**Automatic Control**
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**Lecture – 29**
**Lag Compensation**

So, welcome to the design via root locus and compensation techniques, so in this lecture we will discuss about lag compensation. So, we discussed about p i compensation our pi controller that is used to design for the desired steady state error performance we said that p i controller or ideal integral integration controller is an active system it requires power source; however, this lag compensation is a passive system that does not required external power.

So, in case of p i controller we were able to find the 0 steady state error by putting the pole at origin, but here in case of lag compensation we put the pole near the origin and a 0 close to that pole to the left side. So, here is not the pole on the origin, but close to the origin. So, in case of lag compensation we can get the finite steady state error, but not the 0 steady state error because the pole is not on the origin, but it is at some finite distance left to the origin, so we can see that this system if we have such a system like.

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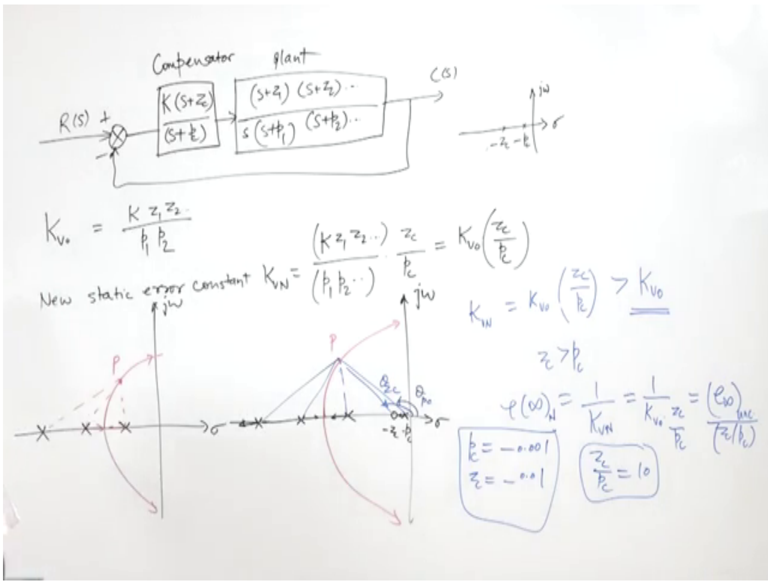


We have this gain and we this g S and g S if we can write in terms of S plus Z 1 S plus Z 2 and so on upon S plus p 1 S plus p 2 so on. So, we have this is our g S and we have C

S, so minus plus this. Now this is here gain and this is your plant we want a compensator here. So, we replace here to add a lag compensator. So, lag compensator here we will have to put S plus Z c by S plus pc.

So, now we can see that here this is not a type 0 system, but this is not type 1 system after adding this compensator, let us take it as here in the beginning type 1 system. So, this plant is type 1 system; because there is 1 pole at origin and we want to add a compensator for this type 1 system. So, because this is a type 1 system we will have the static error constant K v that is velocity static error constant and here we will have K v be say K v 0 equal to K z 1 z 2 upon p 1 p 2.

Now, the new static error constant K v N equal to, K z 1 z 2 upon p 1 p 2 into, so here z c by pc. So, this is equal to K v 0 into z c by pc. So, we see that the original system that has a root locus, if we take here these three polls, this was the root locus of the system and we had some point p here and this was the angle.

We want to improve the steady state error we want to reduce the stage steady state error of the system, but at the same time we want to retain the transient response. So, therefore, if there is some point p with certain tangent response, we want that the root locus passes through this point p even after adding the these compensator. So, the new root locus will be we have added this we add this pole and this 0.

Let us take the minus sign here S plus z c equal to 0, so S equal to minus z let us say z c. So, when we are adding this compensator what will be the root locus now the new root locus. So, we will have 1 pole near the origin that is minus pc then there will be 1 0 minus z c near to this pole and then these are the three points 3 open loop poles here. So, this root locus will move like this will move and breakaway and this will go to finite.

So, now this root locus will be and here it will again the point p is on the root locus. So, now, the angles here this is angle theta pc and this angle is theta z c.

