Automatic Control
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Lecture – 25
Numerical Examples and Second Order Approximation

So, welcome to the lecture on Root Locus Technique. So, we will discuss in this lecture, one numerical problem and then some theory about second order approximation. So, we discussed several rules using that rule, we can draw the sketch of root locus. So, we will discuss one more numerical problem, how to draw the root locus. So, let us take this is the problem.

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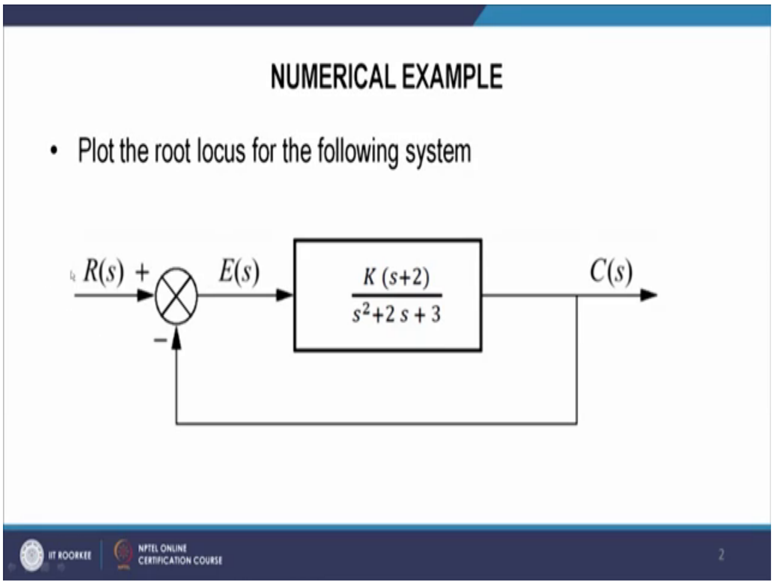

So, we have to plot the root locus for the following system. So, here is this input R S and there is error from E S. This is the K G s, that is K is the gain and this is G s and this is the output here unity feedback system. So, H s is 1.

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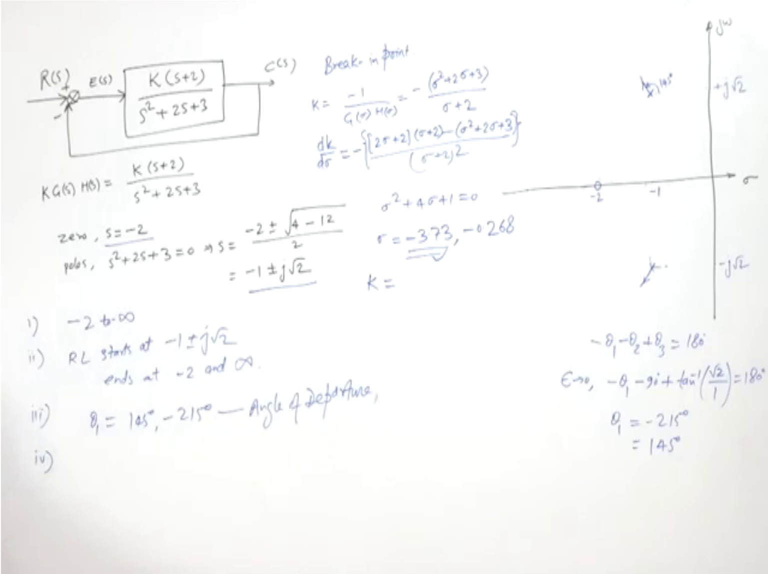

So, we have here this G s is K s plus 2, s square plus 2 s plus 3 and this is C S and unit feedback system here. So, we have to plot the root locus for this system. So, here let us, we have to plot here. So, here sigma this is j omega axis. So, the first thing is we have to locate. So here, K G S H S equal to K s plus 2, s square plus 2 s plus 3.

So, here we have one 0 that is at s equal to minus 2 and poles, 2 poles here with s square plus 2 s plus 3 equal to 0. So, we will get the poles and that is s equal to, so we can get, so we are getting minus 1 plus minus j under root 2 and we have to locate these zeros and poles because they are the zeros and poles of open loop transfer function. This G S H S a is open loop transfer function and so we put 0 s equal to minus 2.

