Automatic Control
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Lecture – 22
Sketching of Root Locus – I

So, welcome to the root locus technique. So, today in this lecture we will discuss about how to, is catch a root locus. So, in the previous lecture, we saw that point on the root locus must satisfy the condition of mod K G S H S equal to 1 and angle K G S H S equal to odd multiple of 180 degree and we saw the example that when we vary the K, we can calculate the poles and we can plot these poles on the S plane and we can see how the poles are moving and we can plot the root locus; however, this process is quite tedious and it involves calculation of poles for each value of K.

There are several some, there are some rules, some, some points, using those we can, plot an approximate root locus and we can refine the root locus to those points, where we are more interested and we can get that the information of tangent response and stability from that root locus. So, we will discuss those rules to that will help to sketch and approximate root locus and these rules are very simple and that will help us to sketch root locus. So, the first rule is that number of branches.

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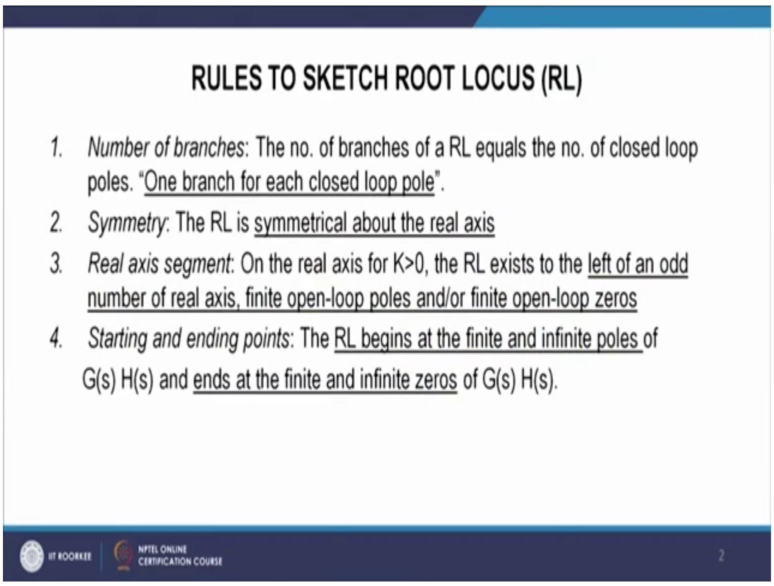

How many branches are root locus must have? So, in the previous example we saw that here,

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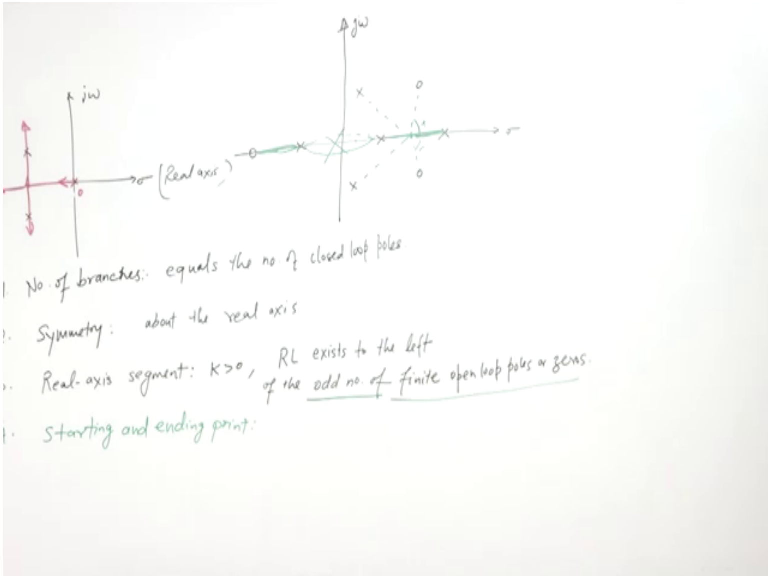

We have this and we saw that the poles started at minus 10 and 0 and we have like this and here we have like this. So, the poles you start moving like this, when we were it K. So, the question is how many branches root locus should have? So, we can, there is the rule that number of branches equals the number of closed loop poles. So, in this problem we saw that, we had two poles closed loop poles and we see that each pole is leading one branch. So, there are two branches of the root locus. So, the number of branches of root locus is equal to the number of closed loop poles.

