Automatic Control

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Lecture – 30
Lead and Lag-Lead Compensation

So, welcome to the lecture on design via root locus and compensation techniques, today we will discuss about lead and lag lead compensation. So, we discussed about in earlier in earlier lectures the lag compensation technique and we know that the lag compensation technique was passive compensator equivalent to the ideal integral compensator that is p i compensator. So, similarly here the lead compensation is passive compensator for the actual active ideal derivative compensator.

So, here the pd compensator that we discussed in active circuit we can use here the lead compensation technique. So, when we remember the active derivative compensator that is ideal compensator p d and we also call it pd compensator. So, we put a 0 close to the origin to compensate that system, because the pd compensator we used to obtain the transient response the desired transient response and. So, we put some 0 and we put some 0s. So, that angle that is made by the desired location of the pole is odd multiple of 180 degree.

In case of passive network it is not possible to put a single 0 rather a 0 and the pole will be resulting in passive network. So, here in case of lead compensation equivalent to the active ideal derivative compensator we will put a pole and a 0. So, that the angular contribution of the compensator is still positive, and thus we will approximate an equivalent single 0 because if we put the pole very far from the 0, then the contribution of that pole in terms of the angle will be very less in compared to the 0 and that can be an approximate compensator for the tangent response.

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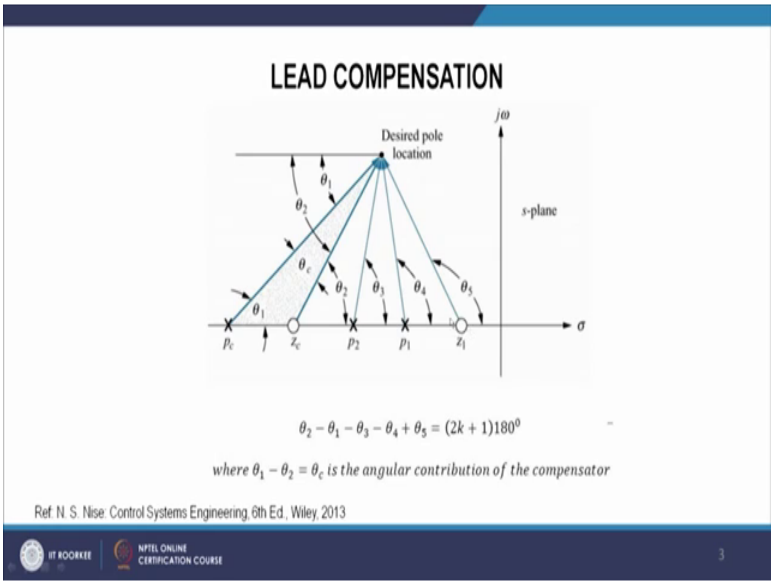

So, here we can see that we have here the 0 of the system p 1 is the pole p 2 is the pole of the system now this is our desired pole location where we want this the new root locus to reach to this point. So, we are putting a 0 here and we are putting another pole that is compensator pole and we can see that the angle that is theta 2 minus, so theta 2 is due to 0. So, it is positive minus theta 1. So, this is negative and minus theta, theta 3 that is negative minus theta 4 plus theta 5 equal to 2 k plus 180 degree.

Now, here theta 1 minus theta 2 is theta c is the angular contribution of the compensator because we are putting a 0 and a pole. So, the angular contribution of the compensator is the difference between these 2 and this we can see here theta c here theta 1 theta 2 and theta c is theta 1 minus theta 2, so theta 1 minus theta 2 we can see that.

