Automatic Control
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Lecture – 23
Sketching of Root Locus – II

So, welcome to the lecture on Root Locus technique. We will discuss in this lecture is caching of Root Locus. So, in the previous lecture, we started to some rules to discuss some about some rules that will help to sketch the Root Locus. So, we know that Root Locus is a graphical representation of variation of closed loop poles, when we change the values of gain or k and it gives the information about the poles at certain value of gain. And when we have a vivid picture of root locus, we can tell that at what value the root locus the system is being a stable, at what value of k the system is being stable or unstable. And we can calculate the transient response at some particular value of gain.

So, here we discussed that to sketch the Root Locus, we discussed 5 rules. So, the number 1 was that the branches of root locus. So, as many number of closed loop poles as many branches of the Root Locus. So, one branch for each closed loop poles, then root locus is symmetrical about the real axis and then the root locus on the real axis lies to the left of the odd number of poles, finite number of poles or and 0, then the root locus always starts with finite and infinite number of poles and poles of G s H s that is open loop transfer function. And it ends at the finite and infinite zeros of the open loop transfer function G s H s.

Then we found that, the root locus approaches to infinity along a straight line asymptote as asymptotes and we found the real axis intersection of that, our real axis coordinate of that straight line when it intersects the real axis. So, we found the sigma a and theta a at what angle this line will intersect the real axis and at what point. So, these are where the 5 rules we discussed and we set some examples.

Now, today, we will discuss few more points, few more rules that will help to even refine the root locus.

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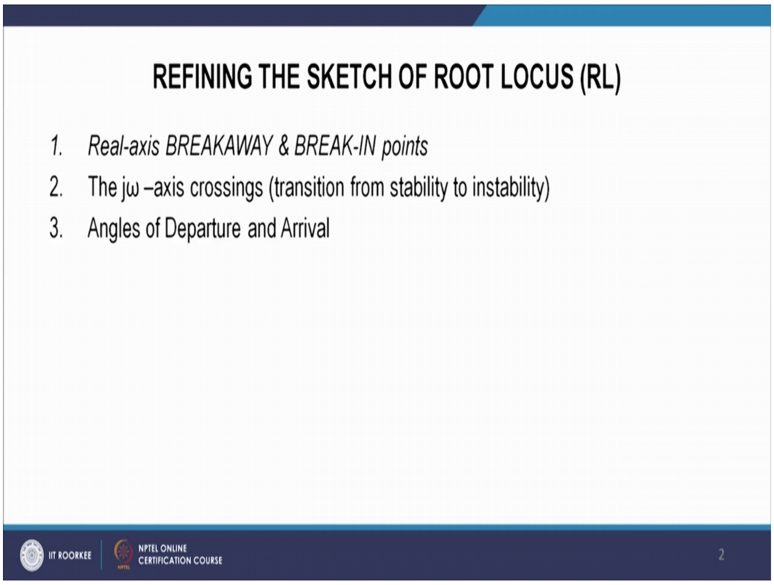

So, we will discuss today first the breakaway and break in points and then j omega axis crossings.

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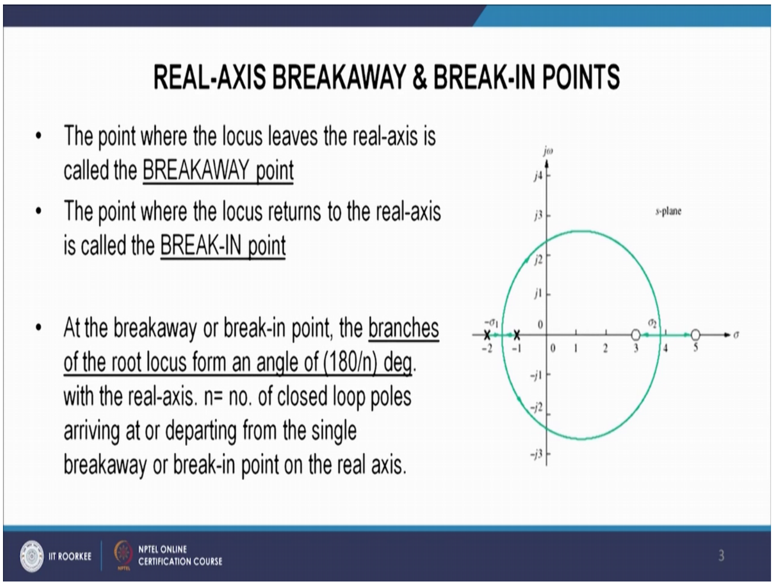

So; what is Breakaway and Break-In points? So, the point where the locus leaves the real axis is called the Breakaway point and the point where the locus returns to the real axis is called the Break-In point.