So, one branch for each closed loop pole. So, here we have two branches. So, this is the first rule, the second rule is symmetry. So, here we can say that root locus is symmetrical about the real axis. So, it is symmetrical about the real axis. So, this is real axis sigma. So, here this is constant omega is 0.

So, this axis is real axis and root locus is symmetrical about this axis, we can see here root locus, here is symmetrical, once it divert, it is leaving this axis, it is leaving symmetrically, why? Because the poles of, closed loop transfer function can be either real or complex, when they are real, they will be on the real axis, when they are complex, they will be complex pairs. So, there will be symmetry, if a pole is here, there will be another pole here. So, they will be always symmetrical about this real axis.

So, therefore, root locus will always be symmetrical about the real axis. The third point is real axis segment. So, third point is real axis segments. So, real axis segments means way on the real axis, if there are a number of poles, where should be the root locus and where it should not be so, this real axis segment for K greater than 0 of course, we are taking K greater than 0. So, this root locus exists to the, the root locus exists to the left of the odd number of, left of the odd number of, odd number of finite open loop poles and, or finite open loop 0 of finite open loop poles or 0s.

So, here we should understand this that why the root locus? If we can see here the root locus is on the real axis existing here, but not here, why this root locus is moving towards this side, not this side? So, therefore, this segment is only between these two points, not this side or this side. So, that rule we are going to discuss the real axis segment. So, we say that the root locus exists to the left of the odd number of finite. So, here is, point is odd number of finite poles or 0s why? So, let us take one example here and let us, we have here some poles 0s. There are some other 0s complex 0s complex poles and so on.

Now, let us take one point here; suppose, that root locus is here. So, if that point is on the root locus, the angle of poles minus angle of 0 is minus angle of poles, would be equal to odd multiple of 180 degree. So, here this is going to make 180 degree; however, the angle that is going to be made by this 0, this is positive and this is negative. So, this angle will be cancelled out. Similarly, the effect of these angles will be also cancelled out, because this, if it is positive, this is negative and the effect of these angles is 0.

So, we can see that if the, this point is will be on the root locus, if it is to the left side, because this is the left side of this first pole, one pole, on the real axis. Now, come to this point, if this point, point is here of course, the effect of these complex poles and 0s will be cancelled and effect of this will be 0, but effect of this is 180 degree and another this 180 degree that is 360 degree. So, this is even multiple of 180 degree and. So, with the condition that, there should be the point is on the root locus, it must satisfy the angle condition with odd multiple of 180 degree.

So, it is not satisfied here therefore, the root locus cannot be, it a between these two points, because here the angle conditions is not satisfied again, once it will be here, then again this another 180 degree, we will add it to be 3 into 180 degree, that is odd multiple of 180 degree. So, root locus will be here in this area and in this area, but not in this area and we see that, because complex poles and 0s are, they have, they nullify the angle effect. So, therefore, only we, we take consider the real odd number of finite open loop poles or 0.

So, we consider only these real axis poles and 0s, we can consider. So, therefore, the root locus here is between left of this odd number of pole that is 1, but not here, because this is left of the even number of poles that is two poles. So, therefore, the, we understand that real axis segment on the real axis, the root locus exists to the left of an odd number of real axis finite open loop poles and or finite open loop 0s.

Now, the fourth point is starting an ending point starting and ending point means where the root locus will start and where it will end. So, this point we have to discuss. So, the rule is that the root locus begins at the finite and infinite poles of G S H S and ends at the finite and infinite zeros of G S H S. So, it begins at finite and infinite poles of open loop transfer function G S H S and ends at finite and infinite zeros of G S H S.

So, let us see, how it is possible that this, they are starting at the finite and open and infinite poles and ending at. So, let us take the closed loop transfer function. So, let us we have T.