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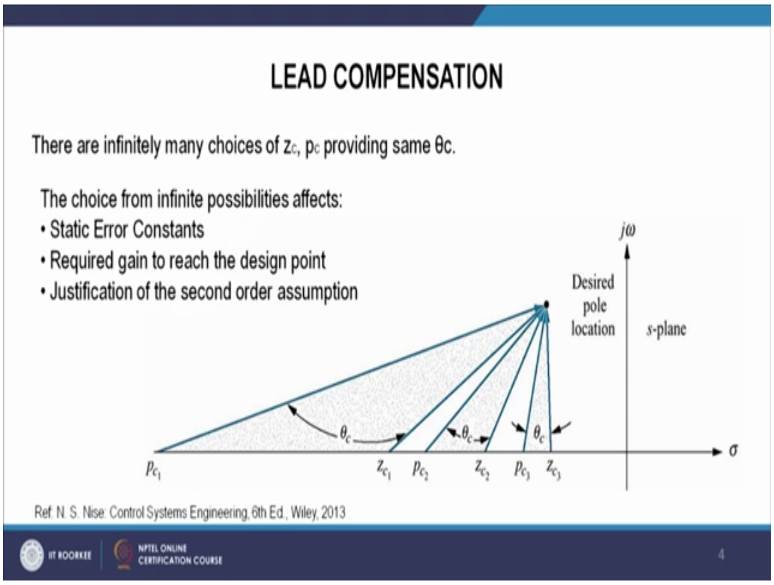

We can find this theta c such that there are several options to find to set this theta c, so that we can reach to the desired pole location. So, we can put like here z the pole the 0 and pole, so we will have this angle and we are going to have some desired pole location and this option also will give us the same desired pole location and similarly this option that is the third option.

So, there could be infinitely many choices for putting 0 and pole of the compensator. So, that we will it will provide the same theta c; however, we will be able to get the desired transient response because we will be able to put the desired pole location, but we will there with the different choices will defer in terms of static error constants required gain to reach the design point and justification of the second order approximation.

So, we although we will get the desired transient response, but we will get different static error constants and, so the steady state error so, here if we want to design for the lead compensation of the system, first we have to find the desired pole location and then we have to select some values of 0. So, we assume some 0 location and for this desired pole location and from this 0 we must compute the angles that is being made by on this point by this 0 and the system poles.

So, we can see this example here in this figure we see that first we put a 0 to get this desired pole location and. So, we calculate that all the angles that theta 2 minus theta, so not theta 1 we have to we have to calculate if we assume that by putting this 0 we are able to get some angle here and that is not 180 degree, but there is some difference and that difference will be fulfilled by putting a 0 at that location.

So, that the theta 1 we will get to can to satisfy this condition. So, here first we will put the 0 and we will put the pole at that location where it will satisfy the angle condition for these desired pole location. So, we will see some example when we will discuss the example for the lead lag compensation. So, now, we come to the lag lead compensation. So, we have already studied the lag compensation and we discussed the lead compensation.

Now, we know that the lag compensation is an equivalent for the transient response sorry, this lag compensation we use for the steady state error desire to get the desired steady state error and this lead compensation we used to get the desired pole location or desired transient response. Now we saw that when we design for the desired pole location our design transient response using lead using lead compensation the steady error constraints may vary and therefore, we should combine these 2 the lag and lead. So, that we can we can design for both that is steady state error as well as the transient response.

So, here lag and lead compensation we will use both the techniques. So, the steps are that first we have to determine the desired pole location based on the specifications. So, what is our design requirement based on that we have to determine the desired pole location.

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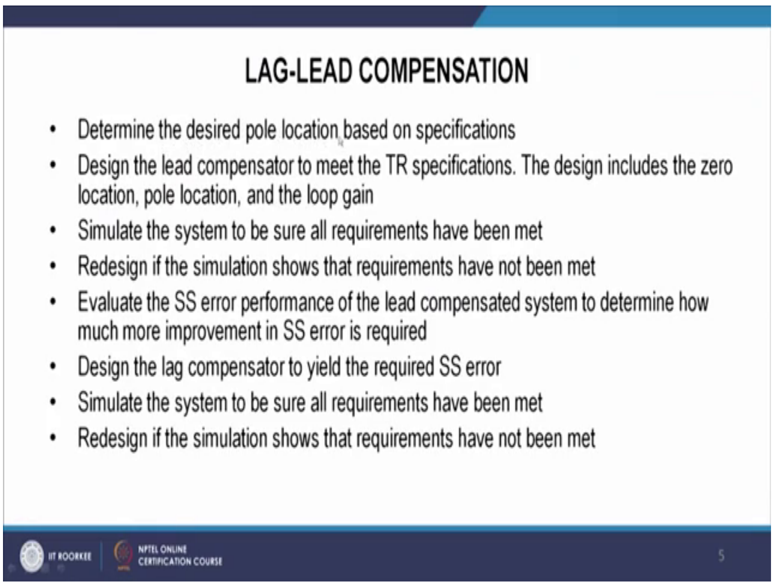