So, we know that, when we have the, we saw this example in the beginning.

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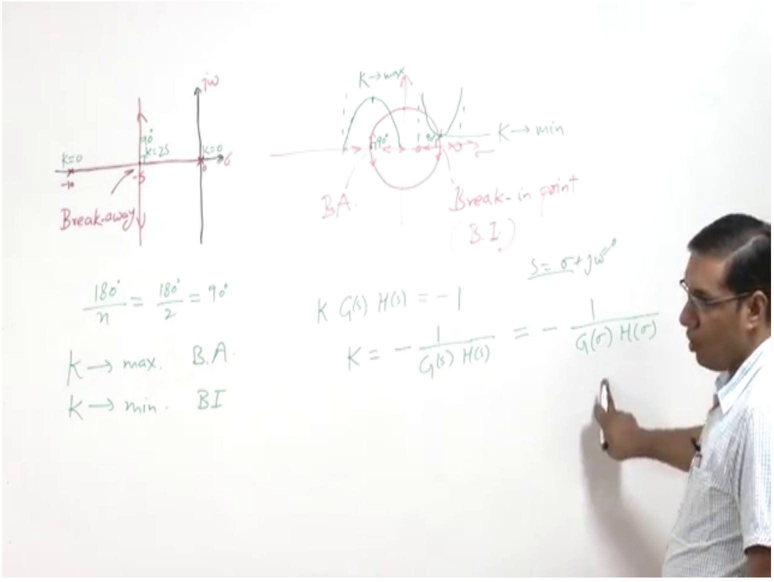

And so, here we saw that the root locus started here and so, minus 10, 0 and then at minus 5, they separates here. So, when the root locus from the real axis is good it goes to complex plane; it means it leaves the real axis and that is called Breakaway point. So, this is this point is called Breakaway point breakaway point. And suppose this root locus, there is some root locus and suppose this is a root locus starting at this point, it separates and then it returns back.

So, we can plot like this. So, suppose it is starting here, it is separating and then it is returning back here and there are some two zeros and going back to end this 0. So, this is Breakaway, Break Away point and this point where it returned back to the real axis from the complex the complex roots and this is called Break-In point BI point.

So, here we can see that when this root locus is started we started from K equal to 0 here and then here it was K equal to 25 when it separated. So, it means when the root locus. So, here is if we see this, the root locus is separates at K, when K is maximum. So, at this point K is maximum. So, here we start with K equal to 0 here K 25. So, on this real axis when K is maximum, it will breakaway and similarly here, because K is start from ends the root locus ends when K is high value of high value. So, here K is minima, here maximum and at the point where it break in is minimum.

So, this point is K is minimum and therefore, Break-In point. So, we can find the Breakaway and Break-In points by considering that K is maximum then breakaway occurs and when K is minimum, the Break-In points occur. Second thing is that the branches of the root locus form an angle of 180 by n degree with the real axis at Breakaway or Break-In points, where any number of closed loop poles arriving at, or departing from the Breakaway are Break-In point.

So, here for example, here there were 2 poles and the number of branches here is make angle with real axis 180 by n. So, that is 180 by 2, so, 90 degree. So, this angle that breakaway this line or branch makes is 90 degree. So, this angle is 90 degree. Similarly here also, this angle is 90 degree and here also 2 closed loop poles arrives. So, here is also this angle is 90 degree.

So, we can find the we have to find that, if we want to sketch the root locus properly or in a more refined position, we must find these Breakaway and Break-In points for the root locus. So, how can we find the Breakaway or Break-In points? So, we have just said that, breakaway points occur when K is maximum. So, here is breakaway points and K is minimum. So, we have Break-In point and we know that K into G s H s equal to minus 1.

