Automatic Control

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Lecture – 26
PI Controller Design

So, welcome to the lecture on design via root locus and compensation techniques. So, in the previous lectures we have discussed how to sketch a root locus and we saw that root locus is the movement of closed loop poles when the gain is varied. So, it is graphical techniques, root locus is a graphical technique that displays both the transient response and stability information.

So, on root locus we can find some specific points, and we can calculate the transient response information about that point. And we can see whether this transient response information is as per the design requirement or not then we can select some other point that better matches the design requirement.

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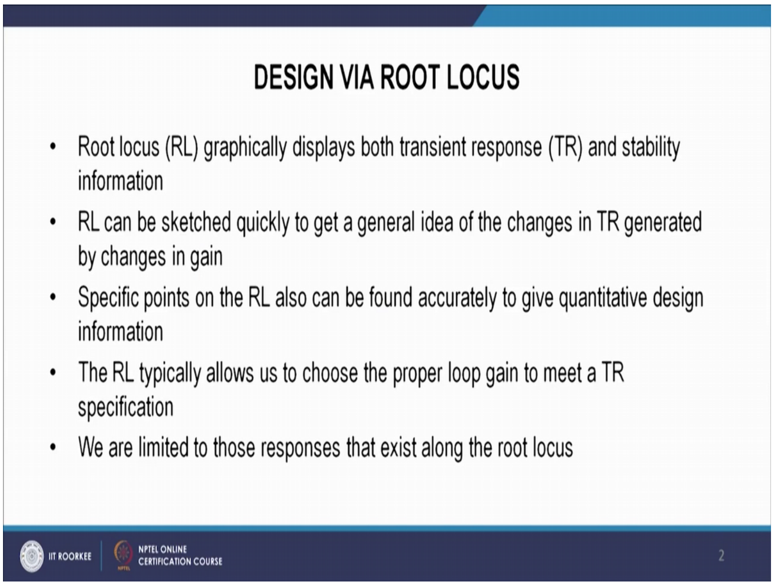

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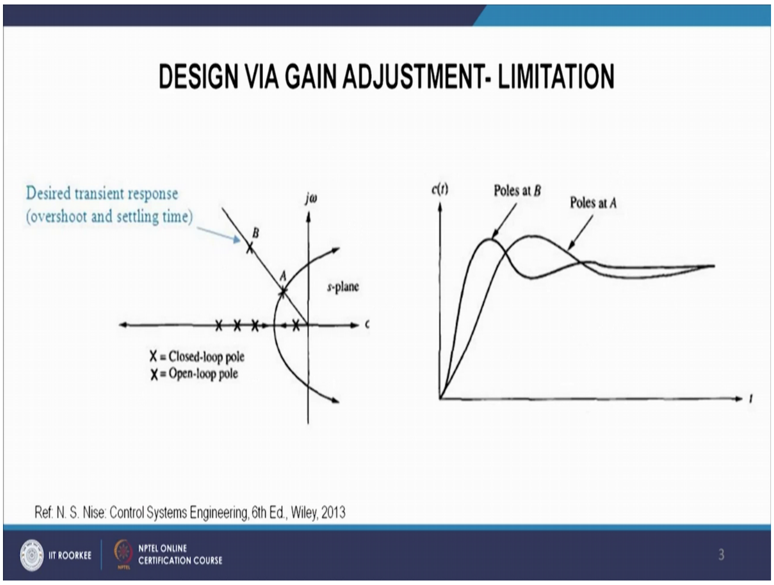

However, in this root locus we are forced to select a point only on the root locus because that root locus is a locus of closed loop poles when gain we varied again; however, what happens if we want to design the system for a point that is not on the root locus, so we can say that.