So, did the pole location is mainly decided by the transient response requirement like percentage overshoot settling time peak time and. So, we will find the desired pole location and for that pole location to obtain we design the lead compensator to meet that transient response specification, and when we use the lead compensator we will get that we add 0 and a pole for the compensator and some gain we will add to this in this design.

So, 1 additional compensator pole and 0 will be added to the system then we simulate the system to see whether we are meeting the transient response requirements or not if not we will redesign if yes we will go to design for the lag compensator, but before designing for the lag compensator we have to evaluate the steady state error performance of the lead compensated system, because our uncompensated system has certain steady state error performance, but now we have designed that system is augmented by designing the lead compensator.

So, the steady state error performance will be changed with respect to the uncompensated system. So, when we come to design for the lag compensator we have to see whether the lead compensator has changed the steady state error performance it has improved or not and with respect to that we will design the lag compensator. So, once we have designed the lag compensator to yield the required steady state error, then we will simulate the system to see whether all the requirements have been met if not we will again go for the redesign.

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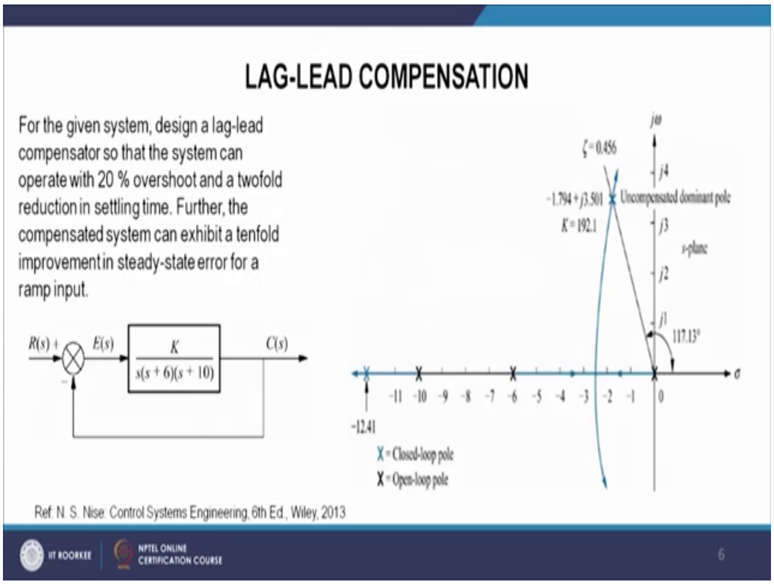

So, here we will take one example for this, so let us take one example and in this example we will use both lead and lag compensation to design for the transient response as well as the steady state error. So, here we have a question that we have this system and this system has this is our g s k g s and this is C s.