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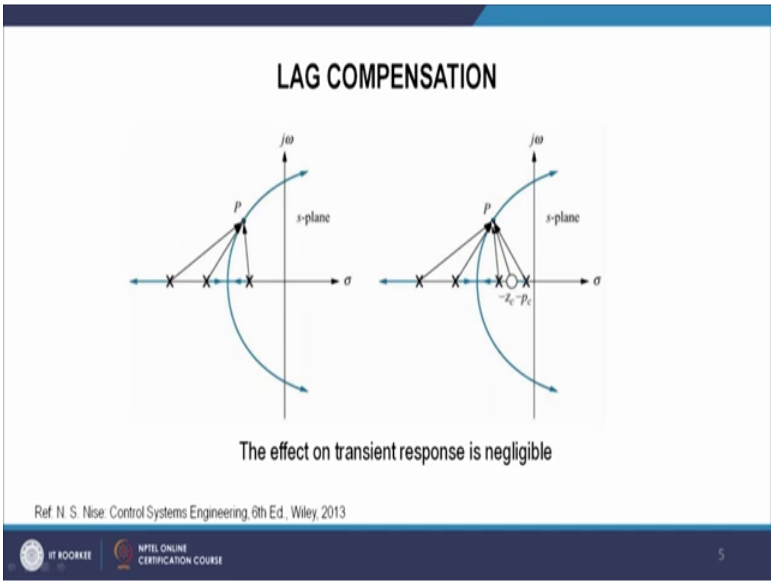


So, here this point we are adding this root locus here we are adding 1 pole and 1 0 here near to the pole. So, that this angle that is with this pole and the angle due to 0 they have a budget sign and they will cancel each other they were there effect will be very less and this point p will be still on the root locus. So, we will not affect the transient response by adding this force and zeros, in order to improve the steady state error and we see that this point p the gain will also not change because we because these points are closure this length of this these 2 polar this peep and this 0 and p to this pole will be same and they will be canceled out as well as the angles are closure. So, they their effect will be cancelled out.

So, here, here we obtain this K v N equal to K v 0 z c by pc and that is greater than K v 0. So, we see that here z c is greater than pc in magnitude, so therefore, K v N is greater than K v 0 and we know that steady state error is 1 by K v. So, if it is greater than earlier K v, so it steady state error will be less than the earlier uncompensated system.

For example if we put pc equal to point 0 1 and z c equal to minus point 0 1. So, z c by pc equal to 10, so here we can see that these pc and z c are very close to the origin and. So, they are and; however, they are they have the 10 times the the 0 is 10 times further than the pole and. So, we will get here K v and 10 times the original and compensated systems K v and. So, the error we will get 10 times reduction in the a steady state error and this will satisfy this angle conditions here.

So, we see that the ideal integral compensator can lead to the steady state error 0 that is p i controller can lead to the steady state error 0, but the lag compensator will reduce the error in this ratio that is z c by pc because here we can write 1 by K v 0 into z c by pc and that is so here K v N. So, here we can write 1 by 1 by K v 0 is infinite of the uncompensated system by z c by pc.

So, here we can see this is if this is 10 we can have the at the steady state error 1 by 10 of the uncompensated system. So, this will reduce the steady state error this lag compensator in the ratio of z c by pc and it will provide the minimal effect on the transient response, because these 2 added pole and 0 are very close to the origin now we will take 1 example to understand this, so let us take 1 example, so here we have a.

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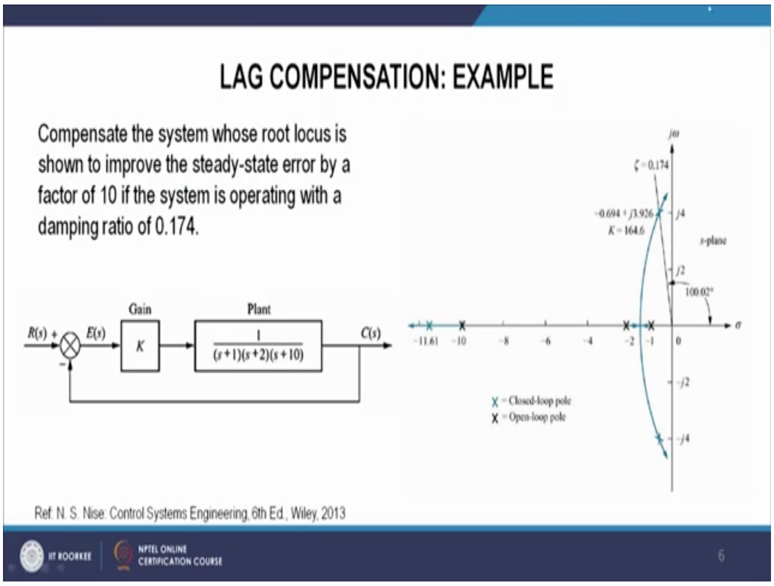


System we have to compensate the system whose root locus is shown to improve the steady state error by a factor of 10 if the system is operating with the damping ratio of 0.174.

