Automatic Control
Dr. Anil Kumar
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture – 27
PD Controller Design

So, welcome to the lecture on design via root locus and compensation techniques. So, in the previous lecture we discussed about PI controller design. In this lecture we will discuss one example based on PI controller design as well as the pt PD controller design that is proportional plus derivative controller.

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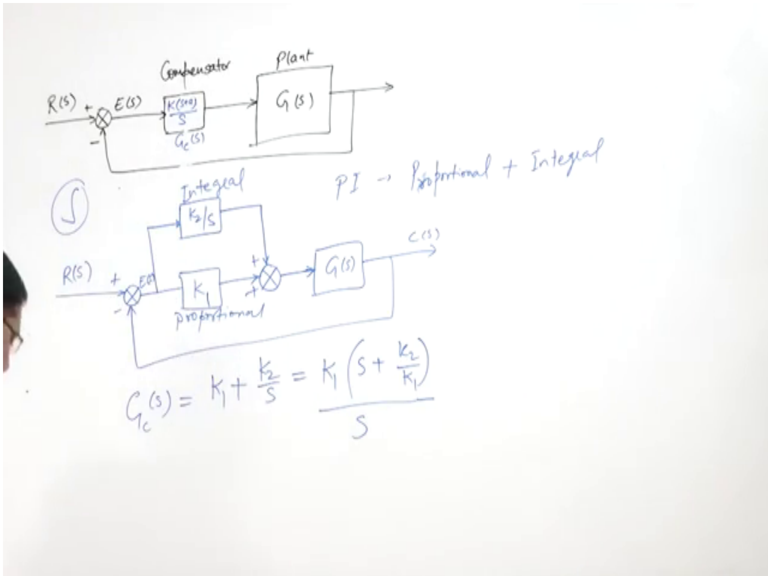

So, here we say that the PI controller, that is have 2 parts proportional plus integral and how with these 2 parts we can get the equivalent transfer function for these PI compensator.

Now, we take one example for this.

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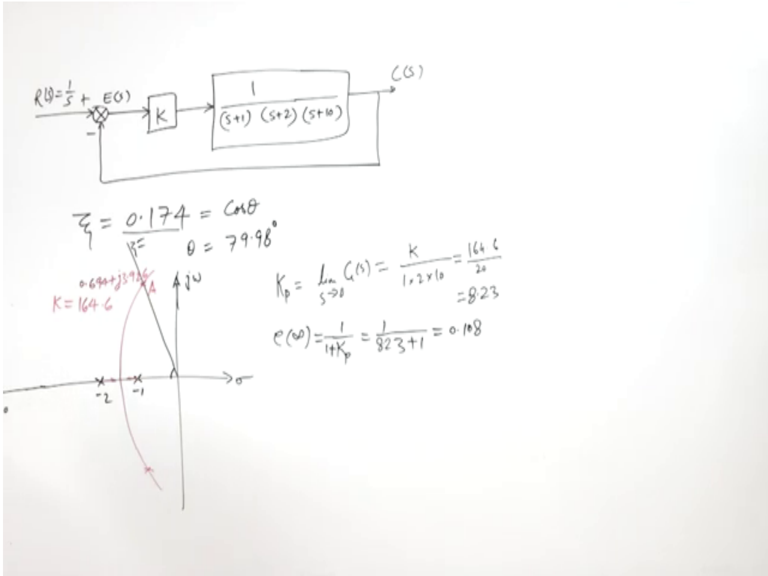

So, we can have let us take the example. So, we have this system here is the gain K. So, here we have we are giving the step input. So, we want that this is the system and we can see that here there is no any in this system there is no any pole at origin because there is no any S. So, this is type 0 system and this type 0 system is subjected to step input.

So, RS equal to 1 by S, now we want to design this system for 0 steady state error and at the same time we want that the transient response of the system remains for damping equal to 0.174 line. So, we want that the system is designed for a steady state remain retaining the transient response corresponding to damping equal to 0.174. So, we know that this cos theta and so, theta equal to cos inverse 0.174 and we can get, here if we plot the root locus for uncompensated system.

So, this is sigma j omega we have minus 1 minus 2 and let us have minus 10 and this is theta equal to 79.98 from here. So, we will have this is the line that is zeta equal to 0.174 and this angle is 79.98. So, we have a root locus will start from the poles and it will go to cross this line at some point and we get that.

