equations; so, output equations are the algebraic equation that expresses the output variable of a system as linear combination of the state variables and the input.

So, it represents the output variable as a linear combination of the state variable and inputs. So, here we will discuss more in detail about this state equations and output equations, because these are the the modeling equations of a system. So, we model a system in state space evening these 2 equations. So, we will discuss more about these equations so, we discuss state equation.

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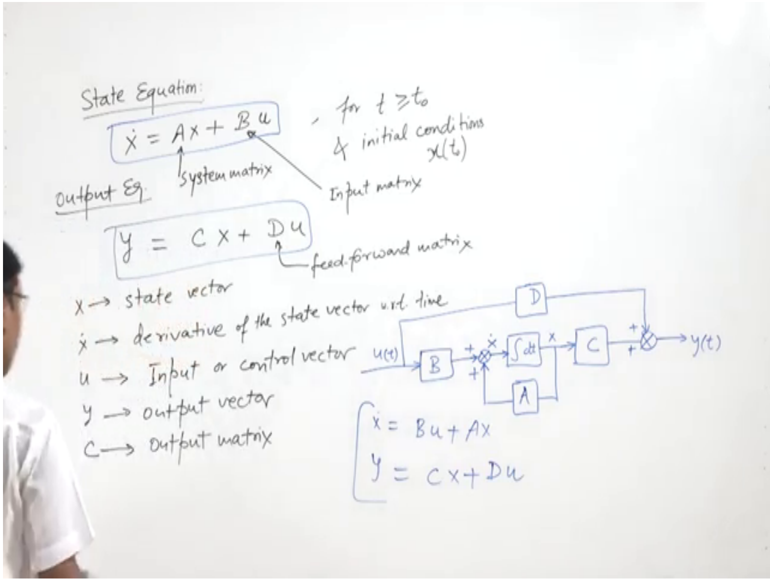

So, we know that a state equation we can write. So, a system can be represented in terms of state equation like x dot equal to A x plus B u for time t greater equal to t 0, and initial conditions and with certain initial conditions x t 0, here and the output equation.

So, we define this state equation that n simultaneous first order differential equations will we then state variables. So, here it is a vector this x and x dot and u, and A and B are the matrices. Now the output equation is represents a linear combination algebraic equation that expresses the output variables as a linear combination of the state variable and the input.

So, here y the output is c x plus d u so, here we can say x, x is a state vector. This is a state vector, and x dot is derivative, derivative with respect to time, derivative of the state vector with respect to time. So, we can see that x dot that is derivative of the state vector is equal to a times x plus b times u. So, here u is the input or control vector. So, we are u this u is input, and y is output vector.

So, this y represents the output vector. Now A, this A system matrix. So, this is A is system matrix. And B is input matrix, then C is the output matrix, the C is output matrix, and D is feed forward matrix so, this D is feed forward matrix. So, here we have shown the state equation and output equation. So, these are the equations that model system in state space.

Now, we can show them on a block diagram like this so, we can see here so, this is input. So, here we can we have represented these both the equations, because these both equations model a system. And this is a block diagram here in time domain we have shown. So, we have an input u t so, this u is u t, now we see that first we say x dot. So, x dot is b times u t. So, here x dot b times u plus, because here is the summation the plus here is x, that is x.

So, we are getting A x plus Bu. Now here x dot we integrate to get x. And so, here y y equal to C into x plus here D into u. So, that is going to be some so this diagram represents this system. Now we take one example to how we can write these equations for a given system. So, we with the help of one example we can see you.

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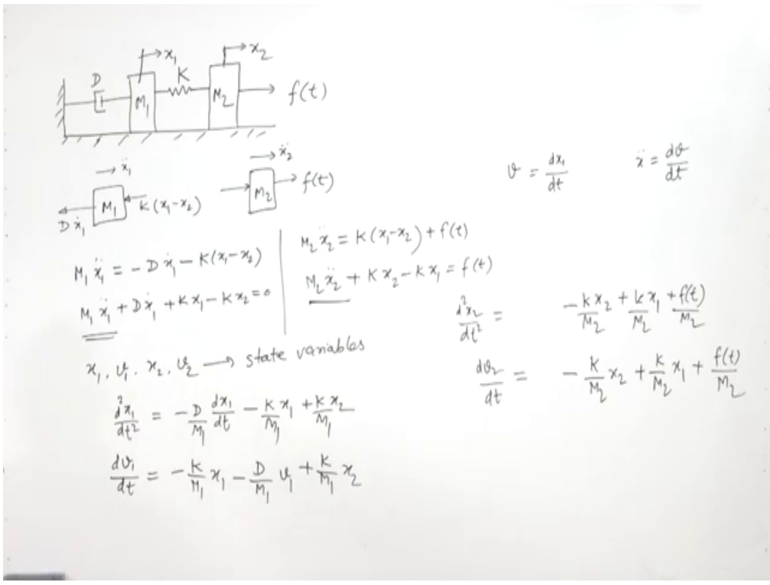

So, here we can see we have a system here. Comprising the 2 masses M 1 and M 2, and a damper d and spring key k now here we have this as x 1 and x 2, these are the displacement with time t. So now, we can write the differential equation for this system first. So, this is the second order system, we can represent the free body diagram here, we can write M 1, and this is we take the acceleration here.

We have here this so, we have this force D x 1 dot, and we have this force k x 1 minus x 2. Similarly, for second mass M 2, we have this force here, and there is a force input. So, ft is the input force, and we take x 2 double dot. Now we apply the newtons second law, and we will get M 1 x 1 double dot equal to minus d x 1 dot minus k x 1 minus x 2.