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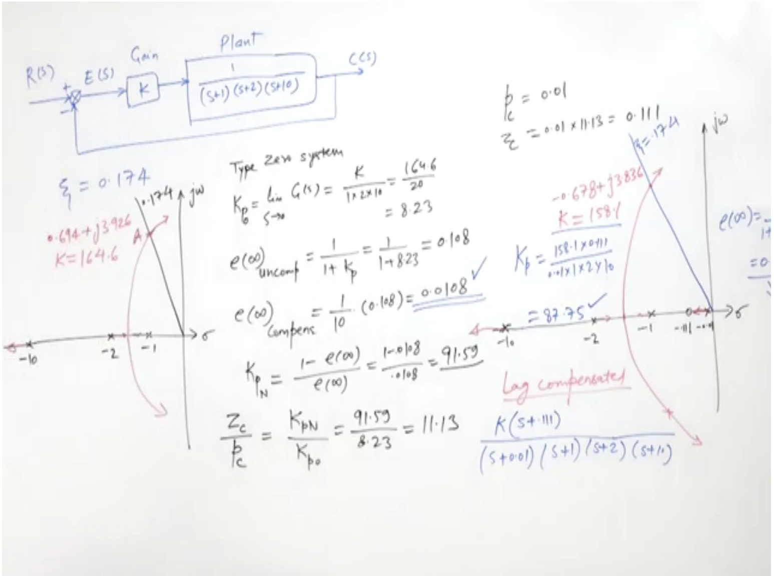


So, we have a system here, so this is gain and this is plant. So, here we have to find the steady state error we have to improve the steady state error by a factor of 10, and the system is operating at damping ratio 0.174. So, first we must find that what is the steady state error of the uncompensated system, then only we can know that what is the objective or of the steady state error that is 1 by 10 of the current steady state error.

So, for this we have to find the K p and we can find that when we have a system, and we have this line 0.174, and we have here minus 1 minus 2 and minus 10. So, we have a root locus that is cutting this line and this root locus. So, here it may cut this line A and that is 0 point this A is 0.694 plus j 3.9 2 6 for K equal to 164.6.

So, this point this root locus is cutting for this damping line at this point, 0.694 plus j 3.926. So, we can find that this K p because this is the type 0 system. So, the K p we have to find the static error constant. So, K p equal to G s here limit S tends to 0 and that is equal to here K the total system is K by S plus 1 S plus 2 S plus 10, so K by 1 into 2 into 10. So, K is 164.4 by 20, so we get 8.23 this K p and. So, uncompensated systems has the steady state error 1 by 1 plus K p that is 1 by 1 plus 8.23 and this is 0.108.

So, this is the steady state error of uncompensated system now the desired steady state error of compensated system that is equal to 1 by 10 of 0.108. So, that is equal to 0.108, so what will be the value of K p for to obtain this. So, K p equal to we can use this here and K p equal to we will have 1 minus e in steady state steady state error, so 1 minus 0 0.0108 by 0.0108.

So, this K p value we obtain is 91.59, so we must have this value of static error constant, if you want to satisfy the steady state error design now we know that z c by z p equal to K p n by K p 0. So, this is K p n n that is K p 0 because we want to know the 0 and pole location of the compensated system, lag compensated system. So, here we will have we know this relation that the ratio is equal to the we develop for earlier case K v n by K v 0.

Similarly, it will be K p n by K p 0 and that equal to 91.59 upon K p 0 that is 8.23 and. So, we will have 11.13 . So, here now we can select some value of sorry here it is pc, we can select some value of pc. So, because this is a design process we can take some value a pc that is close to the origin. So, if we take p c equal to 0.01. So, if we take here pc equal to 0.01 we can find z c equal to 0.01 into 11.13.

So, we get 0.111, so now, we can plot this compensated system root locus. So, here we can have this system root locus. So, now, we have here pc equal to at 0.01 minus 0.01 and this is z c equal to minus 0.111, and the other system poles are at minus 1 minus 2 and then minus 10 and we have here damping equal to 0.174 line, and we can see the root locus here he starts and ends to this 0 here it will start here start here and this will start and go to infinity.

