Automatic Control
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Lecture – 20
Static Error Constants

So, welcome to the lecture on steady state error. So, we will in this lecture we will also discuss about the static error constants. So, we were discussing about the; that steady state error is the difference between the input and the output when times time tends to infinite. So, we found that there are some prescribed test inputs against which the we check the steady state error for their step input ramp input and parabolic input.

So, for the step input we found that, there should be at least one integrator in the forward loop or we are more than 1 it could be. So, n is greater or equal to 1 where as for the ramp input, we found that n should be greater or equal to 2. So, there should be at least 2 integrator in the forward loop so that the steady state error is 0. If there is n is equal to 1 for ramp input, then there will be some finite steady state error and if n equal to 0 we will get infinite steady state error.

Now, we check we formulate the; that expression for parabolic input. So, here if we have parabolic input.

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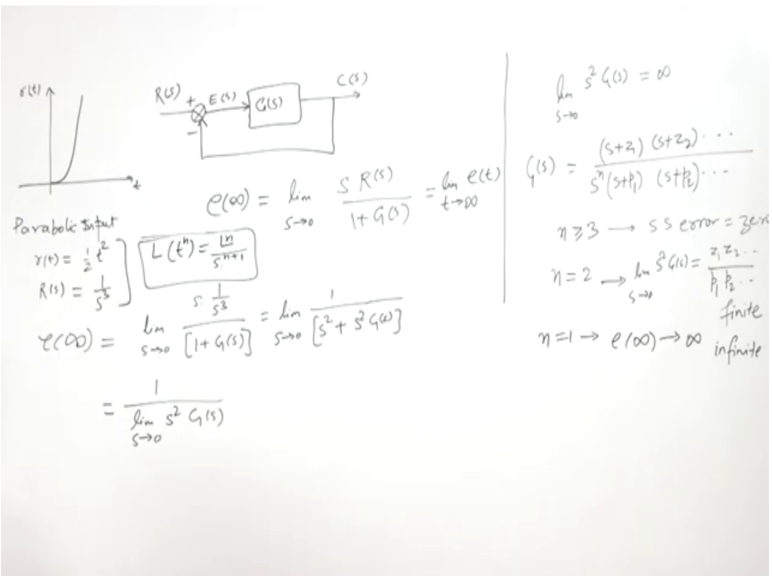

We have parabolic input and so, we have this here G s this is R s, C s and there is a unity feedback system. So, here this is E s plus minus. And we find that E steady state error equal to limit S tends to 0 S R s upon 1 plus G s. So, this is the how we calculate the steady state error? We know the transfer of this R s we know G s, then we can calculate the steady state error. So, if we have parabolic input parabolic input, they then we have rt equal to half t square and so, R s equal to 1 by S cube. So, we can write this expression of that series like e infinity that is a steady state error.

So, we will get. So, this is equal to et when time tends to infinity. So, this error is equal to limit s tends to 0, s into 1 by s cube 1 plus G s and this will be equal to limit s tends to 0, 1 by s square plus. So, here it will be s square G s and this will be equal to. So, this is 0. So, we will have 1 by s square G s and limit s tends to 0. So, if we want this steady state error to be 0 this term, this denominator term should be tending to infinity when it s tends to 0. So, therefore, we will have this limit s tends to 0, s square G s that should be tend to infinite and so, if we assume that G s is s plus z 1, s plus z 2, s plus p 1 s plus p 2 and so on.

So, here z 1, z 2 they are they will 0 am magnitude and p 1, p 2 poles. So, here if we put S power n. So, n should be greater or equal to 3, then we will get a steady state error equal to 0. Because if n equal to 3 we will have here S cube and then this s cube will cancel s square there will be 1 s at least here and that tend to 0. So, this will tend to infinite. So, now, if n is equal to 2. So, n equal to 2. So, s square will cancel out with s square and we will get this limit s tends to 0, s square G s equal to. So, here we will get z 1, z 2 by p 1, p 2 and this error will we get p 1, p 2, p 3 by z 1, z 2, z 3. So, that that is finite error finite a steady state error.

