Automatic Control

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Lecture – 21
Define Root Locus

So, welcome to the lecture on root locus technique. Today, we will discuss about the definition of root locus and the importance of root locus in control engineering. So, here root locus is a graphical representation of the closed loop poles as a system parameter is varied, it is a powerful method of analysis and design for stability and transient response, it can be used to describe qualitatively the performance of a system as various parameters have changed. For example, if we want to see the effect of very varying gain, we can see it and it gives also the graphical representation of system say stability. So, we can say that root locus is the representation of the paths of the closed loop poles as the gain is varied.

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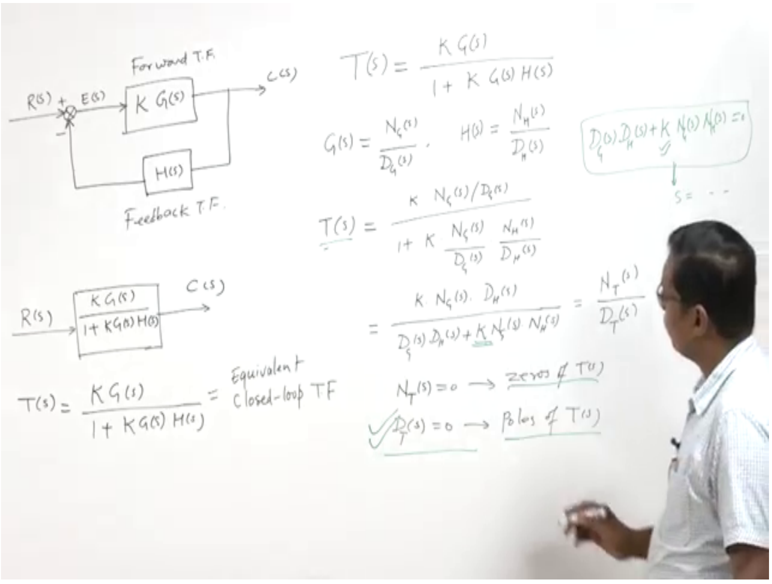

So, let us see here, system. So, we have the system and we here we represented at K G s. We include one parameter that is gain, we take it out specifically also this gain could be the part of G s, but here we write separately K, because we want to see how varying the K? We can see the, the change in the poles of the system. So, here we have C s and here is the feedback transfer function H s.

So, this is forward transfer function and this is feedback transfer function. Now, we want the equivalent closed loop transfer function of this system. So, we know that the equivalent transfer function, we can write here as R s, C s and this is K G s upon 1 plus K G s H s. So, this formula, we have already derived and we call T s equal to K G s upon 1 plus K G s H s and this is called equivalent closed loop transfer function or simply closed loop transfer function. So, this we can call equivalent closed loop transfer function or closed loop transfer function, we can call.

So, these T s can be written as. So, T s is K G s by 1 plus K G s H s. Now, here G s is a transfer function, it can have numerator and denominator part. So, we can write here G is equal to N G s by D G s. So, here we are saying that this G s is numerator by denominator. So, numerator of G s and denominator of G s. Similarly, H s can be written as numerator of H s and denominator of H s.

So, we can write T s as. So, we put these here. So, K into now G s, we can write N G s by D G s upon 1 plus K into N G s by DG s into N H s by D H s. So, we can simplify this and we can write here K into N G s into. So, here D G s will cancel out and D H s will go here so, D H s by D G s D H s plus K N G s into N H s.

So, we can see that the. So, here we can write numerator of T s by denominator of T s. So, we see that the numerator of T s or we get 0s from here, if we put this equal to 0. So, N T s equal to 0 will give the 0s of T s and D T s equal to 0. We will give the poles of T s. So, we see that N T s that is the 0s of T s is function of the N G s and D H s and. So, the 0s of T s; consists the 0, because here is 0 of G s and poles of H s, while the poles of T s or equivalent transfer function comprise the poles of G s H s 0s of G s H s and also here is function of K.

So, we can see that the equivalent transfer function spools is function of K, as well as it is function of the poles of G s and H s and 0s of G s and H s. So, if we want to find the poles of, for T s, we have to solve this equation. So, we have to solve this equation for some given value of K. So, suppose if we want to see the effect of K for some values of K, what is the location of poles every time, we have to solve this expression that is D G s D H s plus K N G s and N H s equal to 0.

So, we have to solve this equation for some value of K. We can find the value of S that is the poles of the equivalent transfer function this and then from once, we obtain the values of poles, we can tell whether the system is these poles are to the left half plane right, half plane ok. So, we can say whether system is stable or not or if we know the location of poles, we can also tell about the transient response of the system, but this is, this process is quite tedious and lengthy and every time for any value of K, we have to do this process.