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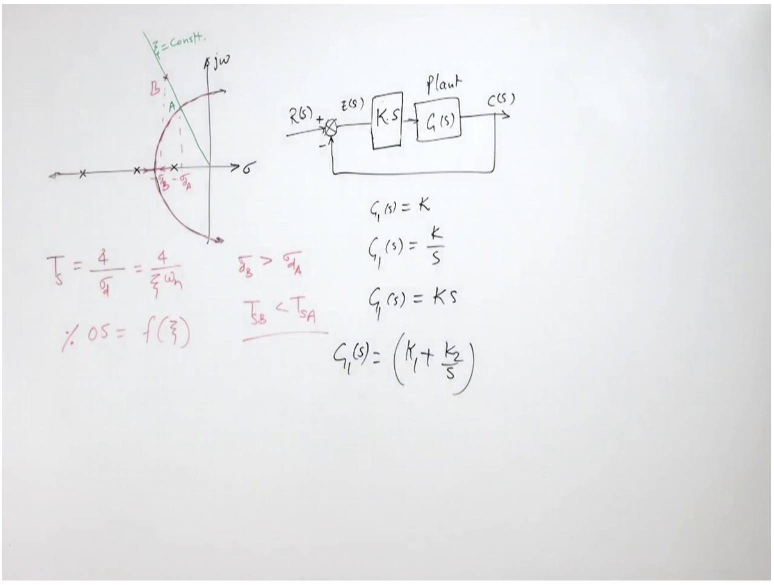

If we have this is our s plane, and we have some point this is the let us say open loop poles and so root locus starts. So, this is our root locus and we want some point on the root locus let us say this is some damping line. So, this is a constant damping line and this is intersecting at some point, let us say a now we are interested to keep the same damping or we can say here the same settling time is 4 upon sigma d or 4 upon zeta omega n and percentage overshoot is function of damping.

So, we want the same percentage overshoot or same damping, but we are interested our system to design. So, that it gives the transient response corresponding to point B, now our root locus follow this path this red line path. So, we can only get, but by changing the gain we can only get a response for those points that fall on the root locus, but if a point is not on the root locus we cannot get that the response of the system.

So, for this point B our objective is that if we want to speed up the response. So, means the settling time is less because if we go to B, we have the same percentage over overshoot because we have the constant overshoot line or damping line, but we have more sigma d because here the sigma d is more.

So, if C this is sigma d 1 and this is sigma d 2 minus sigma d minus, so sigma d 2 is greater than sigma d 1. So, therefore, Ts 2 will be less than Ts 2, so we can say T. So, here B and this is A, so sigma d B is greater than sigma d A and therefore, Ts B is less than Ts A. So, we get a smaller settling time means our response is faster.

So, we want to design our system for this point, not a point on the root locus and this we can see here the response we can see that for this system the response for poles at A. So, here this is the response and here you can see that peak time is also larger and settling time is also larger, but when we take the response corresponding to point B we have less peak time and less settling time. So, we have faster response and this is our requirement.

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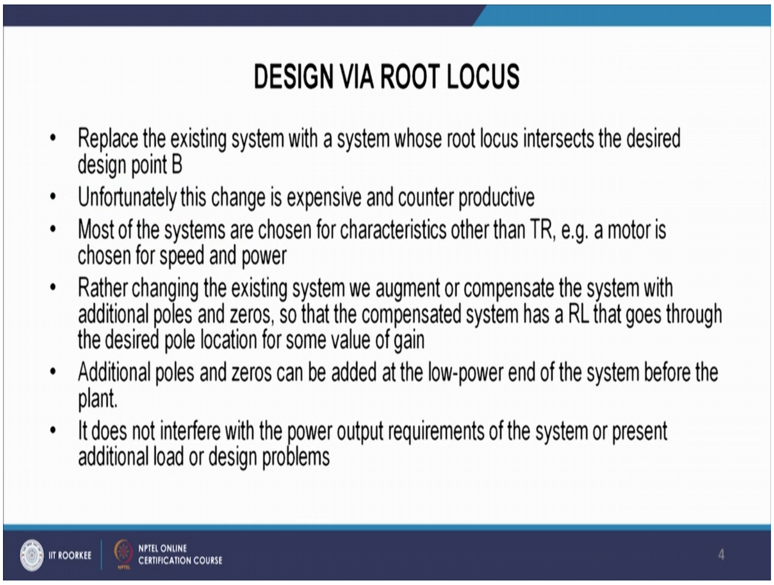

So therefore, when this is the condition we have we cannot get this response only by changing the again therefore, we need to change the system because if we want this root locus to pass through this point B. So, that we can get the response corresponding to point B, we need to change the system because only a system change system can have a root locus that may pass form through this point B, but changing a system is quite expensive and counterproductive process because most of the systems are chosen for characteristic other than the transient response.

