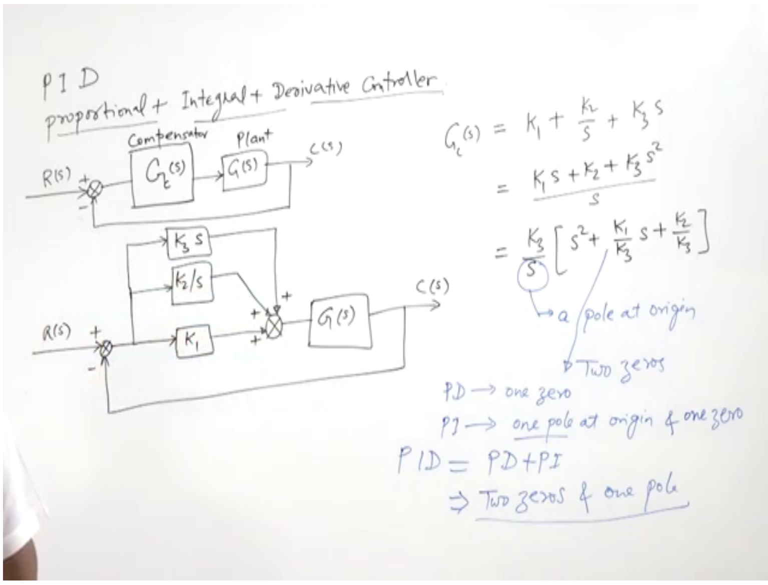
Automatic Control
Dr. Anil Kumar
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture – 28
PID Controller Design

So, welcome to the lecture on design via root locus and compensation techniques. So, in previous lectures we discussed about PI controller and PD controller. So, the PI controller we used to find the desired steady state response steady state error for the response and the PD controller we used to find the desired transient response of the system.

So, now we can obtain both the desired steady state response as well as the desired transient response using the controller that is called PID controller. So, this is an active controller and this active PD controller followed by an active pa controller is called the PID controller and that is proportional plus integral plus derivative controller.

(Refer Slide Time: 01:48)



So, it is PID controller. So, this is proportional plus integral plus derivative controller.

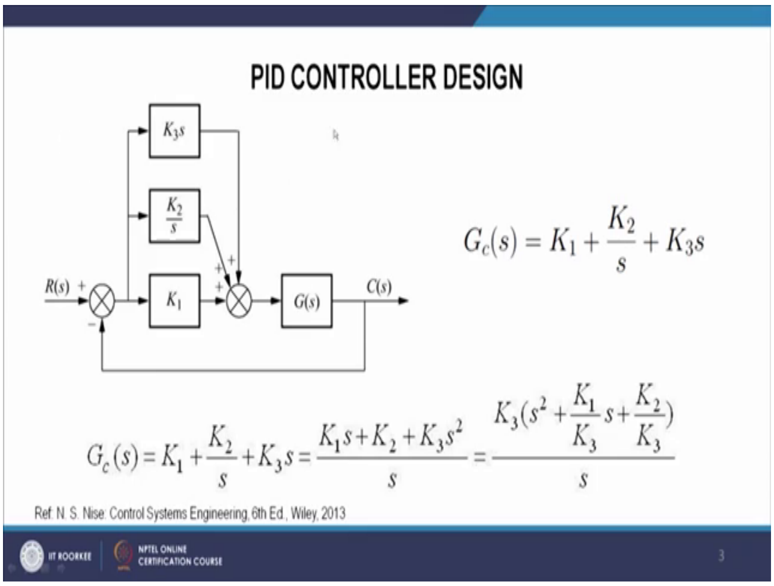
So, as we discuss we can have a controller like this. So, if we have a system like this K g S we want to design a compensator. So, we replace here with some compensator and this is plant. So, this compensator is a PID controller and if we say G C S the compensator transfer function. So, this we can represent as. So, this is the proportional part K 1this is integral part K 2 by S and this is the derivative part K 3 S.