So, we know that how to find this point that intersect some particular damping line and. So, this point we can find with the condition that if this point we can select some points on this line and the point that satisfies the angle condition that is the angle is odd multiple of 2 K plus 1, 180 or odd multiple of 180 degree.

So, that point is on the root locus as well as on this line. So, we find this point is we can find this point as 0.694 plus j 3.92. So, here we are getting this 0.694 plus 3.926 j and similarly the other point will be here that if with minus j 3.926 and the third point we will get here and, that is minus 11.61 minus 11.61 and we are getting gain for which this point is on the this damping line this case 164.6. So, this is the uncompensated system where we are getting this point on this damping line that satisfies this transient response conditions.

So, this point let us say a now we want to that we want to know that, what is the steady state error at this point? So, a steady state error here is. So, we first calculate K P equal to limit G S S tends to 0. So, we will get this is G S is this complete system. So, K upon 1 into 2; into 10 because S is 0 so, and K is 164.6 by 20. So, we are getting 8 as 8.23.

So, we are getting K P as 8.23 now we can find e infinite that is the steady state error that is 1 by K P. So, 1 by 8.23 and 1 by, sorry; 1 by 1 plus K P so here equal to 0.108. So, this is the steady state error currently. So, there is some finite error in the system, now if we want to design compensate in spite of just gain. So, that the steady state error is 0. So, let us take one compensator. So, we have here we can have in this circuit we can have a compensator. So, let us have K upon S. So, one pole and we put some 0 in close to the pole that is 0.1.

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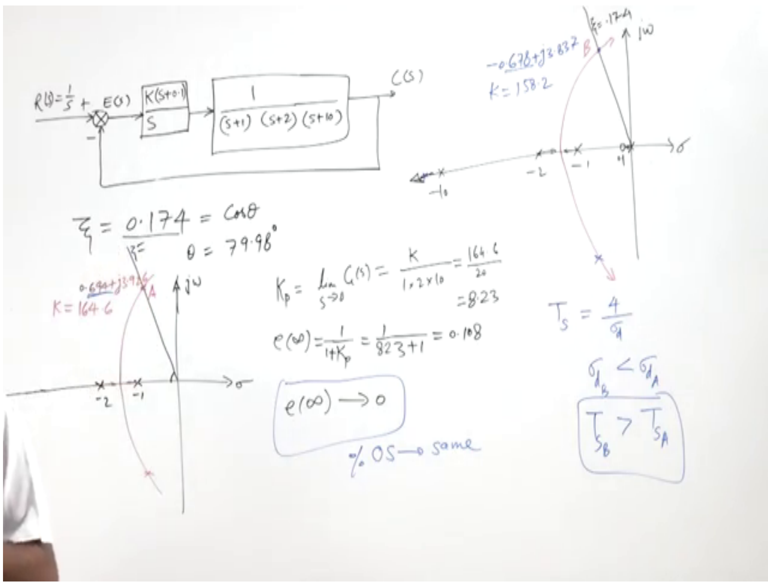

So, because this pole is at origin and we put at 0.1 close to this. So, that now the system type is increased, because this complete transfer function will have one pole at origin. So, the system type is increased and therefore, when we have 1 S here this 0 will keep K P infinite and K P infinite will keep this steady state to tend to 0. So, we will get this steady state now, what happens to root locus. So, we have this point. So, here is minus 1 minus 2 and here is minus 10.

Now, we have one pole here at origin and there is at 10 at 0.1. So, we this root locus will start and end to this 0 this will start here and this will go to this point. So, the root locus will and this is the zeta equal to 0.174 line and the root locus is going to cut this line let us say this is point B and we can find this point B and we get that it is minus 0.678 plus j 3.837.

So, it is cutting this point here and gain is 158.5 and similarly here this second point is here and third point here and so, on. So, we are getting that here TS equal to 4 upon sigma d and we see that here we have sigma d earlier uncompensated system this we have got the steady state 0 we have got by putting one pole at origin, but due to the effect of this 0, we will get a very small change in the transient response characteristics.

So, we see that here it was 0.694. Now it is 0.678. So, sigma d at B is greater than less than sigma d at A this 0.678. So, TS B is greater than TS A. So, we can see that we will have the faster response. So, settling time is increased with the same percentage overshoot. So, here percentage overshot is same, but settling time increased and still. So, here we see that there is minor increase in the settling time and. So, we are able to keep the transient response closer to the earlier one. So, it is we can see without much affecting or less with less affecting this settling time we have found a steady state 0.