For example a motor is chosen for speed and power requirement. So, if we change the system for transient response its power and speed characteristics might change. So, therefore, we do not need to change the system completely, but we need to augment the system or compensate the system with additional poles and zeros, we can add some additional poles and zeros to the system and we can augment the system and we can add these poles and zeros to the before the input to the plant.

So, that is the low power requirement they will need low power requirement they are low power end of the system, so we will not affect the output power of the system. So, this process is called the compensation or augmentation and we will use the root locus method to discuss the compensation techniques how we can compensate a system how we can augment a system existing system. So, that it will give the required transient response and required steady state response.

Now, when we discuss about compensation techniques we have two type of compensation one is that we add the compensator poles and zeros in series of the in the forward loop that is in the series of the plant and before the plant. So, we can see here this we are going to add here this.

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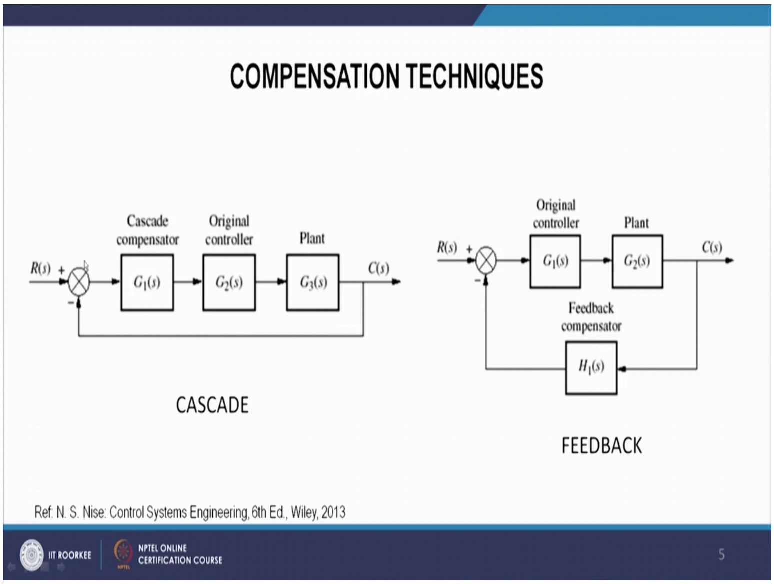

Compensator this is our original this is plant and if there is some original controller it could be a gain k, now here we are adding the cascade compensator cast that is in series to this plant that is G 1 s. So, this is called cascade compensation and here we can add the compensator in feedback loop. So, this is suppose there is earlier there is no any unity feedback we are going to add some feedback compensator here. So, this is called feedback compensation now there are some basic elements of ideal compensators, so here when we say ideal compensators.

So, ideal compensators are able to give us the accurate design requirements and they are active systems for example, we add amplifiers give some day some integrator or some differentiated circuit. So, these are the, you need some external powers, so they, they are active circuits and therefore, we call it ideal compensators because they give us very accurate results.

There are other passive also passive compensators or we call it lag or lead compensators. So, they do not need external powers, so in this lecture we are going to talk about ideal compensators. So, here ideal compensators we can see here there is proportional controller, so proportional controller gives a feedback.