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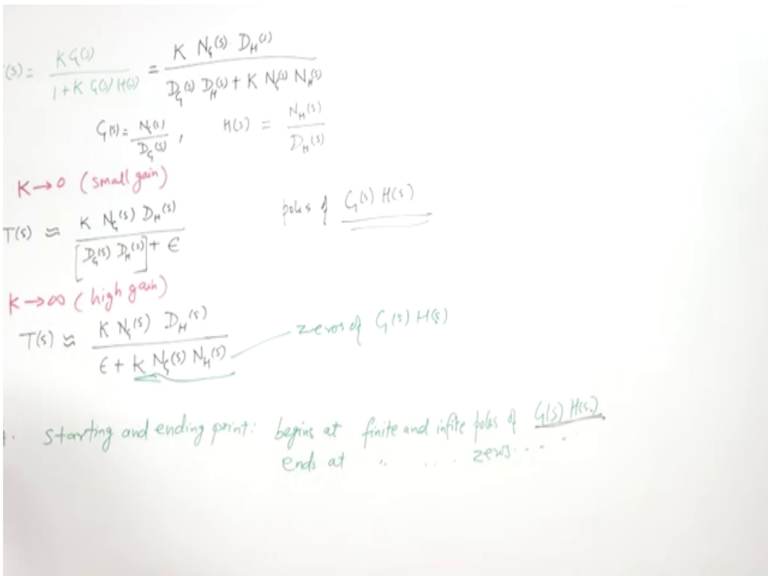

S equal to KG S upon 1 plus K G S H S so, this is the closed loop transfer function and this we can also represent as K times N G S into D H S upon D G S into D H S plus K and G S into N H S here, we have G S equal to N G S by DG S and H S equal to N H S by D H S.

So, we represent this these two transfer functions in terms of numerator and denominator now. So, we evaluate T S for evaluate T S for K tends to 0 that is for a small gain, for a small gain, why? Because the root locus starts at a small gain and progresses at large gains so, here K tends to 0. So, for a small gain, we can write that T S is, we can approximate here. So, at a small K gain, this quantity will be negligible in compared to this.

So, we can write K N G S D H S by DG S D H S plus Epsilon. So, Epsilon is some small quantity, because K is small, it is small and. So, now, you are N G S by D G S A G S, this is approximate and this is H S, because now, it is a ; so, this is negligible, we see that here, we have this is more more effective here, and this is negligible. So, these are what? These are poles of poles of G S H S ok.

So, poles of G S H S. So, that is why we say that the at a small gain, we are getting this equivalent transfer function, is governed by the poles of G S H S and therefore, at starting the gain is small. So, the root locus will start at poles. So, it begins at finite and infinite poles of G S H S. Now, when K is tending to, K is very high. So, when K tends to infinite, let us say high gain, we will get T S, is we can approximate, we can neglect this part.

So, we will get K N G S D H S by this is negligible plus K N G S and N H S. So, we can see here that this part is dominating here, this is negligible and this N G S N H S are the zeros of the G S H S. So, here this is the zeros of G S H S. So, that is why we say that, at high gain, the root locus ends. So, therefore, we say that it ends at finite and infinite zeros of G S H S.

Now, we should try to discuss one more point about finite and infinite poles or zeros. So, finite poles and zeros, we understand, because we can compute them, what about in finite poles and zeros. So, we saw the finite poles and zeros that we were able to compute from the numerator equal to 0, we get zeros and we put the denominator part equal to 0. So, we get poles, they are finite.

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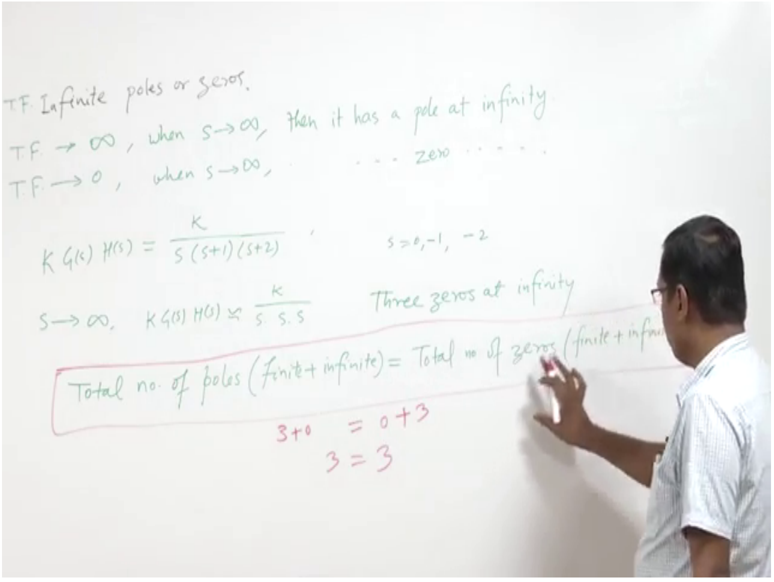

Now, we know we should know that there could be infinite poles or zeros possible. So, if a function, a function can have infinite poles or zeros function means a transfer function can have a transfer function can have infinite poles of zeros, when the transfer function tends to infinity, when S tends to infinity, then it has a pole at infinity and when the transfer function approach to 0 when, when S approach to infinity then it has a 0 at infinity.