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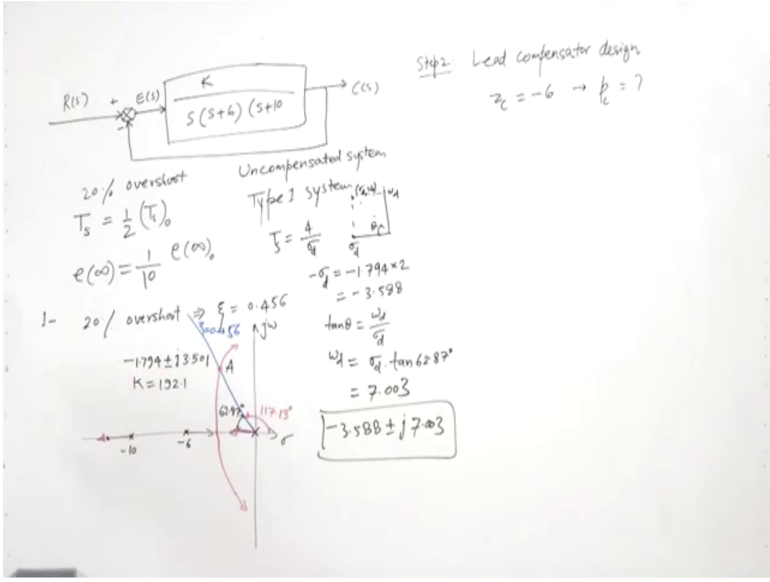

So, we have to design a lead lag compensator, so that for 20 percent over suit, we get the dominant poles at 20 percent overshoot and the settling time is half of the uncompensated system.

So, if I say T s 0 is the settling time of uncompensated system this is uncompensated system. So, this is uncompensated system, so we design, so, that we get reduction in the settling time. So, that is half and we want that the steady state error is one-tenth of the steady state error of the uncompensated system. So, here we can read this problem that for the given system, designs a lag lead compensator so, that the system can operate with 20 percent overshoot and a twofold reduction in settling time.

Further the compensated system can exhibit a tenfold improvement in steady state error for a ramp input. So, here this is a type 1 system, so the test input will be ramp, so this is type 1 system. So, now, when we want to design the system first for these conditions, first we have to calculate this settling time and steady state error for the original system. So, here, so we will calculate the n, so 20 percent overshoot. So, this is equivalent to damping that is equal to 0.456. So, this we can calculate because there is the formula between damping and overshoot.

So, we can calculate the damping and this will this value of damping we have to. So, the root locus must cut this line of damping line, then only we will get a point that we satisfy both the condition that will be on the root locus as well as on this damping line. So, here let us plot this, so we have open loop poles at S equal to 0 at origin, at minus 6 and here at minus 10, and this is a line for damping equal to 0.465, 465 damping equal to 0.456.

Now, in these root locus will start, so let us use it will start this angle is so this is the angle and this angle is 180 minus this angle. So, root locus will start at poles and at this will break away here, and this 1 will start and go to infinite here there is 0 in finite the no any finite 0. So, all these three poles will lead to infinite as a root locus. So, here they will bifurcate at some point between these 2 and this will go and n cut this line at some point.

So, this point we can find as we know how to find these points and this point where it cut is minus 1.794 plus j 3.501. So, if we sue, so we will get this other point here and this happens for k equal to 192.1. So, this is the gain and we will get the third, so these are the 2 poles and here we will get somewhere the third 1 pole. So, this is the uncompensated system giving the 20 percent overshoot for these dominant poles at a.

Now, we can calculate we will go further, so this angle is 62.87 degree now we will go for a step 2 that is the lead compensator design. So, we will design the lead compensator here. So, here we have 2 we know that that settling time should be half and we know that settling time T S equal to 4 upon sigma d that is zeta omega n sigma d is the real part of the pole. So, here if we want T S half this sigma d should be double. So, here sigma d equal to, so here is the real part 1.794, 1.794 into 2.

So, minus 3.5 double 8, so we will get this sigma d now we do not we know sigma d we know here sigma d and this is omega d. So, here is sigma d omega d this is omega d and this is theta. So, here tan theta equal to omega d by sigma d, so this is tan theta that is this damping equal to 0.456 line that is 62.87. So, here omega d equal to sigma d into tan 62.87 and sigma d we put this value 3.5 double 8 and we will get 7.003, so we got omega d.

So, the to obtain this transient response condition with 20 percent over suit and half the settling time we must obtain the pole location at minus 3.588 plus minus j 7.003. So, this is our required design condition, so the dominant pole must be here. So, this is design point, so let us now when we for the lead compensator design we have to design a 0 and a pole.