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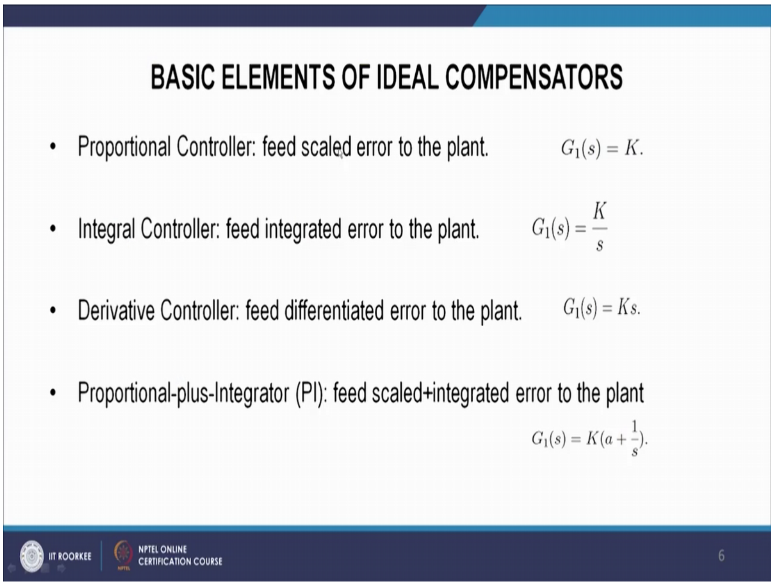

By scaling the error of the plant, so if we have here this is the our input this is our E s. So, here if we add some proportional compensator let us say k. So, K times E s will enter it to the input of the plant, so this is plant and here a gain is proportional controller.

So, we are just giving a scaled input of the error to the plant. So, this is here we have G 1 s equal to K. So, the transfer function of this compensator is K we can also give the integral controller. So, we feed the integrated error to the plant, so we give some here in spite of K we can have integrator. And so if we have integrator and we represent the transfer function we can write K by s because transfer function of integrator is by A divided by s. So, here we can say G 1 s 4 into integrator or integral controller is K by s.

We can also give a derivative controller, so we give differentiation to the error and give to the input and then this will be K into s because if it is first derivative its transfer function is we have K s. So, here G 1 s equal to K s. now we could have also proportional plus integral integrator or proportional plus derivative controller, we feed the scaled plus integrated error to the plant and, so we have, so K 1 plus K 2 by s this could be the transfer function of proportional plus integral controller.

So, here we can see what I discussed we have those compensators transfer function and we can add these systems before the plant. So, we can compensate our system, so let us discuss the ideal integral compensates and we also call a pir proportional plus integral compensation, and the objective of these compensation is to improve the steady state error without affecting the transient response of the system here we have to improve the steady state error and because we are talking about the ideal compensator.

So, we should be able to achieve the 0 steady state error also without affecting the transient response, because if we improve the steady state error and we affect the tangent response then we are we are one design requirement we are fulfilling, but other design requirements we are affecting. So, here we have to improve the steady state error without affecting the transient response of the system.

So, to achieve this we can have a compensator that is called p i compensation proportional plus integral. So, we can have a pole at origin and a 0 close to the pole, so we replace an open loop pole at the origin and because this increases the system type, because we know that if we want to improve the steady state error, then we if we could increase the system type then we can reduce the steady state error, because we remember that when we had a type 0 system, for that we said that n should be greater or equal to 1 for a 0 steady state error, means if we put at least one pole at the origin then for a type 0 system the steady state error will be 0 and if it is type one system. So, for the ramp input we can have n greater or equal to 2.

So, there should be two poles at the origin, so now, here if we take a type 0 system and we want the improved the steady state error we can keep one pole at origin. So, it will improve the steady state error to 0, but it may affect also the transient response and that we have to stop, because here the objective is only to improve the steady state error without affecting the transient response of the system therefore suppose if we have this system.