So, if this is minus 1 and this is minus 2, so here is 0 at minus 2 and we have poles. So, here, this is the pole and this is this second pole and this is minus j root 2 and this is plus j root 2 ; so 2 poles and one 0. Now we know that the root locus on the real axis will exist to the left of the odd number of poles and zeros. So, it cannot exist here because here we have these 2 poles; this is the third one; so, 2 plus 1, 3. So, it will exist in this region, not here.

So, the root locus on real axis will exist minus 2 to infinity. So, the here is infinity. So, the root locus will exist between minus 2 to infinity or minus infinity one. Now root locus will start at poles, finite and infinite poles and it will end at zeroes. So, we see that we have 2 poles and 2 finite poles and one finite zeroes; so total number of finite plus infinite poles equal to total number of finite plus infinite zeros.

So, therefore, one 0 is at infinite. So, here poles finish at the root locus will start at these poles and end at this 0 and other infinite 0.

So, here so root locus starts at minus 1 plus minus j root 2 and it ends at minus 2 and infinite. Now, because root locus will start from here, so it will depart from these points. So, we have to find the angle of departure. So, we can find angle of departure by taking some point here and we can find this. So, this angle, this angle and this angle; so, if we say theta 1, theta 2, theta 3, so here we can have minus theta 1, minus theta 2 and plus theta 3 equal to 180 degree.

So, here we assume K equal to 1. Now when epsilon tends to 0 minus theta 1, minus theta 2; theta 2 is unity 0 theta 2 is 90 degree and plus theta 3; theta 3 is tan inverse root 2 by 1 because here it is minus 1. So, tan inverse root 2 by 1 that is equal to 180 degree. So, we get it. So, theta 1 equal to minus 215 degree and we can add 360, so we get 145 degree.

So, it means that root locus will have theta 1 equal to 145 degree and minus 215 degree. So, this is the break sorry angle of departure. So, we have obtained angle of departure. So, it means that the root locus for these will start at 145 degree from here. So, let us say this is 145 degree and this is minus 215 degree, so here or minus 145 degree. So, it will start from here. So, this is the root locus starting this is minus 145 degree.

So, it is start with that at this angle of departure. Now we know that the these root locus will when going from here they will reach to because the root locus it started, but it helped to come to this point because there is a 0. So, they must reach to the real axis some point and that point is called break in point because, so we have to find some break in point. So, break in point we have to find and we can find the break in point when we write k equal to minus one upon G sigma H sigma and that is equal to minus s square plus sigma square plus 2 sigma plus 3 upon sigma plus 2.

Now, we find d K by d sigma. So, we find. So, this is the first derivative. So, we solve it. So, we finally get this equation sigma square plus 4 sigma plus 1 equal to 0 and we get sigma equal to minus 3.73 and minus 0.268. So, these are the 2 points we are getting.

So, now we know that the root locus cannot exist before between minus 2 and 0. So, it the point that is beyond minus 2 that will that is the possibility. So, this is the possibility because these point is here and root locus is not existing in this range. So, this is the likely point where break in can occur. Now we have to find the value of K for these 2 points. So, if we put the value of sigma 2 sigma here we can find K.

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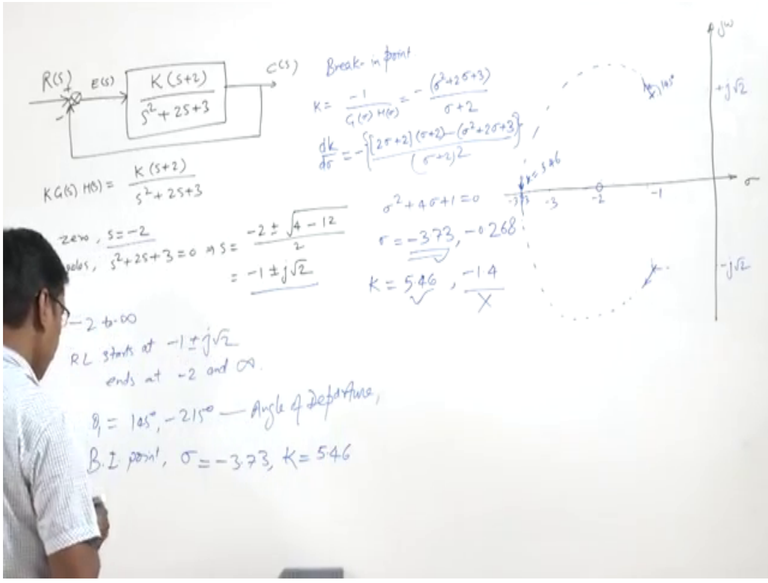

So, the K we are getting 5.46 for this and for other we are getting minus 1.4.