So, we have designed this system now we come to another system that is the PD controller.

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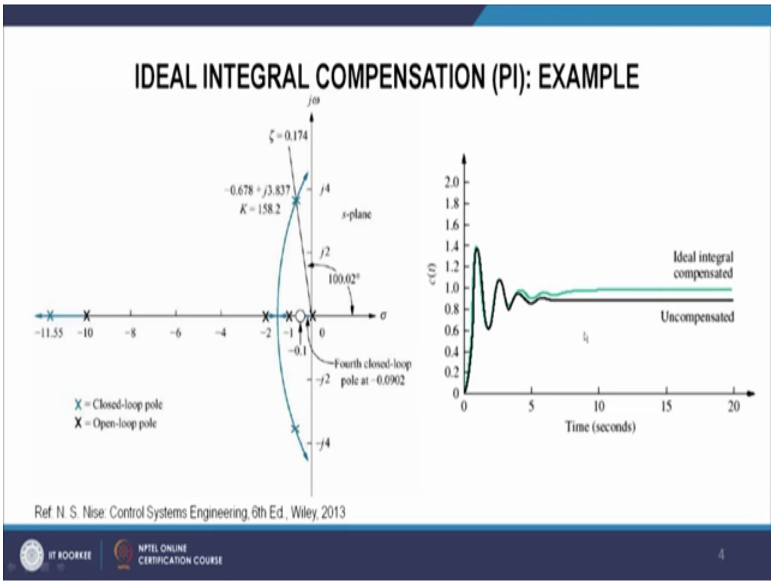

So, here we can in this diagram we can see the response of the 2 system. So, this is uncompensated system and we can see that uncompensated system is of course, there is this is the transient response characteristics, but the steady state error is large; however, with the ideal integral compensation we have increased the we have improved the steady state error because steady state error is tending to 0 and at the same time we see that settling time is larger.

So, here it settles earlier, but now we are saying that the settling time is increased for this system. So, we have got larger settling time. Now we come to the ideal derivative compensator. So, ideal derivative compensator we used to improve the transient response, because the PI controller we used to improve the steady state error. Now, this ideal derivative compensation technique we used to improve the transient response. So, we find we want to find some desirable percentage overshoot and a shorter settling time than the original system.

So, it means we want to improve the transient response we want sort fast response, because we want shorter settling time. So, this we can do by choosing an appropriate closed loop pole location on the s plane. So, if we want to improve the transient response we can select a point on the root locus or on the s plane. So, that we can get the desired transient response; however, if the point, that is giving the desired transient response is not on the root locus.

Then we need to use this PD controller if it is under root locus, then simple gain can we can obtain some value of gain that will give that response, but if it is not on the root locus. So, we have to augment PD controller or PD compensator to the system and that to obtain the desired transient response. So, in this case we add a single 0 to the forward path.

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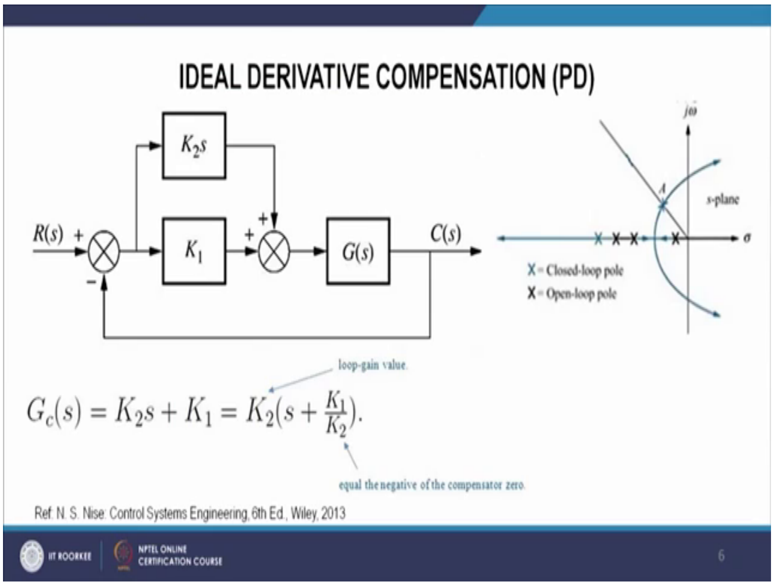

So, ideal PD controller we can see it is similar to. So, here we say that we are going to add a 0 to this plant.