So, here we can use the technique of root locus, because root locus is a graphical representation of the closed loop poles, when some parameter is varied like; K is varied the, it will show where these poles are for the values of K, are varied and we do not need to solve for each k, but the locus will tell whether this poles are going to the right half plane or not. So, the system is stable or not. So, root locus is the representation of the parts of the closed loop poles as the gain is varied here.

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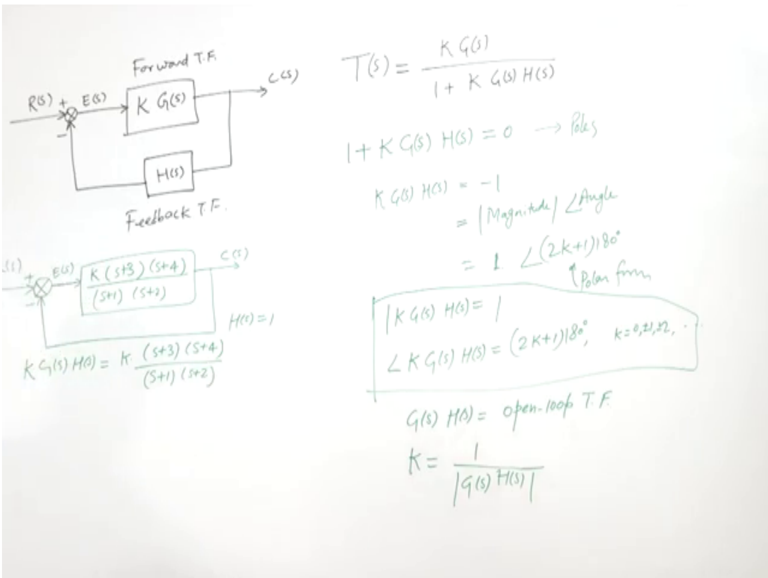

This 1 plus K G s H s equal to 0 will give us the poles; that is closed loop transfer function poles of closed loop transfer function. So, it means that K G s H s equal to minus 1. So, for any value of K, if this relation is satisfied that; pole s is the pole of this closed loop transfer function, and because root locus is the locus of the closed loop poles. This means that pole will be on the root locus for some value of K. So, if we represent this in polar form. So, this we can write as magnitude and angle. So, magnitude is 1 and angle is minus so, 180 degree odd multiples.

So, 2 K plus 1 180 degree so, means here we can say that mod KG s H s equal to 1. So, this is 1 and angle is 2 K plus 1 180 degree. So, mod K G s H s equal to 1, and angle K G s H s equal to 2 K plus 1 180 degree, where K equal to 0 1 2 plus minus 1 plus minus 2 and so on. So, here this is polar form.

So, the root locus a pole is on the root locus, it must satisfy these two conditions, and here G s H s in each called the open loop transfer function, open loop transfer function. So, we should understand the difference between G a; this open loop transfer function, and closed loop transfer function or equivalent cool. So, this T s is equivalent closed loop transfer function, but G s H s is called the open loop transfer function, and so we can find the gain for which gain for which, if there is some pole on the root locus, we will find the gain for which this is on the root locus.

So, these are the some properties of the root locus. Now let us take one example that here. So, this is R s plus minus E s C s. So, here your G s is equal to K. So, H s is 1, because here is unity feedback. So, H s equal to 1. So, here K G s H s equal to K into s plus 3 s plus 4 upon s plus 1 and s plus 2. Now, let us represent the poles of the open loop transfer function G s H s on the s plane.

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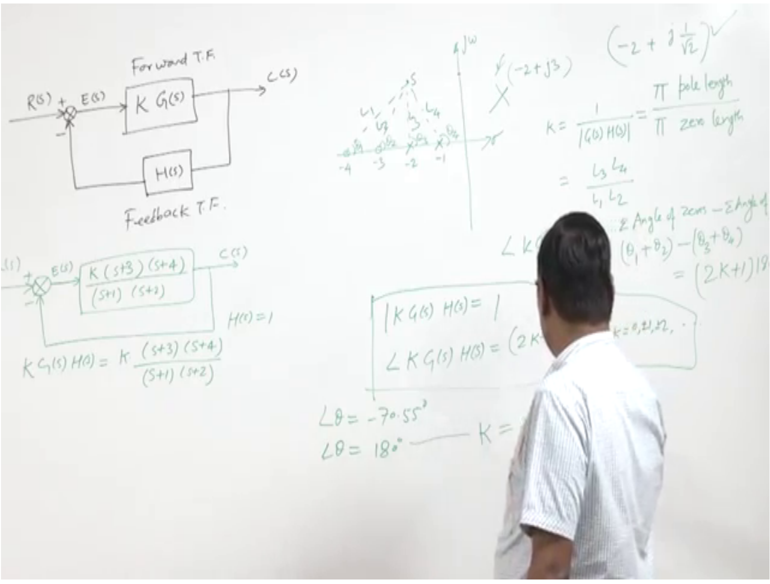

So, this is sigma j omega. Now the 0 is minus 3 and minus 4 the 0s. So, here is 1 0 is at minus 3, other is at minus 4 and poles are at minus 1 and minus 2. So, this is pole minus

1 and minus 2. So, these are the open loop transfer functions poles and zeros; that is of G s H s, so we plot here.