So, this is negative gain we discard this and this is the gain at which this break in will occur. So, these is minus 2 and let us say this is minus 3 and so break in may occur here at minus 3.73. So, they will come here at minus. So, these root locus going from poles will return back to the real axis at minus 3.73 and K equal to 5.46. So, we have got this break in point. So, here we have break in point sigma equal to minus 3.73 and K equal to 5.46.

So, this is the break in point information we have got. So, we do not need to compute the j omega axis crossing because we know that this root locus is coming to this side; it is going away from the j omega axis. So, there is no need to check for j omega axis crossing. And so, we can get some idea of the root locus.

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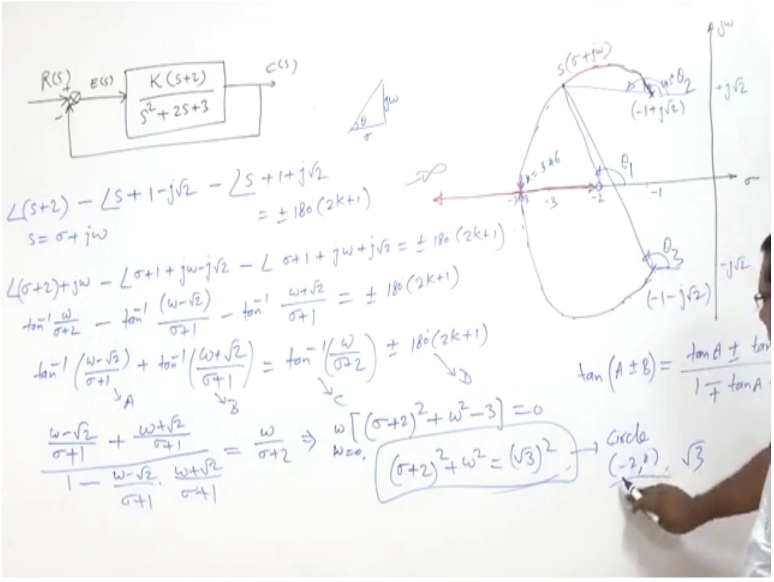

So, the root locus is starting from this point, it is going with following some curve here, reaching their starting at this, reaching there. Now they will one branch will go end to this 0 because they have to end at 0 and other one is going to infinity 0. So, other one will go towards infinite. So, this is the root locus. Now we have to check that, if we want to know that what type of curve it is going to follow or approximate path between this point and this point, we can take maybe 2, 3 points for some sigma and we can check the condition of some point s assuming on the root locus that satisfied the condition of odd multiple of 180 degree that point will be the root locus and 2, 3 points we can obtain and then we can more precisely approximate this path of the root locus.

Another method is, we can use in some by taking some general point s here on the root locus. Let us take here is s is some general points sigma plus j omega on the root locus. So, we can find this equation of this curve. So, let us find this. So, if s is this point on the root locus, we can find here is 0, there is this pole and here is this pole; so these angles, this angle and this angle. So, we have to find this angle. So, let us say this is theta 1, theta 2 and theta 3.

So, that is angle theta 1, that is s, angle s minus. So, let us say theta 1, theta 2, theta 3; so s minus minus 2, so s plus 2, so this is 0 pole. So, minus s, minus minus 1 plus j root 2 so plus 1 minus j root 2. So, we are doing angle s minus this 0. So, s plus 2, angle s minus here minus 1 plus j root 2. So, s minus minus is plus 1 and this minus j root 2 and then minus angle s plus 1 plus j root 2; this third pole because these points are minus 1 plus j root 2 and this is minus 1 minus j root 2 and this is minus 2 and this must be equal to plus minus 180 degree 2K plus 1.