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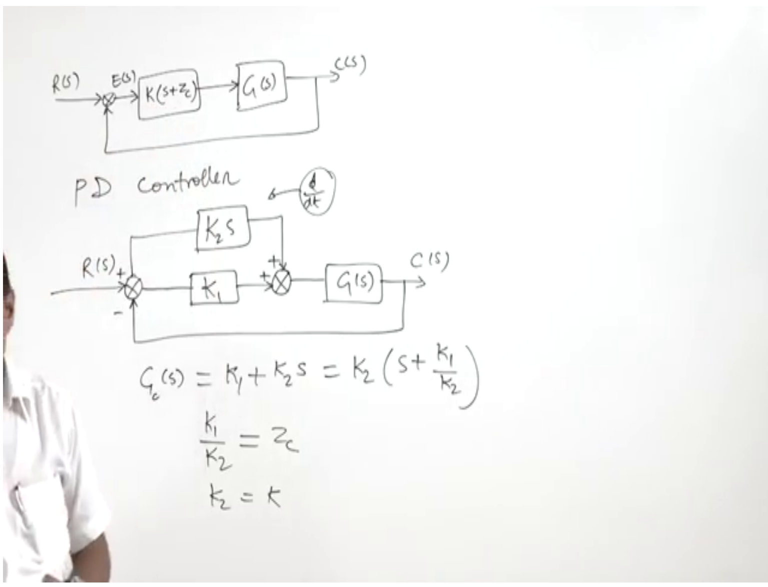

Now, this PD controller is proportional plus derivative. So, we have this RS here is proportional part let us say that is K 1 and then here is the derivative part and here is the G S and here is C S and derivative means we have K 2 into S. So, Laplace transform of a derivative. So, we have a derivative. So, d by d T function. So, the Laplace transform is K 2 S. So, in time domain it is derivative d by d t, but in s domain it is K 2 S.

So, now if we want to find G C S, we have K 1 plus K 2 S. So, this is the transfer function now we can have K 2 S plus K 1 by K 2. So, K 1 by K 2 will help to select the location of the 0, that is z c and that is equal to the. So, K 1 by K 2 is equal to z c; if we compare here and K 2 is equal to K. So, K 2 is selected as the gain proportional. So, here K corresponding K is K 2 and K 2 by K 2 is z c.

So, we can get K 1 and K 2 and we can get this PD controller circuit. So, now, let us take one example for PD controller. So, we have a system here.

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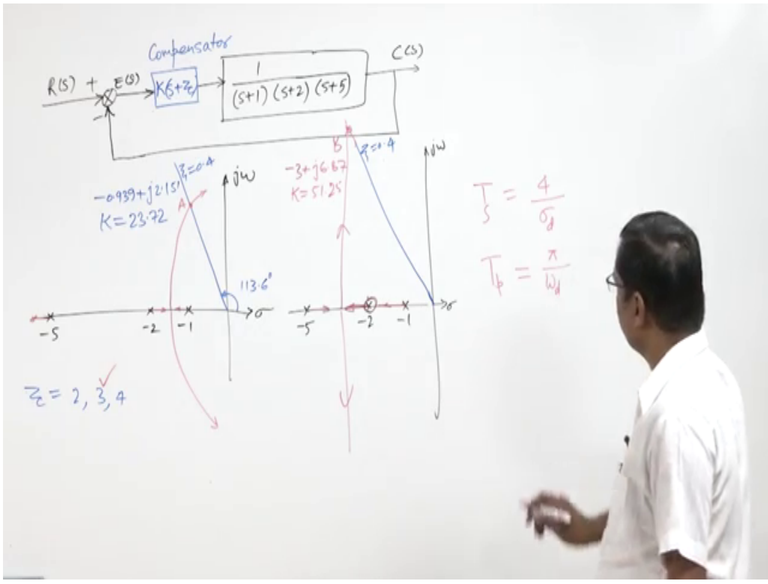

So, R S. So, we have this system and this system we have to this is an uncompensated system and we have transient response corresponding to the damping equal to 0.4 line.