So, proportional integral and derivative these three parts added here on the summing junction and then they are input to the plant and here is the output of the plant then there is a unity feedback to this system. So, we can see that G C S is equal to K 1 plus K 2 by S plus K 3 S. So, proportional and integral and derivative part of this controller, we can write K 1 S plus K 2 plus K 3 S square by S and we can write K 3 by S K 3 by S S square plus K 1 by K 3 S plus K 2 by K 3.

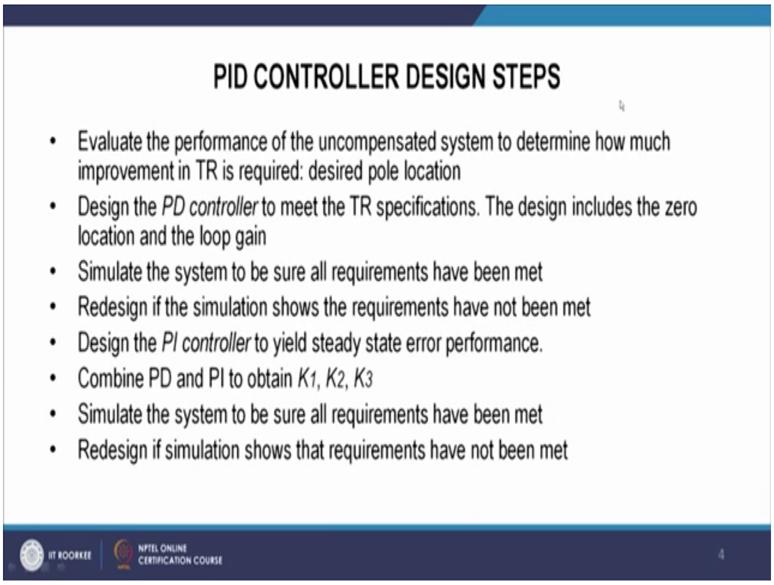
So, here we have this S is a pole at origin and this term is in the numerator and. So, this is two zeros. So, here we see that the transfer function of this PID compensator or PID controller is putting a pole at origin and two zeroes on this system. Now, we know that here we have for PD controller we have 10, near the origin and 4 PI controller we have one pole at origin and 10 near the pole and 10 near the pole.

So, we have PID that is PD plus pi and. So, therefore, we have here 10 and 10. So, 20 and one pole. So, we are getting two zeros and one pole here in the PID controller. So, if we want to we can see from here also the same circuit we have proportional integral and derivative we have obtained this poles n zeroes.

(Refer Slide Time: 08:07)



(Refer Slide Time: 08:17)



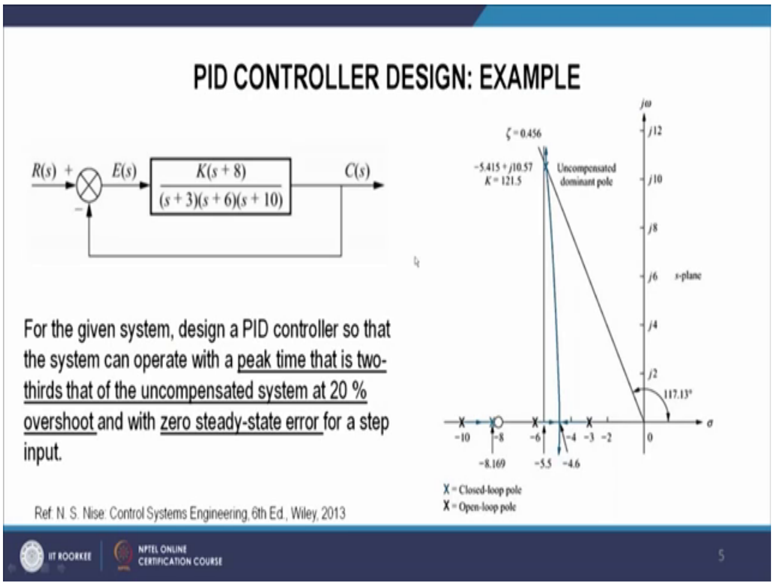
Now, if we want to design a system for PID controller we have to follow the certain steps of design. So, what we follow that we have to evaluate the performance of the uncompensated system to determine how much improvement in transient response is required. So, based on our desired performance we have to find the location of the pole desired location of the pole and then we have to first design the PD controller to meet that transient responses specifications and in that step we will find a 0 location of a 0 to. So, we add a 0 to the system in PD controller to find the desired tangent response.