So, here, K equal to minus 1 upon G s H s. Now, we know that breakaway and break in points are only on the real axis because they form on the real axis either the point we are talking is on the real axis because here the branch is separating or here they are returning back. So, here we can, our omega part is 0, if we represent S equal to sigma plus j omega. So, this omega is 0 at real axis. So, we have only sigma. So, we can write this expression as minus 1 upon G sigma and H sigma.

So, we have to only find the sigma, the real axis point. And so, if we want to find the break away or break in point, we see that here this k is maximum or minimum.

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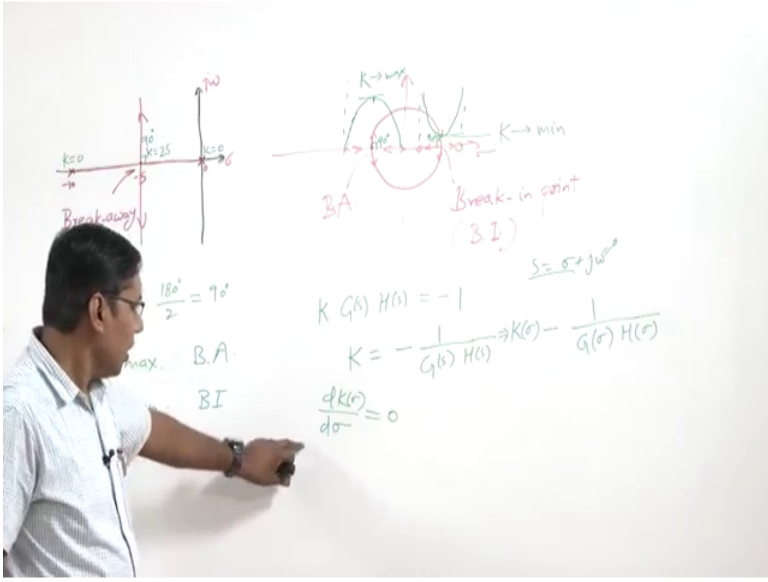

So, the slope d k by d sigma equal to 0 so, here k sigma we can write this is k sigma equal to this. So, here we can write d k sigma because now here k is function of sigma and so d k by d sigma equal to 0 and this will give us the breakaway or break in point. There could be another method also. So, this is first method.

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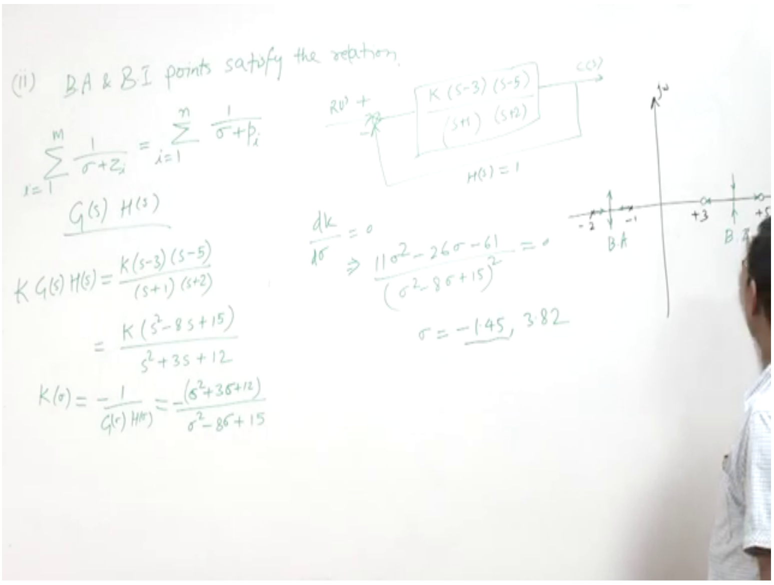

There could be one more method, second method. So, breakaway and break in points, points satisfy the relation sigma 1 to m, 1 upon sigma plus z i equal to sigma 1 to n, 1 upon sigma plus p i. Here, we know that this z i 0 and p i are pole and m is the number of finite 0 and n is number of finite poles of G s H s, so open loop transfer function. So, we can solve for from the here we solve for sigma and we will get the breakaway and break in points.