So, let us take one example K G S H S equal to K upon S, S plus 1 S plus 2. So, we know that here, we have three finite poles. So, S equal to 0 and S equal to minus 1 and minus 2. So, we have three finite poles. Now, let us put S tends to infinity. So, when H tends to infinity, we will get K G S H S is approximate K upon S into S into S, because if S is tending to infinity, we can neglect this 1 and 2 in compared to S.

Now, each S in the denominator causes this transfer function open loop, transfer function 0, when X tends to infinity. So, when S will tend to infinity, each S value will cause this transfer function to be 0 and therefore, transfer function is tending to 0, when X is tending to infinity therefore, there are a 0 in infinity. So, each S will cause 0 to transfer function. So, there are 3 S and these 3 S will cause 0 to these transfer functions. So, there are three zeros at infinity.

So, there are three zeros at infinity. So, total number of poles that is finite plus in finite equal to total number of zeros finite plus infinite. So, this is the rule. So, because in this transfer function, we saw that there were three finite poles and there was no any finite zeros.

So, here number of poles finite is 3 and we did not find any finite pole, because when S was tending to infinity, this transfer function is not tending to infinite. So, 3 plus 0 equal to total number of zero is finite. So, finite zeros, there is no any finite zero. So, that is 0 plus infinite and we are getting three infinite zero. So, 3 equal to 3.

So, here we are getting that total number of pores, finite plus in finite equal to total number of zeros finite plus infinite. So, if in any this function, there is suppose, one zero is finite then, there will be two poles, that will be in finite. So, now, one more point, we will discuss that is this fifth point, that is behavior at infinity.

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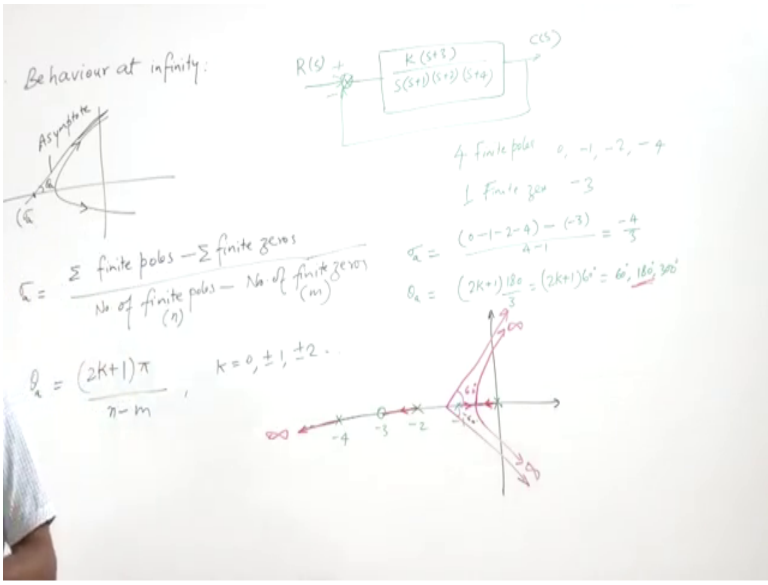

So, behavior at infinity. So, here the root locus aapproaches straight lines as asymptotes as the locus approaches infinity and.

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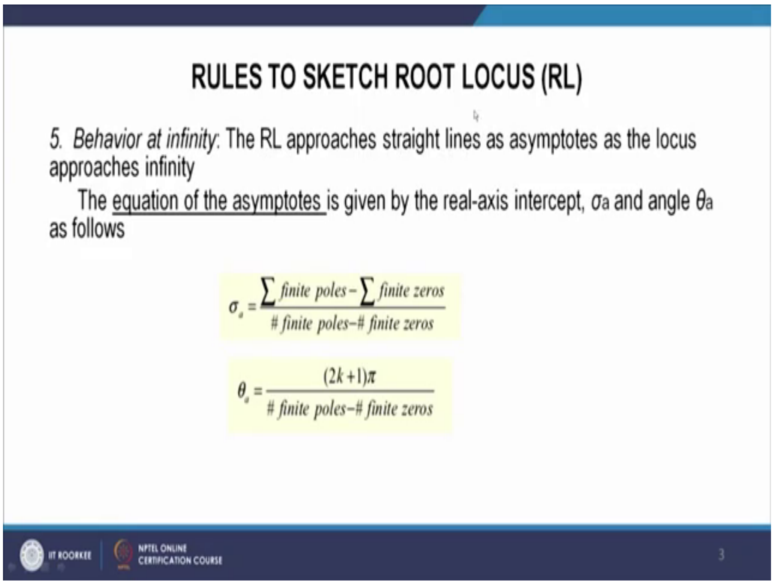