So, a 0 we select let us select arbitrary, so we select z c equal to minus 6, so we select a 0 at minus 6 and then we calculate we try to find the pc. So, we have to find the compensator pole to find the compensator pole. So, let us say we have selected here, so now, let us we plot this point B, so let us say this is minus 1.794 and now we are double of this, so.

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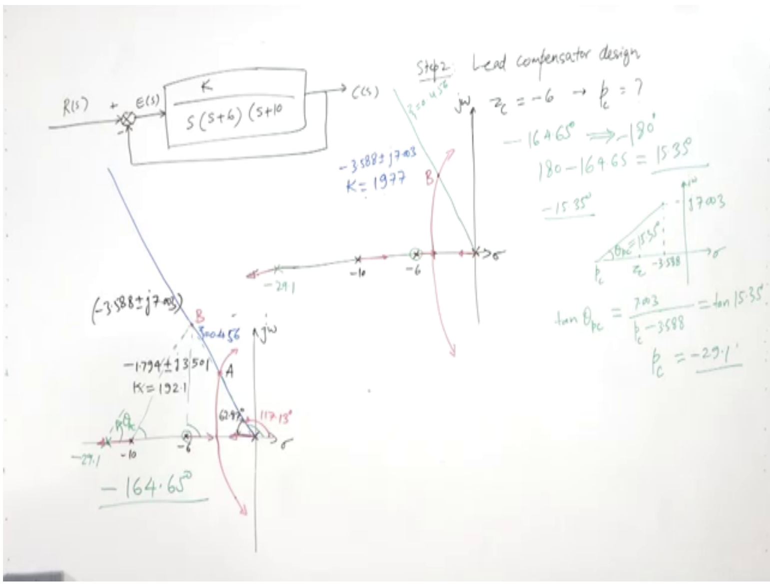

So, this is the damping line and our point is here somewhere this point that is point B minus 3.588 plus j 7.003. So, this point we have to we located a 0 here, now we have 2 that is z c there is already a system pole here we put the compensator 0 here now we have to calculate these angles. So, here this angles this angle and there is one system pole here, so this angles, so these angles. So, we know that these angles will cancel because there is a pole and 0 here, and this angle and plus this angle we can calculate. So, this angle is coming equal to minus 164.65 degree.

So, this angle, so some of angles to the design point from the uncompensated system poles and 0 and the compensator 0 is coming minus 164.65 degree, now if we want this point to pass through the root locus this point must make angle of minus 180 degree. So, we must put a pole somewhere, so let us we put here some pole pc. So, that the angle contribution these angle contribution this let us say theta pc.

So, this angle contribution will make the, this angle to 1 minus 180 degree. So, here the difference is 180 minus 164.65 and that is 15.35 degree. So, we know that when we add a pole it will add some negative angle. So, we must add this minus 15.35 degree, so that in this angle and it will make minus 180 degree.

So, from here we can find the pc, so pc is we have this sigma j omega axis here we have put this z c and this is our pole location and this is our pc. So, here this is minus 3.588, and this is j 7.003 and we want that this angle theta pc. So, tan theta pc equal to 7.003 by so pc minus 3.5 double 8, so this upon this so this is pc and this is here 3.588.

So, we are going to get this pc minus 3.5 double it and that is this theta p c each time 15.35, so this is 15.35 degree. So, from here we will get pc and that is minus 29.1. So, we are getting this pc at minus 29.1 this third pole compensator pole sorry. So, when we have obtained this we have a lead compensated system and the lead compensated system is this. So, we have now this B that is minus 3.5 double 8 plus minus j 7.003.

We have here system pole here system pole compensator 0 system pole and here is the compensator pole and so this is the and now this root locus must pass from so we can plot here. So, here now our system is one pole at this point then we have here 1 system, system pole here and system pole here and we have compensator 0 here and compensator pole here at minus 29.1 and this is the damping line.