So, let us take one example. So, if we have, suppose we have K G s H s equal to K s minus 3, s minus 5, s plus 1, s plus 2. So, we have a system like this. So, we have K s minus 3, s minus 5 and here s plus 1, s plus 2. So, if we have this system. So, we can find K G s H s is 1 here. So, K, G s H s equal to this. So, we can write it as K s square minus 8 s plus 15 upon s square plus 3 s plus 12.

Now, we write as. So, K sigma equal to minus 1 upon G sigma H sigma. So, your G s H s is this part. So, we can write minus s square plus 3 s plus 12. So, here we represent s as sigma. So, we replace this as sigma upon sigma i square minus 8 s plus 15. So, this is our K. Now we can take d K by d sigma equal to 0.

So, d K by d sigma equal to 0. So, we can take the derivative here and so, we take here this derivative with respect to sigma. So, we will get here so, we get 11 sigma square minus 26 sigma minus 61 equal to upon this denominator square equal to 0 and we solved for sigma and we will get the value of sigma minus 1.45 and 3.82.

So, if we plot this. So, our open loop transfer function G s H s. We have poles at minus 1 minus 2 and these are at 3 and 5; so, 3 and 5 so, here 3 plus 3 plus 5, here minus 1 minus 2. Now, they will start here these poles will move here because pole will the root locus will start at poles and end sorry here is 0, this is 0 and this is 0.

So, and they will end here. So, here we have break, we see that break this point is this side negative side between this and this. So, at minus 1.45 here, they will break away. And at 3.82 here they will return back. So, here is the, this is breakaway point and this is break in point. So, the same values we can also get by using the second method that is this method.

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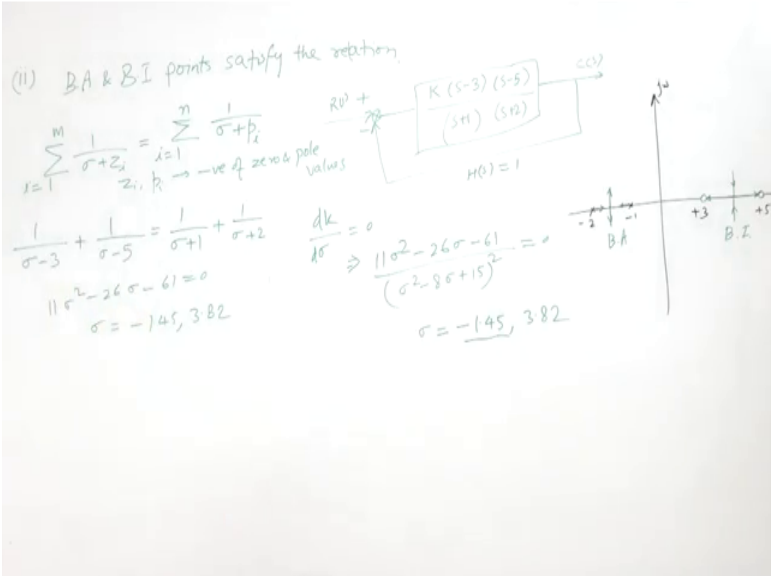

So, we can write 1 by so, 0 sigma minus 3 so, here the pole is plus 3 and. So here, so z i and p i are the negative of the 0 and pole values. So, here z i and p i negative of the 0 and pole values. So, sigma minus 3 plus second 0 so, sigma minus 5 equal to, so 1 upon sigma so, first pole is minus 1 and. So, negative of this is plus 1 and plus 1 upon second is minus 2; so sigma plus 2. So, here we should note z i and p i or negative of 0 and pole values.

So, now we solve this equation and we will get directly this equation. And then we will get minus 1.45 and 3.82. So, we see that by using this breakaway and break in point, we currently find the root locus; we can know exactly at what point it will leave and at what point it will return to the real axis and here also we can see these formulas, this real axis breakaway and break in points.