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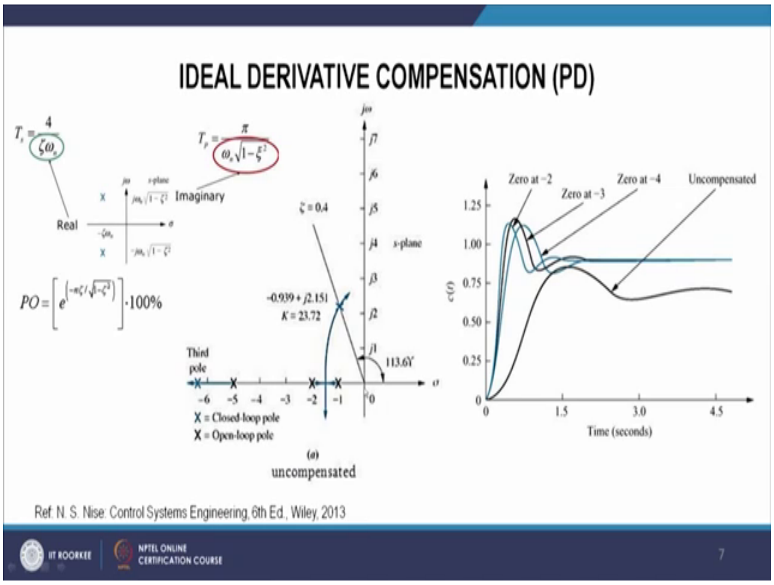

So, we want to keep the percentage overshoot or damping corresponding to this line at the same time we want to reduce the settling time.

So, we want to improve the speed of the response. So, faster response we want. So, here we can see the settling time is 4 upon zeta omega n and therefore, if we want to improve this settling time; however, percentage overshoot is function of the damping and. So, the damping will be constant if we want to move with 0.4 lines. So, if we plot the root locus for this system we can see here. So, here minus 1 minus 2 and minus 5 there is no any all the zeros are at infinite open loop 0.

So, we have this root locus and we have this is the line corresponding to zeta equal to 0.4 this is a line and from this angle from here is 113.6 degree. So, now, the root locus that will pass here and it will pass through this path and it is going to cut here at point a and this point a is minus 0.939 plus j 2 0.051 and the gain K we are getting is 23.72.

So, in this diagram we can see here we have this pole minus 1 minus 2 and minus 5 and the third this corresponding to K equal to 23.72 we are getting the location of these closed loop poles as minus 0.939 plus minus j 2.151 and the third pole is at this location beyond minus 6 and this line uncompensated this is showing the response of this system this line black line uncompensated response.

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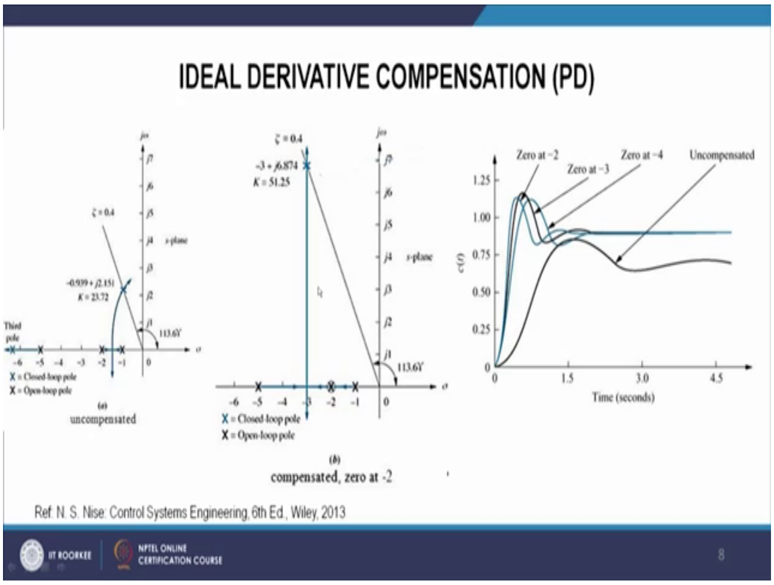

Now, we want to because if we want to have a PD controller we have to add here zero. So, here it should be K S plus z c this is the compensator PD compensator we are going to add 0. So, let us we try three conditions we add and check whether we are adding 0 at minus 2 minus 3 and minus 4. So, we are adding this z c is 2, 3 and 4. So, z CS plus 2. So, we are getting S equal to minus 2 here this zero. So, first condition is we are going to add a 0 at this location itself.