So, we can write it as M 1 x 1 double dot plus D x 1 dot plus k x 1 minus k x 2 that is equal to 0. For this equation we can write as M 2 x 2 double dot minus k plus k x 1 minus x 2 so, this is equal to plus f t. So, we can write M 2 x 2 double dot plus k x 2 minus k x 1 equal to f t.

So, we can write, now we have to select the state variables. Here if you want to write this equation in a state space, we have to find the state variable. Now we see that here we have second order differential equation here also the second order differential equation. So, the minimum number of state variable must be the order of the differential equation.

So, here we have we can have 2 plus 2 4 state variables. So, here let us select x 1, v 1 and x 2 and v 2 these are the 4 state variables so, here we can write. So, we can write here M 1 D square x 1 by dt square equal to minus D d x 1 by dt minus k x 1 plus k x 2.

And this equation we can write as M 2 D square x 2 by dt square equal to minus kx 2. So, here or we can write plus k x 1 plus f t. So now, here we know that D x 1 by dt equal to v, and x 2 x 1 double dot x double dot equal to dv by dt. So, here we can write M 1, again here we can take M 1 here. So, by M 1 here by M 1 and by M 1.

So, we can represent this as D, v 1 by dt equal to minus k by M 1 x 1 minus d by M 1 v 1 plus k by M 1 x 2. This we can write M 2 d v 2 y dt equal to minus. So, here we again do this M 2 we take here by M 2 by M 2 by M 2. So, minus k by M 2 into x 2 plus k by M 2 into x 1 plus f t by M 2.

So, here we have represented the equation in terms of each state variable x 1, x 2 and v 1 v 2 and the derivative of this state variable.

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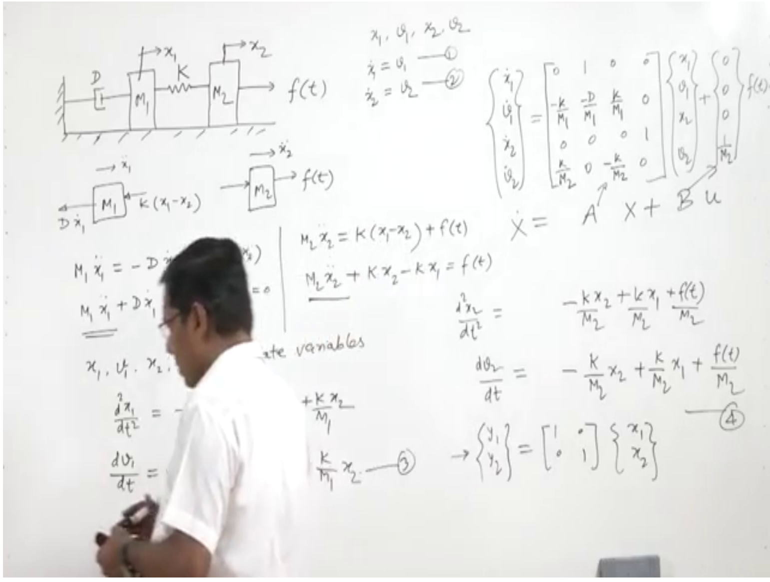

So now we have 4 equations; that is, if we have x 1, v 1, x 2, v 2 these are the state variables, we can write as so, x 1 dot equal to v 1, and x 2 dot equal to v 2. So, this is 1, equation this is 2, equation this is third equation, and this is 4 equation, 4th equation. So, this equation 1, 2, 3, 4 we have to collect in a matrix. So, we can collect this in a matrix; so, we can write x 1 dot, and v 1 dot, and x 2 dot and v 2 dot. So, these are the, and we can write here x 1, v 1, x 2, v 2 plus ft. So now, we have to collect this term so, first equation x 1 dot equal to v 1. So, we have here 0, 1, 0, 0.

Then we will get x 1 dot equal to v 1. Then v 1 dot equal to here we have x 1 term. So, minus k by M 1 then v 1 so, here is minus t by M 1, then x 2, x 2 term is k by M 1 and v 2 is no any term so, it is 0. Then come to x 2 so, x 2 dot equal to v 2 so, x 2 dot equal to v 2. So, only this will be one and other will be 0, then only x 2 dot will be v 2, and v 2 dot will be written from here v 2 dot equal to minus.

So, x 1 is k by M 2, and v 1 is here 0 term then x 2 is minus k by M 2, and v 2 is 0. Plus, here in this equation the last equation is 1 by M 2 here. So, 1 by M 2 into f t and these will be 0 because. So, so, this is we have written this is like x dot equal to A x plus B u. So, this is B, this is A, this is x is the vector and x dot t the derivative. So, this is a is the system matrix.

And you can see that, this matrix contains the parameters of the system kd M 1 M 2. So, the output equation we can write y 1, y 2, y 1, y 2 equal to x 1 x 2, because here we so, we can write output equation here. So, this could be also 1 0 0 1. So, we can also take so, y 1 equal to x 1, if we want output only these x 1 and x 2. If we also want v 1 and v 2 we can add this this vector and we can write this matrix.

So, if you only want this output as the displacement we can write this as output. So, here we discussed about the state space representation of a system that is the modeling as system in time domain. And we found this state equation and output equation. And mean minimum number of state variables must be they have the order of the differential equation. So, here we stop and we will continue in the next lecture.

Thank you.