So, we can write S equal to sigma plus j omega and we can get sigma plus 2 plus j omega angle minus sigma plus 1 plus j omega minus j root 2 minus sigma plus 1 plus j omega plus j root 2 equal to plus minus 180 2K plus 1. So, we can find this angle that is tan inverse omega by sigma plus 2 minus tan inverse omega minus root 2 by sigma plus 1 minus tan inverse omega plus root 2 by sigma plus 1 because these are a real part and imaginary part and this is the tan theta.

So, this is sigma and this is omega. So, tan inverse is equal to imaginary part; this magnitude up on this part. And that is equal to plus minus 180 2K plus 1. So, we can write tan inverse omega minus root 2 by sigma plus 1 plus tan inverse omega plus root 2 by sigma plus 1 equal to tan inverse omega by sigma plus 2 plus minus 180 degree 2K plus 1.

Now, we take tan to the both side. So, tan this is let us say A, this is B, this is C and this is D. So, time we take tan A plus B. So, tan A plus B, here this formula tan A plus minus B equal to tan A plus tan B upon time 1 minus tan A tan B. Here we can have this if it is minus. So, we can write tan A plus B. So, this is A and B. So, tan A plus B equal to tan A plus tan B. So, tan A means tan inverse will gone, this will be remaining.

So, here taking tan to this equation, we will get omega minus root 2 upon sigma plus 1 plus omega plus root 2 upon sigma plus 1 upon 1 minus omega minus root 2 upon sigma plus 1 into omega plus root 2 upon sigma plus 1 and that is equal to that part and that is omega by sigma plus 2 because this part will make 0 and so we will solve this, we will get omega.

So, we get this equation. So, omega equal to 0 is one solution and second is sigma plus 2 whole square plus omega square equal to root 3 square. So, this is a equation of circle. This is circle with center minus 2, 0 and radius root 3. So, means this root locus will follow a circular path with which center will be minus 2, 0 and radius is root 3.

So, here this is this is root 3 this radius and this path is centered to this. So, that is how we understand that how to plot the root locus and how can we taking general point, we can find the equation of that path. So, now we come to another important point to discuss about second order approximations.

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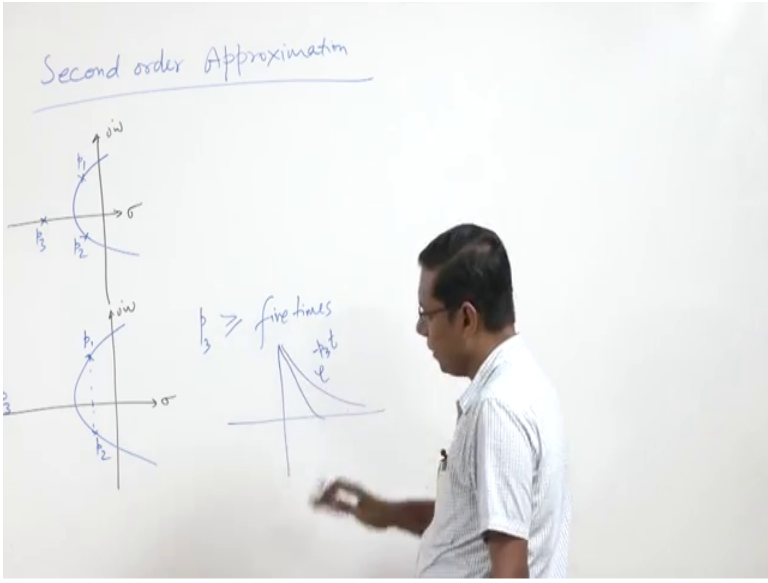

So, here we discuss second order approximation means if we have a system of higher order, more than second order, how can we approximate it as a second order. Is it possible or not?

So, let us have a condition when. So, this is sigma, this is j omega and we have root locus and there are these second order points and at this gain there is p 3. So, this is p 1, this is p 2 and this is p 3. So, for certain value of gain, this is the condition. We have 3 poles. So, means this is a third order system. Can we approximate it as a second order means we can, can we neglect the role of the third pole or importance of the third pole? Yes we can do this, but only in certain conditions. We can do this if this pole p 3 is far.