So, here this root locus we can plot sigma we are going to add here. So, this is minus 1 minus 2 and minus 5 and we are going to add here 10 at minus 2. So, we can see the root locus will be here it will start and end to this 0 and to this 0 and root locus will start here it will start here and here is this zeta equal to 0.4 line and. So, this line will cut here to this line somewhere some point it will cut let us say B and. So, here we can see that at 0 is minus 2 this line is going to cut zeta equal to 0.4 this root locus is cutting at minus 3 plus j 6; 0.7 plus j 6.8 at K equal to 51.25.

So, here what we have done because this point we can see earlier the it was minus point 0.939. Now it is 2.1 and plus j 2.15. Now on the real axis part is increased. So, it is now minus 3. So, the settling time because this real part is increased the settling time will decrease the response will be faster. Now come to the next one that is we are adding this 0 at minus 3. So, we are now taking this part. So, we are adding this at minus 3 and we can see when we are adding 0 at minus 3.

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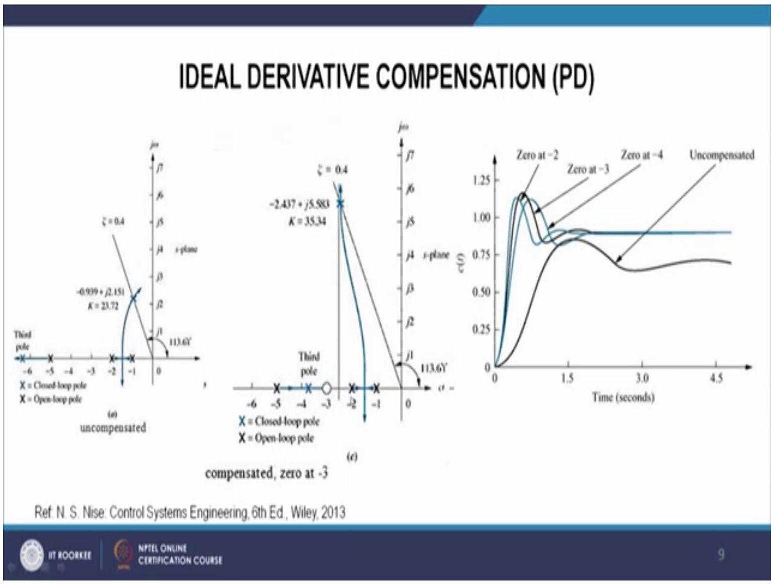

The root locus will always form at the left of the odd number of poles are 0. So, it will be here. So, and it will be here.

So, we can see this locus starting at these poles and going like this the other lower part start here and ends here this part third. So, here it is cutting this damping line at minus 2.437. So, again it this real is more than the uncompensated system. So, in this case also the settling time is improved? Now come to the compensator 0 at minus 4 when we are putting here at minus 4 again this root locus cutting at minus 1.869 that is almost double of this real part of uncompensated system and here also the settling time will improve. So, now, we can see the responses of these three and we said see that here this is p percent overshoot each constant we have keep constant percent over suit and the more negative real and imaginary parts smaller will be TS and T p.

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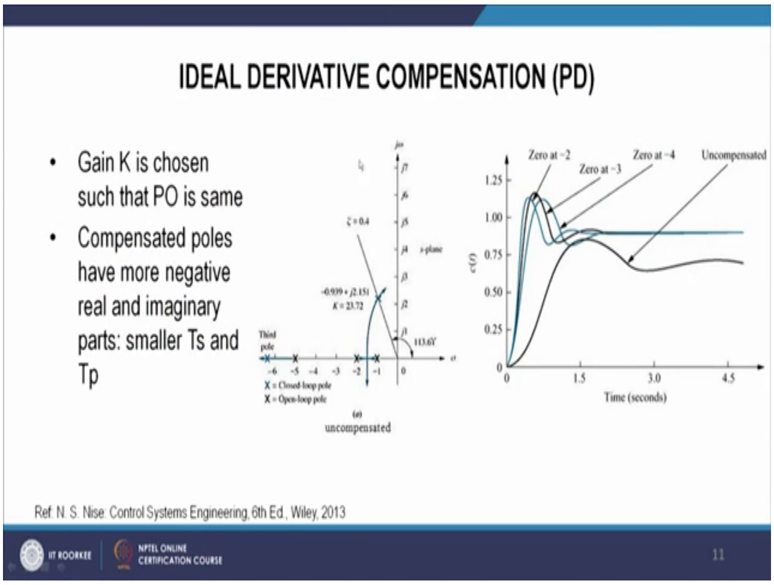

So, because here TS is four by sigma d that is real part and peak time T p is pi by omega d that is imaginary part. So, if they are more than larger than the uncompensated system the corresponding TS and T p will be smaller and so, the response will be faster. So, here we can see that in compared to uncompensated system for all these three zeros that we have selected or compensated schemes we have retained the same percent overshoot, because the damping line is constant all these points are on this damping equal to 0.4.