Now if we assume that there is some pole s on the root locus or it is a closed loop poles of closed loop transfer function T s, then it must satisfy these two conditions. So, let us say that here is one point, this point that is minus 2 plus j 3 is on the root locus or that is a closed loop pole. So, here we know that here K equal to 1 by mod G s H s.

So, if here is a pole that is here, we have some s. So, and this we can write as. So, this is a multiplication of the 0 length; and multiplication of pole lengths for the number of zeros or poles, because here G s H s, each we can write represent as 0s upon poles. So, poles will go in the numerator and 0 will remain here, and so the difference between this, this 0 and this pole is the magnitude, is the length.

So, that is why we take the length. So, here if we say that this is l 1 this is l 2 this is l 3 and this is l 4. So, pole length these points the length from the pole that is l 3, l 4 by the 0 is length L 1 and L 2. So, if this is on the root locus or on the closed loop or it is a closed loop pole, this condition must satisfy, and so we can get the length, the value of the gain; however, whether it is on the root locus or not, it must satisfy this condition also. So, if we take the angle K G s H s. So, let us say this is theta 1, this is theta 2, this is theta 3, this is theta 4.

So, the angle should be. So, sum of angle of 0s minus sum of angle of poles. So, here theta 1 plus theta 2 minus theta 3 plus theta 4 so, it must also satisfy this angle condition, because here we have satisfied the, we have calculated the, just the length or magnitude. Here we, it must satisfy the angle condition. So, if this pole is on the root locus, it must satisfy this as 2 K plus 1 180 degree.

So, it must be odd multiple of 180 degree, then only this pole can be said to be on the root locus, or it can be a closed loop pole for the certain values of K, and when we calculate this angle, let us say this is angle theta for this pole. So, angle theta equal to, we can calculate this angle. So, theta 1 is equal to this length L 3 by L 1. So, tan theta 1, so we can find the angle. Similarly we can find theta 2 these upon L 3.

So, we can find that sin theta 1 equal to L 3 by L 1, and then we can find sin theta 2 equal to L 3 by L 2 and so on. So, we can find this theta 1, theta 2, theta 3 and theta 4. And when we find, we find that this theta is coming minus 70.55 degrees. So, if this angle is not odd multiple, either it should be 180 or 3 times 180 or 5 times so on. So, this point is not on the root locus, or it is not a closed loop poles.

However; if we take another point; that is minus 2 plus j 1 by root 2 for this point we get this theta equal to 180 degree, and so this point is. So, this point is on the root locus and for this we can calculate the gain K, using this method, and so we will get K equal to 0.33. So, thus we understand that the root locus must satisfy these two conditions. If a point is on the root locus, it must satisfy these two conditions. So, let us take another example.

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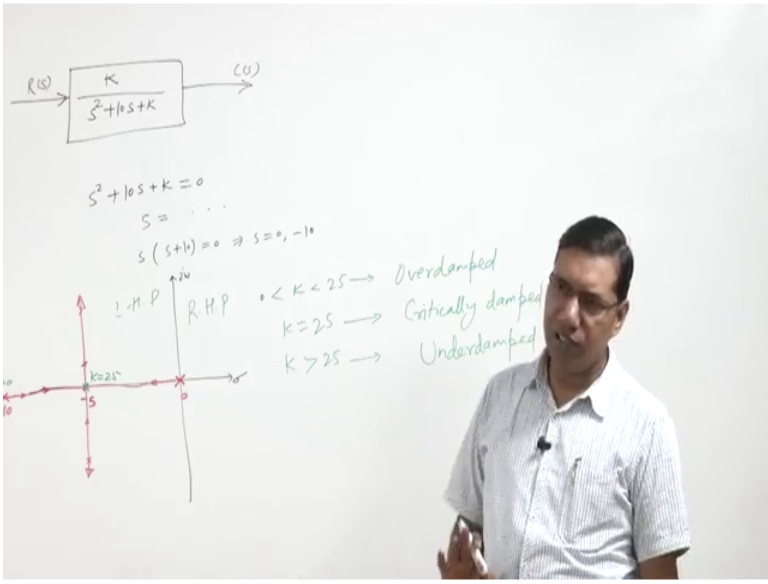

So, here we take one example; that is. So, we can see here in this figure that we have an equivalent closed loop transfer function for some system, and we are getting this.