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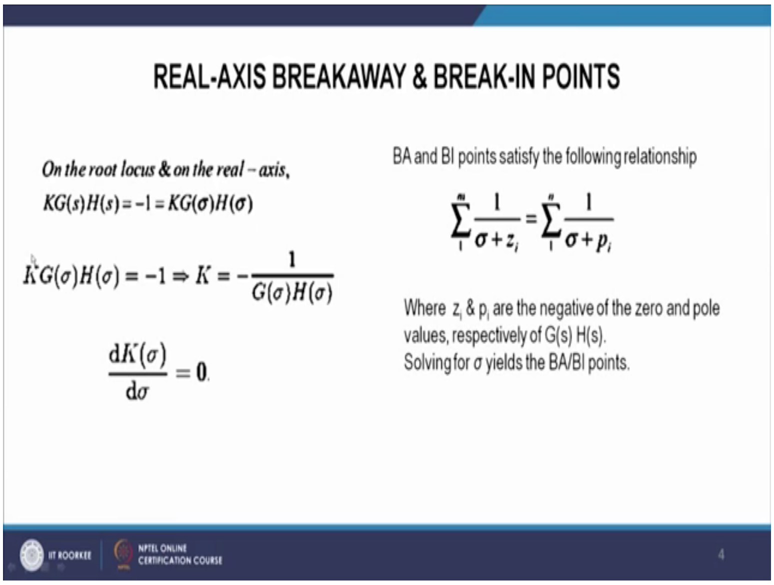

And those formulas that I derived there we can see here. Now we come to the next point that is j omega axis crossing. So, j omega axis crossing will tell about the stability and instability and instability and stability transition. So, where there is transition from stability to instability region, so, we have the points. So, j omega axis we know that the j omega axis.

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Here left half plane and this sorry right half plane and this is left half plane. So if the root locus is in this side or poles are in this side they are stable, but if even a single pole is this side, this is unstable. So, here if the root locus is crossing at some value of gain and at some particular point, it means at this point, beyond this point or beyond this value of gain it is going to j j omega axis cross crossing the j omega axis I am going to unstable region.

So, therefore, we must know that when we are plotting the root locus and we know that if root locus is going to the right half plane, then at what value it is going and at what values of gain it is going to the right half plane or it is crossing to j omega axis. So, we have already discussed about the stability and Routh Hurwitzcriterion for stability and we know that when there is even polynomial when we make the route table and we find there is even polynomial, this means there are some roots on the j omega axis.

So, we will take the help of route table, we will make the route table and we will search for some row that is all the elements are 0. The complete row is 0 and the polynomial above that row will give us the even polynomial and from there we will find the that at what value of omega this j omega axis crossing is taking place. So, here let us take one example directly to find this and apply this. So, let us we have this system.

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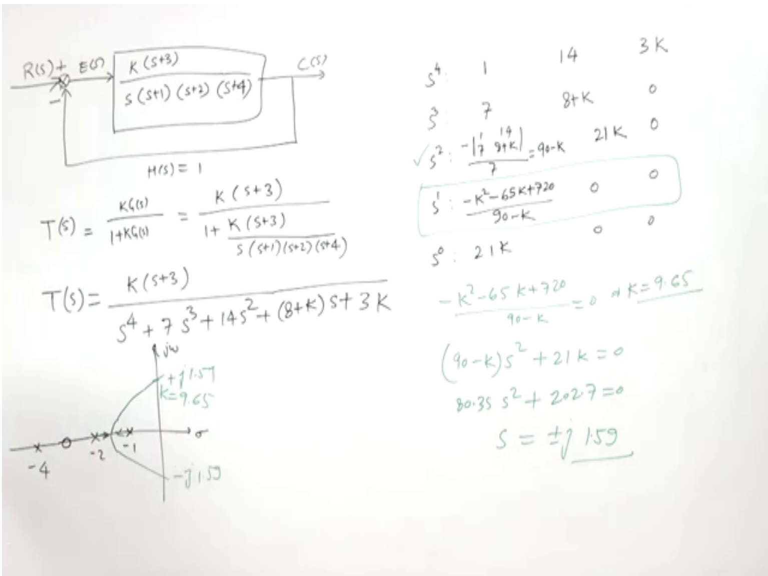

So, here again H s equal to 1 and we are going to get the transfer function T s because to find the stability we have to find the T s and T s equal to we can find G s upon 1 plus G s H s and G s H s is 1 and of course, here k will come. So, here K s plus 3 upon 1 plus K and here s plus 3; as s plus 1 s plus 2 and s plus 4.