So, percentage overshoot is the same; however, we can see that at 0 at minus 2, we have faster response than at minus 3 and then at minus 4 we are getting first the peak time less in case of 0 at minus 2.

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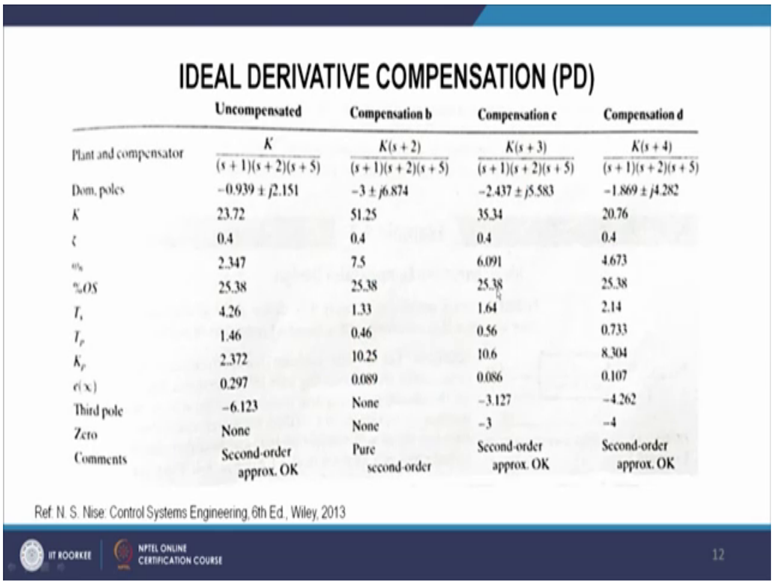

IDEAL DERIVATIVE COMPENSATION (PD)

	Uncompensated	Compensation b	Compensation c	Compensation d
Plant and compensator	$\frac{K}{(s+1)(s+2)(s+5)}$	$\frac{K(s+2)}{(s+1)(s+2)(s+5)}$	$\frac{K(s+3)}{(s+1)(s+2)(s+5)}$	$\frac{K(s+4)}{(s+1)(s+2)(s+5)}$
Dom. poles	$-0.939 \pm j2.151$	$-3 \pm j6.874$	$-2.437 \pm j5.583$	$-1.869 \pm j4.282$
K	23.72	51.25	35.34	20.76
ζ	0.4	0.4	0.4	0.4
ω_n	2.347	7.5	6.091	4.673
$\%OS$	25.38	25.38	25.38	25.38
T_s	4.26	1.33	1.64	2.14
T_p	1.46	0.46	0.56	0.733
K_p	2.372	10.25	10.6	8.304
$e(\infty)$	0.297	0.089	0.086	0.107
Third pole	−6.123	None	−3.127	−4.262
Zero	None	None	−3	−4
Comments	Second-order approx. OK	Pure second-order	Second-order approx. OK	Second-order approx. OK

Ref: N. S. Nise: Control Systems Engineering, 6th Ed., Wiley, 2013

So, this we can understand from this table this is uncompensated systems transfer function and they is compensated with 0 at minus 2 and this is compensated 0 at minus 3 compensated 0 at minus 4, we are getting the dominant poles corresponding to the gains that are passing through the damping line zeta equal to 0.4 and percentage overshoot 25.38 we are getting here the natural frequency changing the system.

So, earlier uncompetitive system as this natural frequency now we have these and we are getting settling time here we are getting 4.26 in all these three cases we are getting a great reduction in the settling time here 1.33 here. So, almost one-third here 1.64 and here 2.14; so, this is almost half settling time similarly we are getting reduction in peak time. So, here 1.46, but here only 0.46, 0.56, 0.73 and here we can see that K P we have calculated because this is a 0 type system. So, this position static error constant is valid.