So, now the root locus will start from this pole it will start from this pole and it will break away, the pole that we start here can lead to this point this 0 and this may lead to infinity because we have these three poles leading to infinite because there are only 1 finite 0 here. So, this locus here cutting at this point B and we get this minus 3.5 double 8 plus minus j 7.003 for our gain K equal to 19 double 7.

So, we are getting this at the value of gain k equal to 19 double 7, now they step five is to design the lag compensator. So, we have designed the lead compensator and the lead compensator has this has transfer function G l lead compensator is 1977, because k is 1977 S S plus 10 and S plus 29.1.

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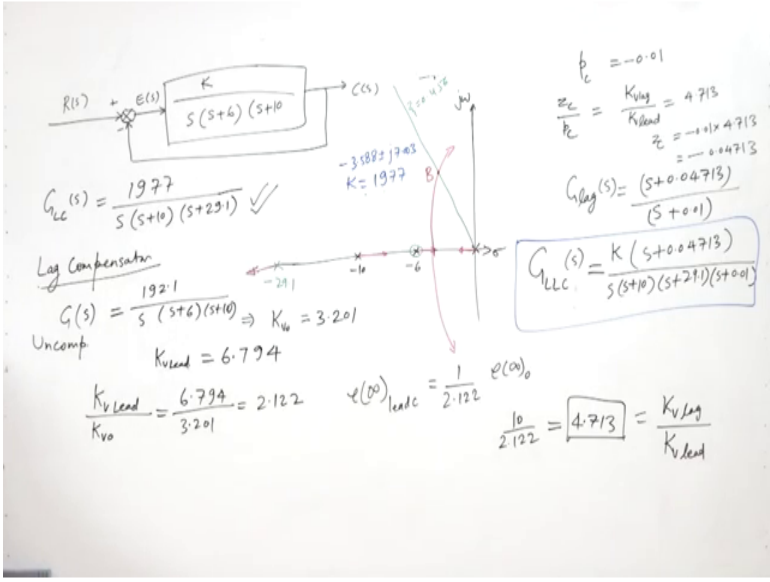

So, here we see that when we put the compensator 0 at this pole location they will cancel out. So, this 0 at S plus 6 concerned this pole system pole at S plus 6. So, this is the compensated lead compensated systems transfer function now we have to go for lag compensator. So, lag we have to design, so we know that the uncompensated system has. So, G S was and we had the gain K equal to 192.1 by S S plus 6 S plus 10 and from here we can find K v 0.

So, we say K v is the velocity static error constant and 0 is for the uncompensated system this is uncompensated system, and when we calculate S equal to tends to 0 we will get this 192.1 by here 60. So, that is 3.2 now for the lead compensated system we will get K v lead, and this we will get we have 1977 by 10 into 29.1 and we will get 6.794.

So, the objective was to design for the steady state error the objective or to design the steady state error 1 tenth of the uncompensated system. So, now, we see that K v lead compensated by K v 0 equal to 6.794 by 3.201, so we get it 2.1to 2, so we have already and this is the ratio this ratio is the ratio of the state a steady state error also.

So, we have already improved the system in terms of a steady state error 2.122 times. So, steady state of the error of the lead compensated system is 1 by 2.1 to 2 into a steady state error of the uncompensated system. So, our target was to design compensator for 1 by 10 of this, so now, for the lag compensation we need only the improvement that is 10 by 2.122 only this much improvement we need by lag compensation because we have already achieved this much improvement by lead compensation in terms of steady state error.

So, that is 4.713, so 4.713 we need and we know that, so this is equal to, so K v, so this is the 4.713 and that is K v lag by K v lead. So, in the lag compensated we will get improvement in terms of lead by 4.713 times. So, now, when we design the lag compensated system, we will select the location of the pole near the origin, we know that to design the lag compensated system first we select the pole at some position close to origin.

So, let us we select that pole at 0.01 that is close to the origin and we know that z c by p c equal to K v this value lag by K v lead. So, equal to 4.713. So, z c equal to 0.01 into 4.713 that is equal to minus 0.04713. So, we got this location of this lag 0 like a 0 of the lag compensator so we have so the compensator, we have S plus 4.713 by S plus 0.01. So, this is the lag compensator only and so the transfer function of lead lag compensated system is equal to. So, this compensator we have this lead and this is lag.