Then we redesign once we have find this PD controller 0 we simulate the system to see if we are getting the transient response as per our requirement if not we will redesign the system for the PD controller part once we have got the PD controller design and we are satisfied with the transient response that we are getting from the PD controller we will go to design the PI controller so that, we can get the desired steady state error performance.

So, once we obtain the PI controller that is 10 and a pole at origin then we can combine the PD and PI to obtain the values of K 1, K 2 and K 3. So, we can get K 1, K 2 and K 3 and. So, we obtain these PID controller parameters and then we simulate the system to see whether we have obtained the design performance the in terms of both the tangent response and a steady state response because when we go for a steady state response using PI controller it may affect the little the transient response.

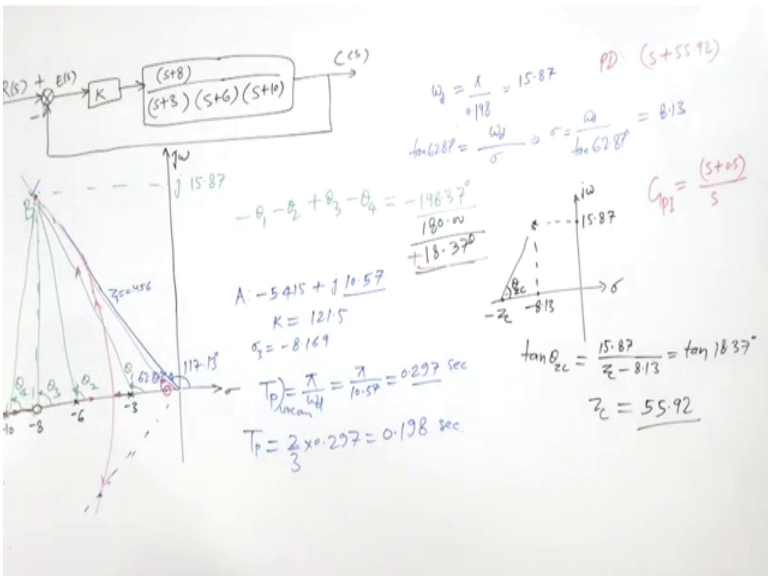
So, therefore, we have to do the simulation to get whether we are co that we are getting what we want if not then we will redesign the system.

(Refer Slide Time: 11:32)



So, here we take one example for this. So, this is the example we can see that here.

(Refer Slide Time: 11:47)



We have this system. So, this is input then we have here this K. So, this is the system K S plus 8 by S plus 3 S plus 6 S plus 10 and we have to find the PID controller design. So, that the system can operate with the peak time that is two-thirds that of the uncompensated system at 20 percent overshoot and with 0 steady state error for a step input.

So, here we want that the peak time is two-third of the uncompensated system. So, T P equal to two-third of T P of uncompensated system at 20 percent overshoot and steady state error is 0 for a step input. So, here we can see that this system is type 0 system, because there is no any pole at the origin. So, this is a type 0 systems. So, here the we have this a step input the test input.

So, now, we can try to see the root locus first that how the root locus of this system is. So, here minus 3 is 1 open loop pole then minus 6 and then minus 8 and here we have sorry this is pole at minus 80 at minus 8 and there is another pole at minus 10 and here we have this 20 percent overshoot line corresponds to zeta that is damping equal to 0.456 that is equal to cos theta.

So, we can get the theta equal to 60 to 2.87 degree. So, we can plot this zeta equal to 0.456 line and this root locus will start here and here and it will start here and end to this 0 and here the root locus will break away and. So, it will hit cut this line somewhere here let us say point A.

So, we can see here that this point A this damping line and this root locus intersect at point A and this point A is we can obtain this point A by the root locus techniques that I discussed to find a point that satisfies the angle condition odd multiple of 180 degree we can find that point A and point 8 is at minus 5.415 plus j 10.57 and at K equal to. So, gain is 121.5. So, at gain K equal to 121.5 this line intersects at a equal to this. And similarly here also this line will intersect this here the other line that is minus j 10.57.