So, if this is p 2, this is p 1 and this p 3 is far from here, the real coordinate, real axis coordinate of the complex poles. So, if p 3 is further than 5 times, at least 5 times. So, 5 times for the far than the real axis coordinate of the dominant complex poles ; if it is not it is closure, we cannot neglect the effect of p 3 and we cannot approximate this third order system as a second order.

However, if this condition is satisfied that it is very far weak and the effect of this will because we know that the in the response, the decay is e power p 3 t of course p 3 here is negative. So, if p 3 is larger it will decay very fast and therefore it is effect will can be neglected. However, if it is closed, it will affect the response and we cannot approximate. Now let us take another case when we have also a 0.

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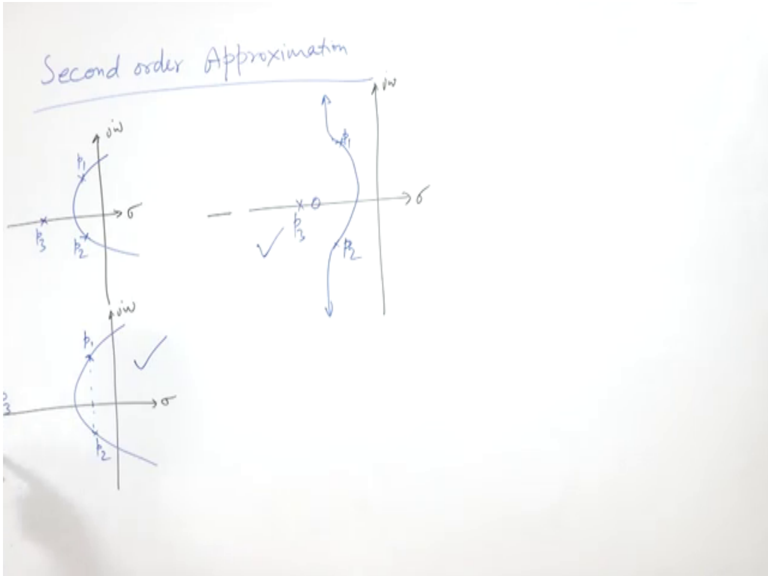

So, if we have a 0 and we have the root locus and here is one pole p 3 and we have here p 1 and p 2.

So, here we can see that the, if there is a 0 here, can we have if there is a 0, this 0 will affect the response of the system.

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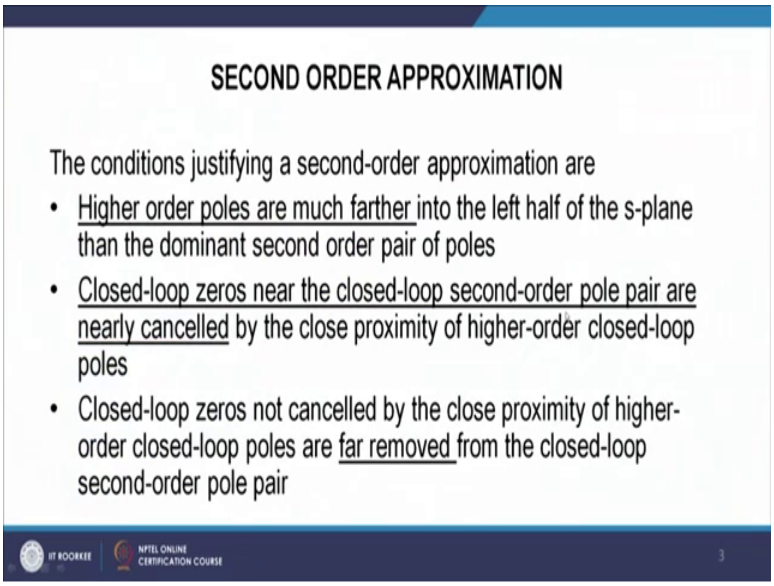

So, closed loops 0 near the closed loop second order pole pair or can be canceled by the close proximity of higher order closed loop poles. So, if we have 0 here, it will affect the response of this system and this role of this 0 can be only canceled if this third higher pole is linear to the 0.