So, we can write this as T s we can write K s plus 3 upon s power 4 plus 7 s cube plus 14 s square plus 8 plus K s plus 3 K. So, we will get this the closed loop transfer function. And so, we will create the route table. So, if we create the route tables for this polynomial. So, we have s 4.

So, s 4 coefficient is 1, then 14, then 3 k. Then here s cube we have 7 then 8 plus K, then 0, then s square. So, s square we can find out as minus 1, 7 and 14, 8 plus k upon 7. So, we will we will get 90 minus K then this value we will get 21 K and the other values we will get 0 here. Then s power 1, we will get minus K square minus 65 k plus 720 and 90 minus k and here we will get two 0, 0.

And here we will get 21 K 0, 0. So, we see that this row, this row if this element is 0, we are going to get this row as 0, complete row is 0 and. So, this will give the even polynomial. So, let us put this equal to 0 and we get the value of K. So, minus K square minus 65 K plus 720 upon 90 minus K equal to 0 and. So, when we solve this. So, we get K equal to 9.65. So, it means that for the value of gain K equal to 9.65, this will be 0. And so, complete row is 0.

So, for this value, we will get this polynomial. So, 90 minus 9.65, so 90 minus K square, 90 minus K s square plus 21 K equal to 0 and so, here we put K equal to 9.65 here and we will get 80.35 s square plus 22.7 equal to 0 and we will get s equal to plus minus j 1.59.

So, it means that this system, when we plot the system, this system we have minus 1, minus 2 and minus 4 here; 3 poles and at s equal to minus 3 is a 0. So, this may cross suppose here the poles will move, poles will move and they will break away and they will cross maybe j omega axis for at this point plus plus j 1.59 and here minus j 1.59 and here the value of K will be equal to 9.65.

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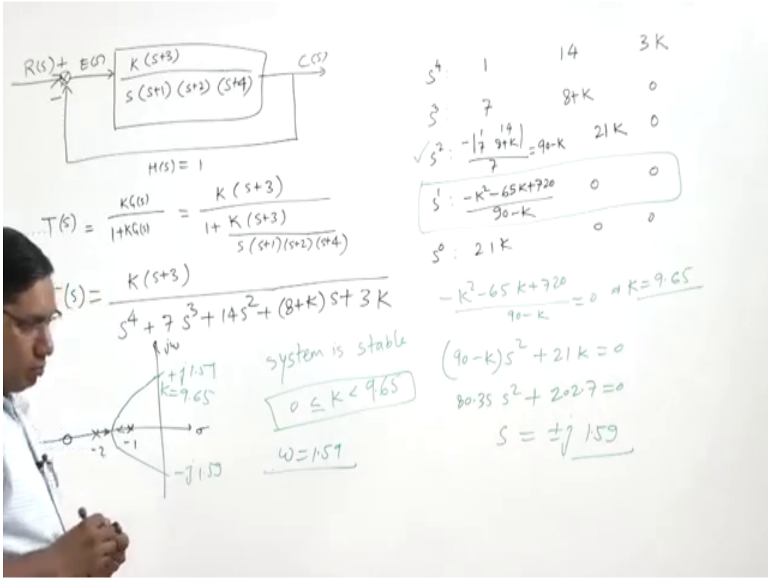

And therefore, the system is stable, system is stable for. So, system is stable for the value of K less than 9.65. So, this is the range of the parameter K when the system is stable because if we go to 9.65, we will be reached to the marginally stable system. And therefore, we are we will be stable only for values less than 9.65 and you know that this frequency of oscillation when the system is on the j omega axis omega is 1.59, this is the frequency of oscillation.

So, these examples were taken from the book of Nise Norman as control systems engineering. So, I thank you for attending this lecture and we will continue this root locus in the next lecture.

Thank you.