The third of course, there will be the third pole and that will come here at sigma 3 equal to minus 8.169. So, that will come somewhere here now we have condition that the T P should be equal to two-third of T P of the uncompensated. So, uncompensated is we have to find this is the uncompensated system and we have to find the peak time and therefore, we have to peak time T P equal to pi upon omega d. So, pi upon omega d is this part the imaginary part; that is 10.57 and we will get this 0.297 second. So, this is the peak time for the uncompensated system.

Now, the peak time for. So, this is uncompensated system now the peak time for the compensated system we want that two-third into 0.297. So, we want that this peak time is now 0.198 we want a faster response. So, for these peak time we need to relocate the pole and. So, we have to find the location of the new pole that is that will give this peak time. So, omega d now we can get omega d for this the imaginary part.

So, omega d for the compensated system is from this formula we can find pi upon T P. So, pi upon 0.198 and we will get 15.87 this omega d that is for the compensated pole and that sigma we will get equal to omega d by. So, this angle we say this is 62.87 and this is 180 minus 62.87. So, that is 117.13 degree. So, if we have some pole at B. So, we have we can get this tan theta equal to imaginary part upon a real part.

So, imaginary part upon real part equal to tan 62.87 this implies we get sigma equal to omega d by tan 62.87. So, we get 8.13. So, we have sigma here at 8.13 sites. So, the new location the new location we want somewhere here 8.13 and 15.87.

So, maybe this line is going to cut here at point b this is the new location that is coming here at 8.13 and coming to this axis j 15.87. So, this is the location B and this is A. Now, we want this response transient response corresponding to the point p. So, this is the point B that we are interested in now. So, we have to design a PD controller because we want to improve the transient response. So, for PD controller we need to put a 0 near to the origin.

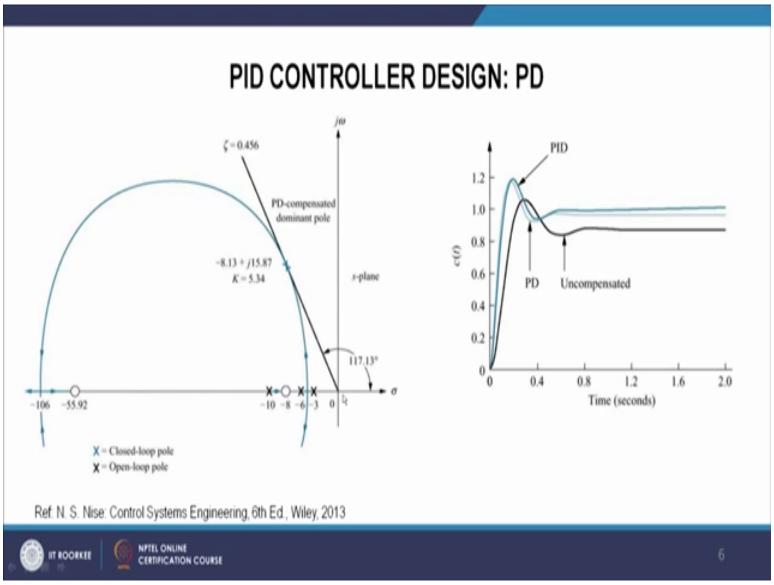
So, what will be the location of that 0 that we have to find now if for this point, if we calculate the angles. So, let us calculate these angles. So, we have these angles this angle and these angle this angle. So, if we say this is theta 1 this is theta 2 this is theta 3 and this is theta 4. So, we have this point how much angle it is making with respect to these uncompensated open loop poles and zeros. So, we get minus theta 1 minus theta 2 minus plus theta 3 minus, theta 4 this angle we can calculate and we see that this angle is coming equal to minus 198.37 degree.

So, this pole is making minus 198.37 degree. So, it is not the odd multiple of 180 degree. So, this could be minus 180 degree if we can add a 0 and that could contribute some small angle that is the difference between this angle and 180 degree that is 18.37 degree. So, if we add a compensated 0 near some point here we add some compensator 0 here. So, that the contribution of this compensator will make this angle equal to 18.37 degree and because this will be plus. So, we add this and this will make this condition to be 180 degree and so, this point will follow the new root locus because this will again start going to the root locus.