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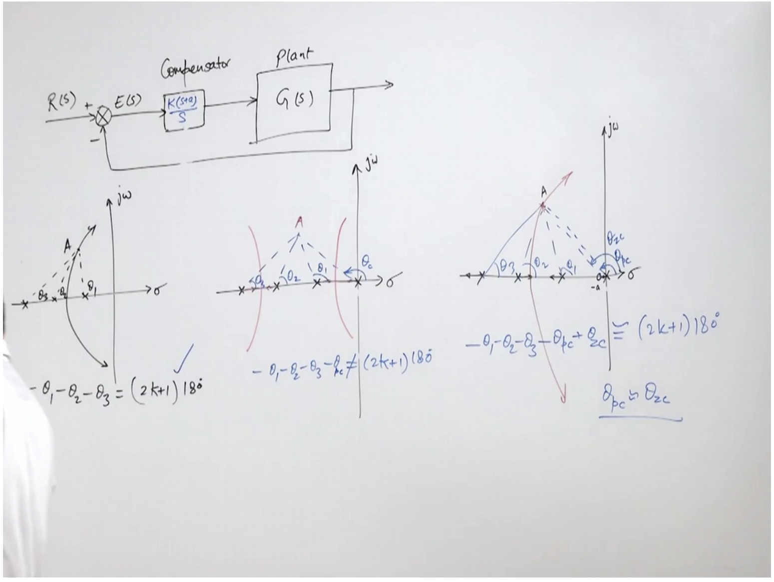

So, we have input we have this E s here K and G s. So, let us say this is gain and this is our plant. So, this is our plant and if we plot this root locus for this let us this G s has three open loop poles here and we have root locus and we have a point here A. So, here these angles theta 1, theta 2 theta 3 and we have these are all 3 poles. So, minus theta 1, minus theta 2, minus theta 3 should be equal to 2 K plus 1 180 degree.

They are because this system for this the root locus we are getting this condition now when we are adding going to add one pole at the origin. So, what will happen we have this is a compensator, so now, this is a compensator. So, this compensator with this compensator we will have we can we will change the root locus and it is possible that this root locus does not pass through this point A. So, we have now this 1 pole at origin and these three earlier G s pole.

So, now the root locus will lie here in this and here. So, this root locus will may move like this and like this, and here is our point A and it does not pass through point A and here we have. So, this is theta C, so theta C is the angle of this compensator pole and these angles are already this theta 1, theta 2, theta 3. So, here minus theta 1 minus theta 2 minus theta 3 minus theta C that will not be equal to 2 K plus 1, 180 because at this point a it was 180 this condition was satisfied, but now the root locus is changed and this condition will not be satisfied.

So, what can we do now this additional angle that is made due to addition of this pole if we add a 0 close to this pole. So, this angle will also be made almost equal and opposite sign. So, this will be cancelled, so here if we add a pole sorry a 0, so if we change this as K s plus A. So, we are going to add a 0 that is at minus A. So, here what will be the root locus for these, so again here we can have this is the pole at 0 compensator pole and here we are putting a 0 nearby this is compensator 0 that is minus A.

So, this minus A is coming here and these three poles, and here is this point A now this pole root locus will start this pole and at this pole, now this will start here and at here and this will go here. So, again this root locus will change and it may start going through this point A. So, we will have this angle, so let us say this is pc pole compensator pole angle now here we will have. So, this is theta pc and the other angle is theta 0 compensator 0 this is theta 1, theta 2 and this is theta 3.

So, again here minus theta 1 minus theta 2 minus theta 3 minus theta pc plus theta z c equal to 2 K plus 1 180 degree, so of course, here this will approximately equal to because this theta pc is approximately equal to theta z c and they are opposite signs.

So, they will almost cancel and so this condition will remain that was here initial condition and therefore, we will we have improved the steady state error by putting this pole at one pole at origin because system type is improved increased and. So, in steady state error is improved; however, at the same time we have by adding a 0 close to this pole we have design retain the transient response requirement of the system.