So, if this is here the if this will cancel because we know that 0 is in numerator and pole is in the denominator and due to close proximity of these 2, this 0 and pole will be canceled and the effect of this. So, even if this p 3 is close to these dominant poles, but it is effect is cancelled by the zeros that is near to this pole and in this condition also we can approximate this higher order system as second order ok. So, these are the 2 conditions when we can approximate the third order system as the second order system.

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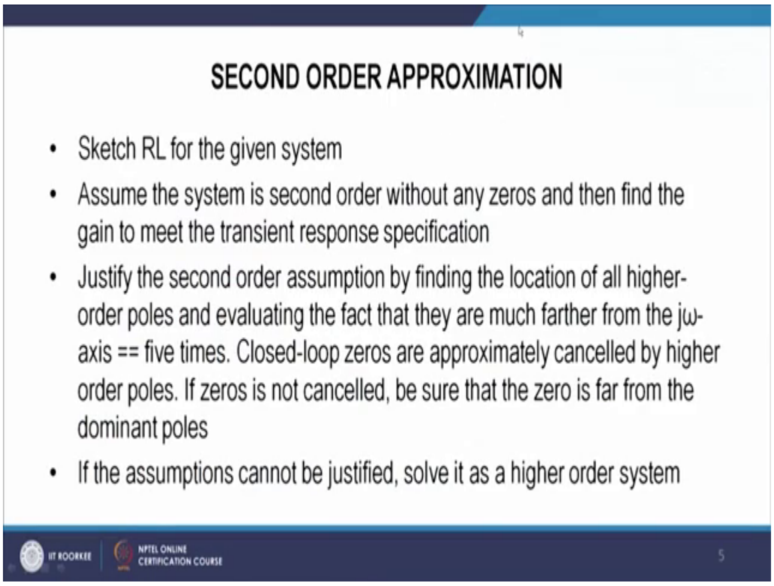

So, here the second order approximations, we have to sketch the root locus for the given system, then we assume that the system is second order without any zeros and then find the gain to meet the transient response specification. Then justify the second order assumption by finding the location of all higher order poles and evaluating the fact that they are much further from the j omega axis or at least 5 times the location of the dominant poles on the real axis.

Close loop zeros are approximately canceled by higher order poles. If 0 is not cancelled b is so that the 0 is far from the dominant poles. If the assumptions cannot be justified, we cannot approximate it as a second order system and this will act as a higher order system.

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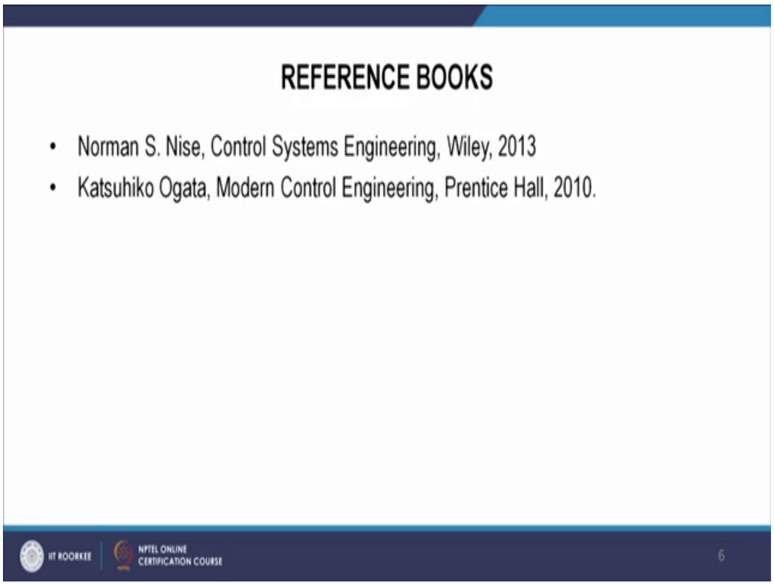

So, we followed the example from the reference book K Ogata, modern control engineering. So, I complete here the root locus technique and we learned how to sketch the root locus and to find the information of transient response and stability.

So, thank you for attending the lecture and see you in the next lecture.