So, here we have to calculate let us we have here is our 0 that we locate and we have minus 8.1 this is minus 8.13 and this is 15.87 and here is the z c. So, now, we have here is this pole location desired pole location and this angle here theta z c that should be 18.37. So, here tan theta z c equal to 15.87 n upon 8.13 sorry 8.13minus. So, it is z c minus 8.13.

So, that is equal to tan theta z c equal to 18.37. So, here we can find z c 55.92. So, we are getting that here z c is even beyond here 55.92 [. So, that it is making angle that is equal to 18.37 and this angle is going to make this equation minus 180 degree angle and. So, this point B will be on the root locus and also on the line that is z equal to 0.456. So, we will get satisfy the condition of the peak time as well as the, we will keep this percentage over shoot.

(Refer Slide Time: 28:40)



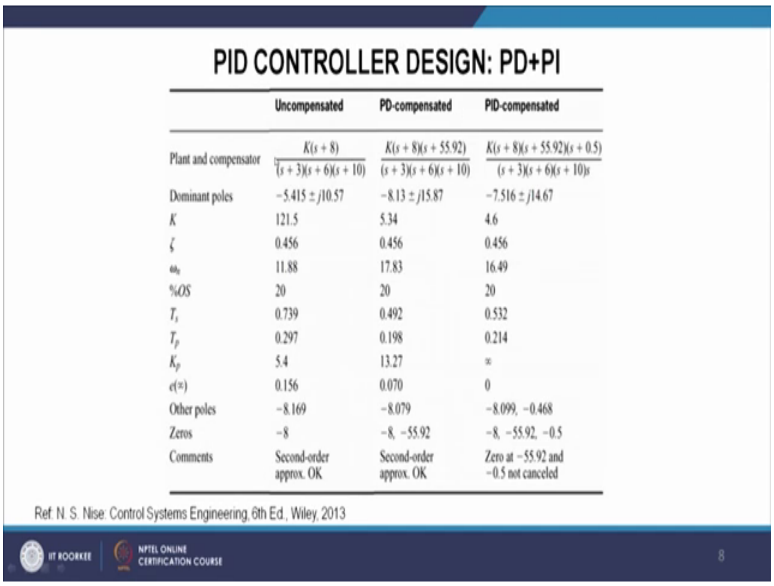
So, here we can see when we are adding this 0, because this is our original system minus 3 minus 6 minus 8 minus 10. Now, when we have added a 0 at minus 55.92 we are getting this root locus passing and cutting this line zeta equal to 0.456 location 8.13 plus j 15.87 and at K equal to 5.34 and this is the response PD response we can see here uncompensated response and PD response.

Now, we have designed the PD response we have to design the PI response. So, for the PI response we have to add a 0 at a pole at origin and a 0 close to that pole. So, let us we add this PI controller as PI equal to because here the PD design PD design was that we added this controller as S plus 55.92 we added a controller at minus 55.92 for PI we add a 0 here and at we add one we add a pole at origin and we add a 0 at S equal to minus 0.5.

So, when we add this PI controller. So, we get the PID and we can see here we added in this root locus here is a, this pole at origin. So, system type is now changed and it is type one system and it will have 0 steady state error and here we will have this 0. So, poles starting here will end at this 0 and the two poles start and go break away and go and cut this line at zeta equal to 0.456 at this point and here this pole starting will end at this 0 and there is already 10.

So, the one pole will come and end to this 0 and other pole will tend to the infinity and this is the PID controller we can see that here uncompensated system this is PD and this is PID. Now, we can see here this PID controller it has improved this steady state error because PD controller is not has not also it is has improved the steady state error with respect to uncompensated system, but PID controller is completely improved this steady state to 0 and from this table.