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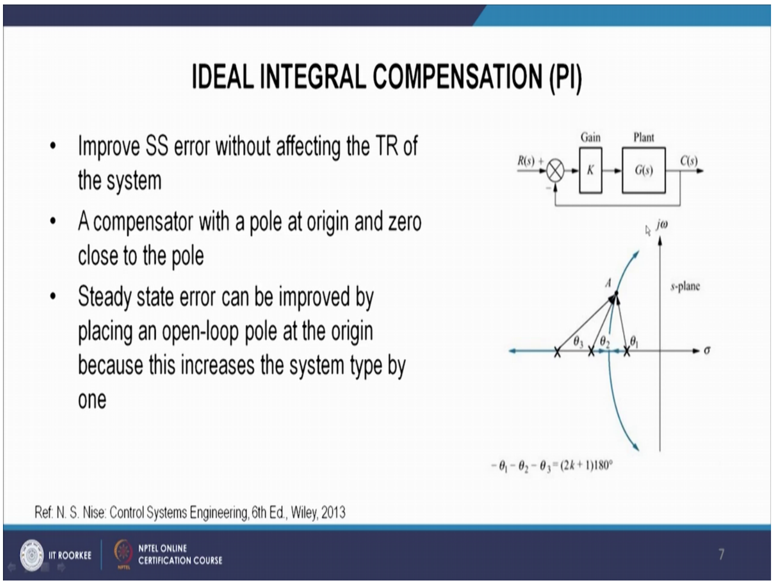

And so here we can see what we discussed here the same thing we can see here.

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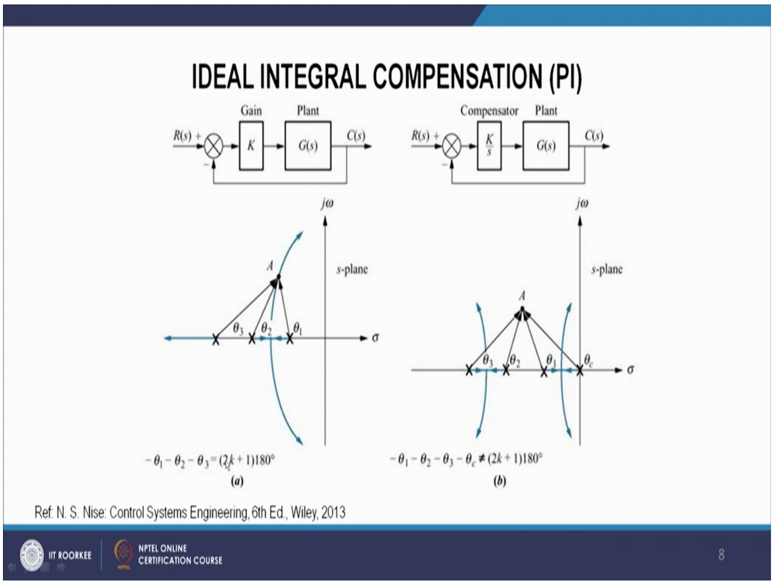

And these are these two other compensated root locus.

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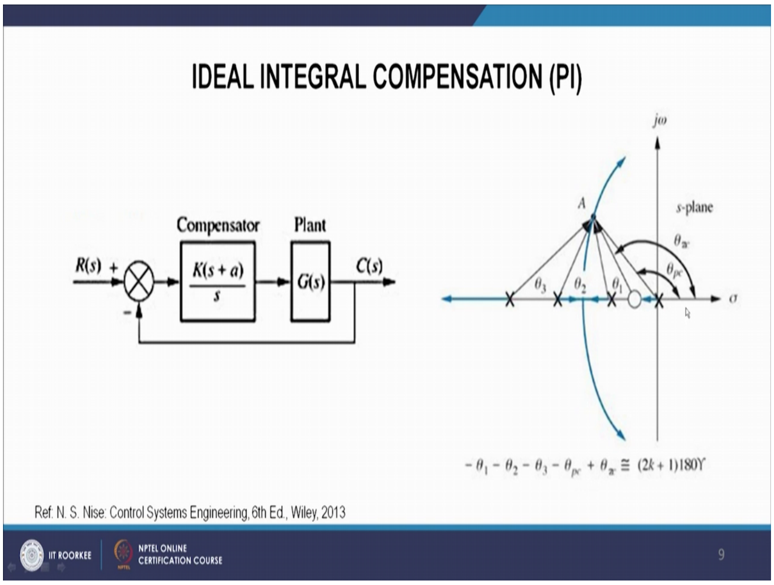

And this is the final one, so we are getting this root locus as original.