So, at infinity the root locus will approach as asymptotes of a straight line and the equation of the asymptotes can be fine with the real axis intercept means there is a straight line and it will intersect on the real axis with some, at some real axis, a real axis point and at certain angle.

So, we can find the intercept. So, if this is a point when suppose, this is the root locus is going like this and. So, it will follow some line as I seem to it and that line is going to intersect both, this is a root locus, going to infinity and this is the line as asymptotes, this is an asymptote straight line. So, it is cutting at sigma a with some angle theta a. So, this we can find. So, sigma a equal to summation of finite poles minus sigma finite zeroes upon number of finite poles minus number of finite zeroes.

So, we can see that when, we have to find here only decent finite poles and zeros numbers and the values of this finite poles and theta a equal to 2 K plus 1 pi upon N minus M. So, this is M and this is M, number of finite poles minus number of finite zero and here K equal to 0 plus minus 1 plus minus 2 and so on.

So, if we have some example here, let us take one example. So, we see that here there are 1 2 3 4, four finite poles. So, finite poles at 0 minus 1 minus 2 and minus 4, because it is 1 here. So, we K S H S is this. So and finite zeroes at minus 3 so, we have four finite poles and one finite zeroes. So, we have 3 and we know that finite poles plus infinite poles equal to finite zero plus infinity zero. So, here our total for finite poles so, we will have one finite zero means three zeros are infinite at infinite and therefore, because they are in finite and we know that the root locus will lead towards the zeros, because they start at poles and ends at zeros and three zeros are at infinite.

So, root locus will go to infinite and. So, at those infinite, how they will proceed will be told by the asymptotes defined by this formula. So, when we want sigma a. So, we can create sigma a. So, here sigma finite poles, finite poles 0 plus 1 is minus 1 minus 2 minus 4 minus finite 0. So, minus 3 by here 4 minus 1 so, that is 1 2 3 4 7 and plus 3. So, that is minus 4 by 3 and then theta a equal to 2 K plus 1 180 by 3.

So, that is 2 K plus 1 and 60 degree. So, that is 60 degree and then 180 degree and 300 degree. So, these are the angles. So, if we see on the S plane this. So, we can represent the poles at 0 minus 1 minus 2 and minus 4 and the 0 is at minus 3. So, we can see that the root locus will exist in this real segment and in this part and this part, because root locus will exist to the odd left of the odd number of poles our zeros.

So, here is this 1 left to the, 1 left to the 3 and left to this 5 1 2 3 4 5. So, we see that it will start here, it will start here, because root locus will start at poles and here this root locus will start and go N to 0 and this root locus will start and go to infinite, because there is 180 degree. So, this root locus is going to 180 degree and other two, they will diverge here to the complex plane and they may lead like this to infinity and they are intercept, we are getting at minus 4 by 3 that is 1.3. So, here this a straight line will tell at 60 degree and this line at 300 degree.

So, this minus 60 that is minus 60 here. So, these asymptotes are going to guide this, moment of the root locus at infinity. So, today, we, this example we took from the book of Norman as nise control systems engineering. So, today we discussed these five rules of two is sketch, the root locus first is the number of branches. So, one branch for each closed loop pole, then symmetry root locus is symmetrical about the real axis then real axes segment. So, root locus exists to the left of an odd number of real axis, finite open loop poles and or finite open loop zeros, then starting and ending points, the root locus begins at the finite and infinite poles of G S H S and ends at the finite and infinite zeros of G S H S then at infinity, it will approach towards a straight line asymptotes and the equation is, was given with intercept at the real axis, that is sigma I and theta and we saw this example.

So, we will continue some more rules to sketch the root locus in the next lecture 6 and I thank you for attending this lecture and we see you in the next lecture.