(Refer Slide Time: 31:56)



PID CONTROLLER DESIGN: PD+PI

	Uncompensated	PD-compensated	PID-compensated
Plant and compensator	$\frac{K(s+8)}{(s+3)(s+6)(s+10)}$	$\frac{K(s+8)(s+55.92)}{(s+3)(s+6)(s+10)}$	$\frac{K(s+8)(s+55.92)(s+0.5)}{(s+3)(s+6)(s+10)s}$
Dominant poles	$-5.415 \pm j10.57$	$-8.13 \pm j15.87$	$-7.516 \pm j14.67$
K	121.5	5.34	4.6
ζ	0.456	0.456	0.456
ω_n	11.88	17.83	16.49
$\%OS$	20	20	20
T_s	0.739	0.492	0.532
T_p	0.297	0.198	0.214
K_p	5.4	13.27	∞
$e(\infty)$	0.156	0.070	0
Other poles	−8.169	−8.079	−8.099, −0.468
Zeros	−8	−8, −55.92	−8, −55.92, −0.5
Comments	Second-order approx. OK	Second-order approx. OK	Zero at −55.92 and −0.5 not canceled

Ref. N. S. Nise: Control Systems Engineering, 6th Ed., Wiley, 2013

NPTEL ONLINE CERTIFICATION COURSE

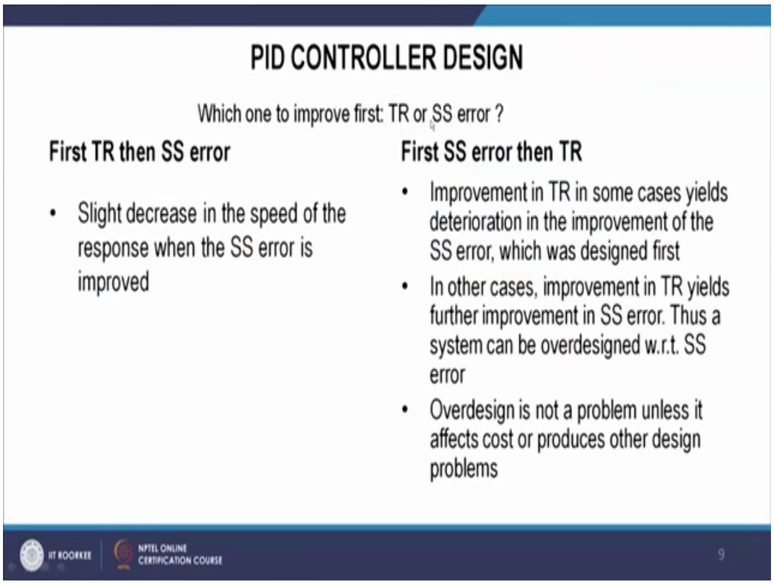
We can see that there are these plant and compensator.

So, here we have original system that is uncompensated system this is PD compensated and this is PID compensated. So, here we added one 0 at minus 55.92 and here we added a 0 at 0.5 and a pole that is S at origin we get these dominant poles here. So, here we see that when we add the PI we changed little bit this dominant poles locations. So, transient response we change little bit.

So, here we see that gain is changed damping is same here percent overshoot is same twenty percent here the settling time we here using PD we improved the settling time 0.492, but here we for PID the settling time is more. So, the steady state will be obtained later than the PD compensated system and similarly here the peak time also occurs later for the PID. So, here after using the first we designed PD for the transient response and then we design pi for steady state response and we lost some of the improvement in case of transient response.

Now, here the question is in case of PID controller design which one to improve first whether first we should follow the transient response or our steady state error.

(Refer Slide Time: 33:44)



So, if we first do the transient response then steady state error that is slight difference in the speed of the response when the steady state error is improved that we saw here and now what happens if we do first S S steady state error like we first do PI then we do PD then improvement in transient response in some cases it is deterioration in the improvement of the steady state error which was designed first.

So, if we first design for the steady state error and then go for transient response we may face some decrease in the steady state errors improvement, but in some other cases the improvement in transient response is further improvement in a steady state error like previous examples we saw in case of pi we saw that and PD that we saw that the improvement in case of PD example of PD controller we saw that improvement in transient response also improves the steady state error.

So, the system is over design with respect to the steady state error; however, over design is not a problem unless it affects the cost or produces other design problems. So, here we took the references from this Norman S nice control systems engineering this examples. So, here we would like to stop and see you in the next lecture.

Thank you.