So, now the total compensation is K S plus 0.04713 by S S plus 6 S plus 10 now S plus 10 here, and S plus 29.1 and here S plus 0.01. So, this is the transfer function of the lag lead compensated system now what is the now we can plot this root locus.

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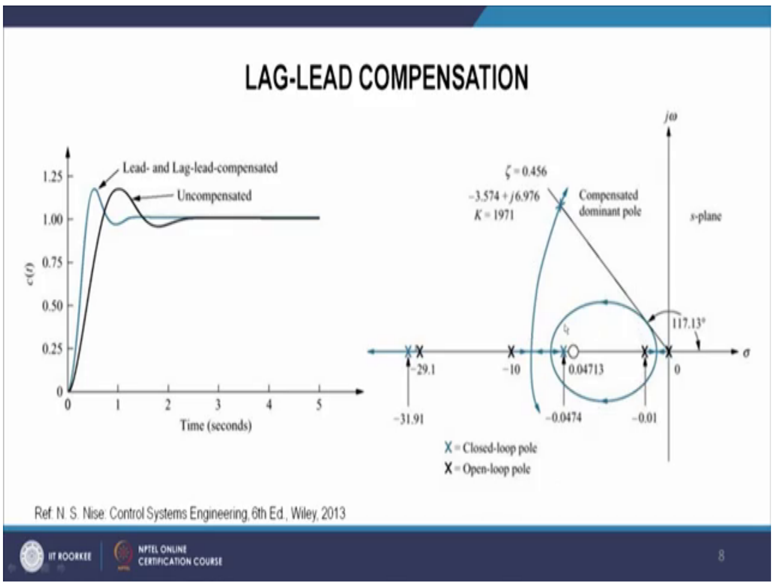

And so here we can see the system here the root locus of the lag lead compensated system we can see here. So, 1 pole at origin then this pole at my minus 0.01 there is a 0, and this and the root locus is passing through the this point and this is the response of lag lead compensated system and this is uncompensated now we can see that we have improved our transient response as well as the steady state error.

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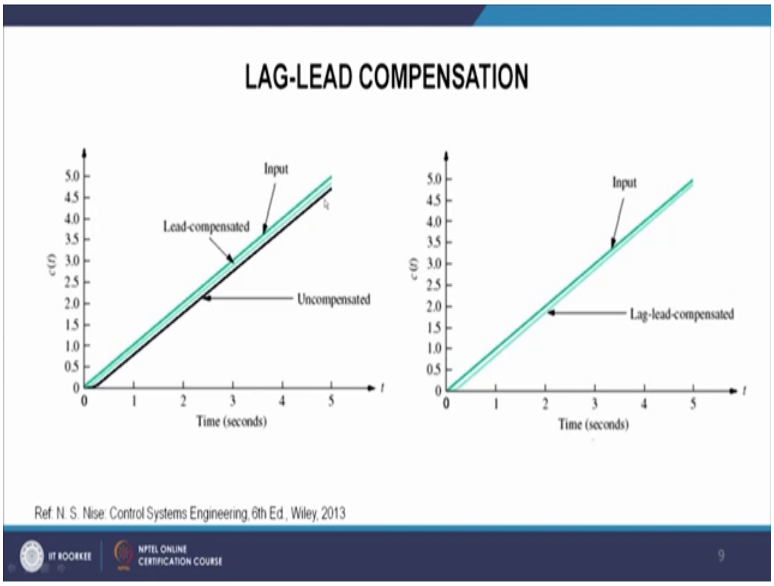

And here we can see better the steady state error for a ramp input we have this lead compensated and when we apply the lag compensation we have further improvement in the steady state error. So, this example was taken from a nise ns control systems engineering, so I thank you for this lecture and let us see in the next lecture.

Thanks.