So, we are getting these values, now if we see the steady state error here it was 0.297, but here it is 0.089, 0.086 and 0.107. So, we see that while we our target was to improve the transient response using PD controller, but at the same time we were able to also improve the steady state error, but this may not be always the case it could be also that the steady state error could be badly affected by improving the tangent response.

But in these cases the steady state error is improved and we see that here second order approximation is ok; because the third pole here is minus at minus 6 in compared to this real part is minus 0.9. So, 5 times far if the pole third pole is then the dominant poles real part then we can approximate as a second order system. So, here it is more than that. So, here we can have second order approximation, but in this case in this case we can see that this pole 0 will be cancelled. So, it is perfectly second order system.

Now, here again it is minus 2.4 and here 0 is at minus 3 say at third pole is minus 3. So, in this case I think there should not be the proper this approximation we cannot take.

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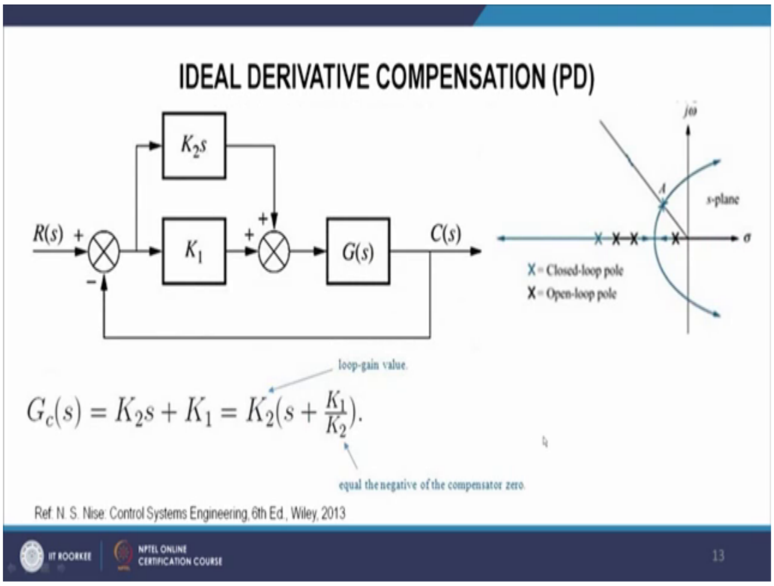

Now, we come to the, this ideal compensation ideal derivative compensation drawback.

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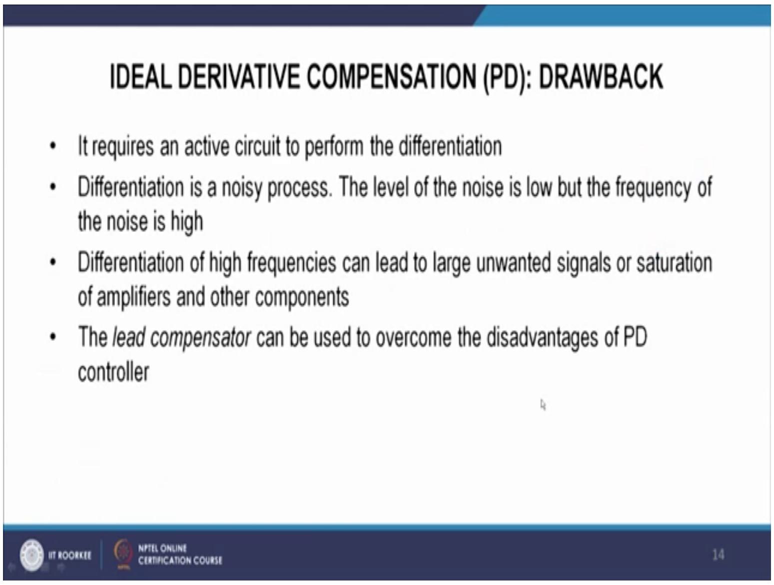

There is some drawback of this ideal derivative compensation we can have this ideal derivative compensation requires an active circuit to perform the differentiation and differentiation is a noisy process and the label of the noise is low, but the frequency of the noise is high therefore, differentiation of high frequencies can lead to large unwanted signals.

Therefore to get this drawback we need to use the lead compensator that is a passive systems and. So, therefore, we will also discuss the lead compensator to overcome the disadvantages of the PD control controller in next lectures. So, these examples were taken from the normal as noise control systems engineering reference book.

And I thank you for attending the lecture and see you in the next lecture.