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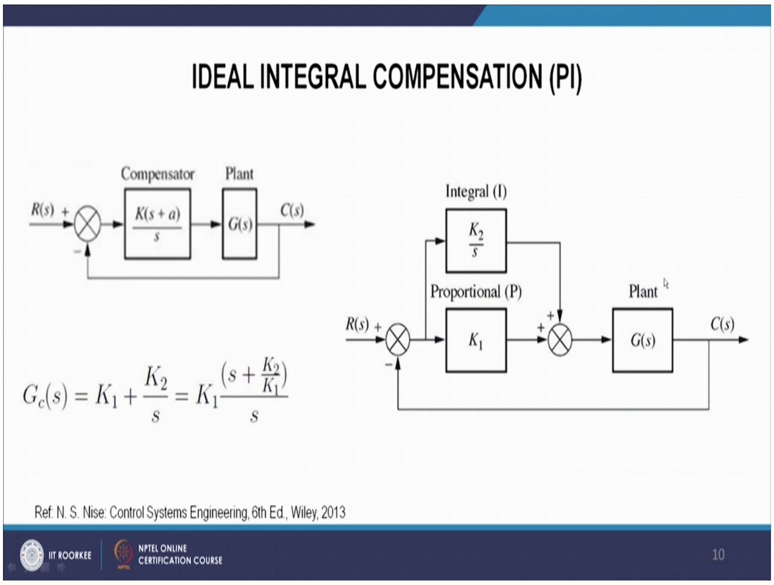

Now, how to deep resent this root locus, how to represent this ideal integral compensator compensation. So, we can see that here we have K s plus a by s. So, the we can represent it as individual circuits of gain and integration, so individual circuits or.

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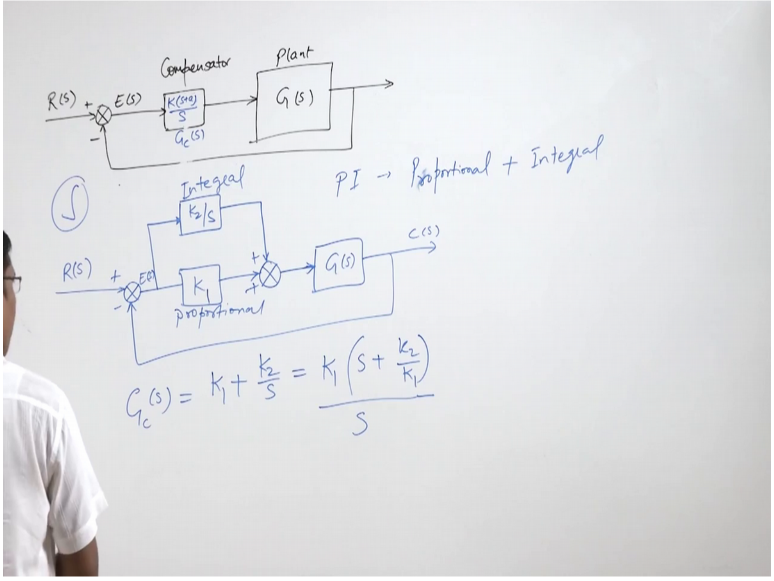

so here we have, so we can have here let us say K 1. So, here we have this we are talking p i controller. So, here this is proportional plus integral.

So, here this part is proportional because here this is E s coming and so proportional to this error. So, scale one going here then this is the integral, because here is an integral integrator and this integrator we represent as a transfer function K 2 by s and so therefore, proportional integral that is going to be sum. So, here G we can have this is if this is G c s; G c s equivalent of this proportional plus integral, so that is K 1 plus because here K 2 by s.

So, this is equal to K 1 1 plus so s plus K 2 by K 1. So, here we can see that here we have a is K 2 by a equal to K 2 by K 1 and K 1 equal to k. So, the A the location of A that is the 0 can be adjusted by wearing this K 2 and K 1. So, here we discussed the ideal integral compensation that is proportional also called proportional plus integral compensation and we will continue the, this compensation and others in the next lecture.

So, I thank you for attending the lecture and see you in the next lecture.