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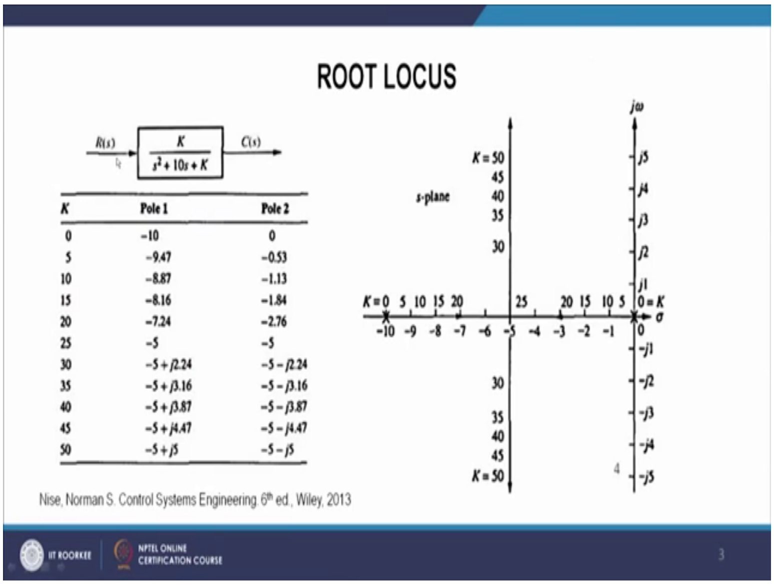

Now as we define the root locus, that root locus is a graphical representation, when the sum for some parameter is changing, how the closed loop poles are going to change or what path they follow that is called the root locus.

So, here we can see that when we are varying the K. So, K is 0 we put K. So, this is the s square plus 10 s plus K. So, this s square plus 10 s plus K equal to 0, will give you the poles s, the closed loop poles. So, if K equal to 0. So, here we will get s square plus 10 s equal to 0 so, s plus 10 equal to 0. So, this means s equal to 0 and minus 10. So, we will get the two poles here; pole 1 and pole 2, south pole 1 is minus 10 pole two is 0.

Similarly, if we put K equal to 5, here we put K equal to 5. So, again we will get some pole here; that is we get minus 9.47 and minus 0.53. Then, if we put K equal to 10 we get minus 8.87 and minus 1.13. Similarly we are varying this K from 0 to with 5 increment 5 10, 15, 20, 25, 30, 35, 40, 45, 50 and we are going to get that the two poles of the system for this varying K, and these poles we are plotting here on the s plane; that is sigma j omega axis is here and we have, we represent these pole poles. So, first we represent for K equal to 0, we represent minus 10 and 0.

So, this is the pole 0 and minus 10 here. Now we are changing the K from 0 5 10 15 20 and these poles are changing. So, minus 9.47 then minus 8.87 and so on, and this is moving. And from here also we can see that. So, our initial poles or here at 0 and then here at minus 10 at K equal to 0. Now when I am changing this, they are the poles are moving like this. So, poles are changing when I am changing case, then at K equal to 25, we can see here at K equal to 25 we see that here the poles are distinct for K equal to 0 to 20.

However at K equal to 25, the poles are equal, but real. So, at K equal to 25 we are here at minus 5, the poles are, both poles are same, they are equal, and so these move, this poles move here then we see that the poles are, when we 25 beyond at K equal to 30, the poles are the complex pair minus 5 plus minus j 2.24 and so on. So, the poles are being complex here. So, the poles are going to here, complex and at K equal to 50, it is minus 5 plus j 5.

So, here we have some pole here at minus 5 j 5. So, what we see, we see that for K equal to 0, we are starting with the real, two real poles, real and distinct poles at K equal to 25. The poles are real buts equal and beyond 25 the poles are complex. So, we can see here that how the; this is called the root locus, these red lines are the root locus, because they tell that how with the changing this parameter K, the poles are moving and we can see that this tells many information when the K is less than 25, we are having real and distinct poles. So, we know that we are in over damped, over damped condition, because this is second order system.

When K equal to 25 we have equal poles. So, we are in critically damped condition, and when we have K greater than 25, we are in under damped system condition under damped. So, the response of the second order system is can be understood from the this; that we are in these three different regions, and so that responses will be accordingly transient response.

Now, we see that the poles neighbor crossed these axes; they are not going to the right half plane. So, this system is always is stable for any value of K, because this root locus is not crossing the j omega axis to go in the right hand side, because this is right half plane, and this is left half plane. So, it is always through the left half plane. So, the system is always stable for, it is not unstable for any value of k, but it is stable. So, we see that from the root locus, we can see the transient response, because we see the location now and movement of poles. So, we can tell about transient response and stability. So, we discussed and defined the root locus. And in the next lecture we will start the how to sketch the root locus.

So I stop here, and I thank you for attending the lecture.