So, this root locus will go and cut this line and it will cut at this point. So, it cutting minus 0.678 plus j 3.836 for the value of K equal to 158.1 of course, here will be the another point and other pole will come to this side. So, the lag compensated system, we have the lag compensated system. So, the lag compensated system will be here. So, we will have 1 K S 0.111 upon S plus 0.01 S plus 1 S plus 2 and S plus 10, here K equal to 158.1. So, this is the lag compensated system and if we find the K p value of K p for this system.

So, value of K p equal to, so 158.1 into 0.111 by 0.01 into 1 into 2 into 10. So, this is K p when S tends to 0 we can calculate this value and we will get that this value of K p is 0.818. So, this is the value of K p and when we want to we can calculate the steady state error from here by 1 by 1 plus K p and when we compute the steady state error we will get this as 0.011.

So, this was our objective and we obtain this steady state value error value by lag compensated system and we maintain this point on the damping equal to 0.174 line, so here the values of K we get 164.6.

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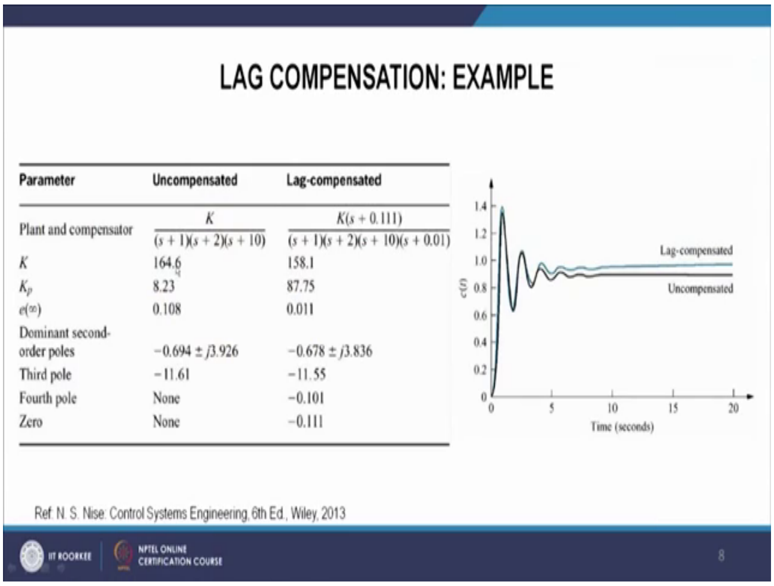


And 158.1 and K p is we are getting for uncompensated system 8.23 and 4 lag compensated system we are getting 87.75. So, I think here is some mistake and we can have this value at 87. 75.

So, there is this mistake that this is not this value, so we can have K p 87.75 and this thing we can see here in the this table. So, we have this system uncompensated this is compensated system we have K here was 164.6, when this point was on the damping equal to 0.174 line here it is 158.1 this is K p and here we can see the response that is uncompensated system and lag compensated system. So, we can see the response of this lag composite that is improved the steady state error, but not the 0 steady state error, but there is some finite a steady state error.

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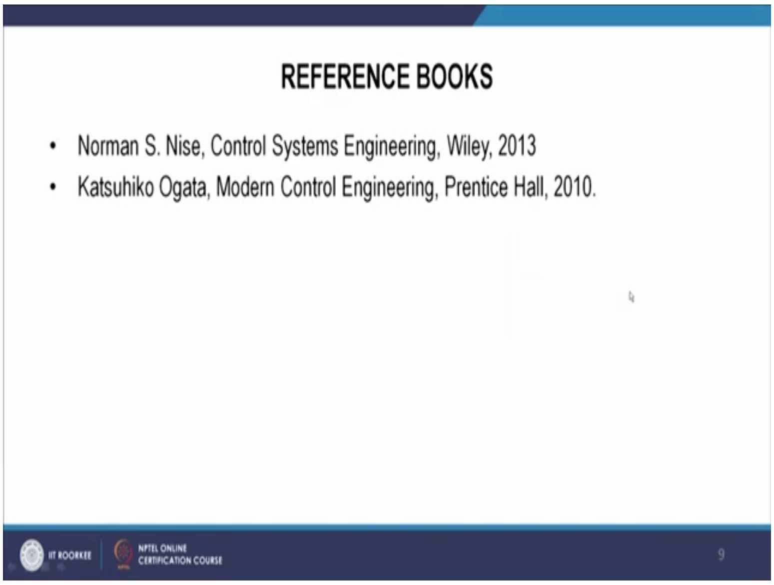


So, this example was taken from the Norman S Nise Control Systems Engineering. So, I thank you for attending this lecture and see you in the next lecture.