However when n equal to 1 or less. So, n equal to 1 or less. So, here we will get 1 s and that is here. So, we will have 1 s here. So, that will be make this quantity 0 and so, we will get e infinite tends to infinite. So, we will get infinite steady state error for n equal to 1 or less than 1. So, we will get the this. So, this these things we wed get this Laplace transform of t n equal to factorial n upon s n plus 1.

So, here we have t square. So, factorial 2 upon s 3. So, 2 will cancel out. So, this we have already discussed this transport tons transform Laplace transform and the formulas. So, now, we will we understand how for a step input for ramp input and for the parabolic input, we can have the desired steady state error that could be 0 finite or infinite what we want we can get it by adding some integrator pure integrator in the forward loop. Now we take 1 example to calculate this error.

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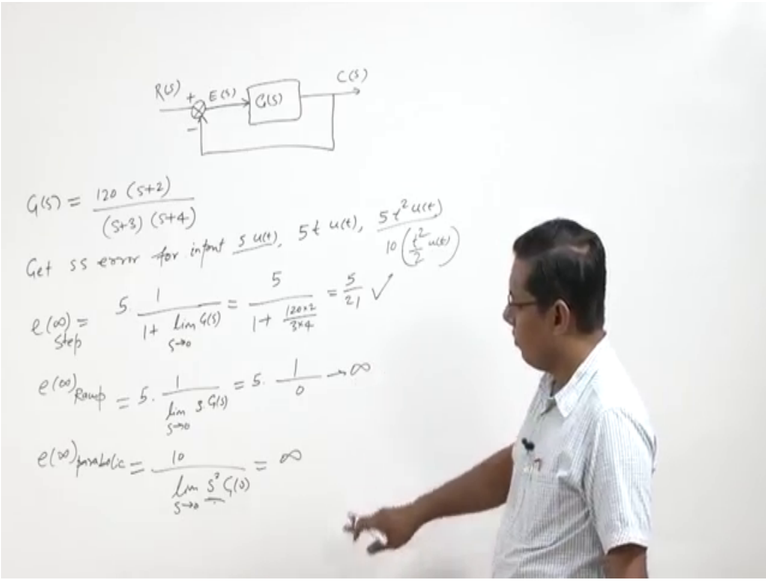

So, let us we have G s equal to 120 s plus 2 upon s plus 3 and s plus 4. So, in this closed loop sa closed loop system, we have G s this. So, now, we have to get the steady state error for input. So, get steady state error for input 5 ut, 5 into t into ut and 5 into t square into ut. So, here we assume that this system is stable we of course, we said that steady state error has only meaning if the system is stable. So, let us we can check, but this system is stable. So, we can get e infinite for this step input. If we know that the formula that, we have derived we derived for unit step input. So, here we will use this is 5 times the unit. So, 5 into the formula that we derived for a step input that is 1 upon 1 plus limit s tends to 0 and G s.

So, here we will get 5 upon 1 plus when s tends to 0 here. So, this is 120 into 2 by 3 into 4. So, that is here 20 this is 20 and plus 1. So, 5 by 21. Now for this is for a step, now for ramp we will get 5 times. So, we got the formula that 1 by limit s tends to 0 and s G s into G s. So, we will get 5 into 1 by here we get when this is 0 ss will make it 0 and gis G s is finite. So, this is 0. So, it means this is infinite and similarly for parabolic we will get we know that the parabolic function we derived was for half t square. So, here we will have 10 upon limit s tends to 0 and s square G s. So, that was what we derived here was for t square by 2 ut. Now here we have to get this we have this is 10 into t square by 2. So, that gives. So, for this the formula was 1 by limit s square G s. So, this is 10. So, again this s square will make 0. So, infinite; so it will tend to infinite.

So, we know that here just by saying this G s, we can know that what type of a steady state error we will get by different inputs whether we will get 0 or finite or infinite, but there is no any integrator here there is no any s term. So, you know that when for the step input to get the 0 steady state error at least there should be 1 integrator. If there is j n equal to 0 we will get finite error where we are getting finite error for a step, but for ramp and parabolic here end should be greater or equal to 2 and here end should be greater or equal to 3 otherwise we will get infinite error.

So, here we have discussed about steady state error for 3 different test inputs. Now we define some more terms that is called static error constants.

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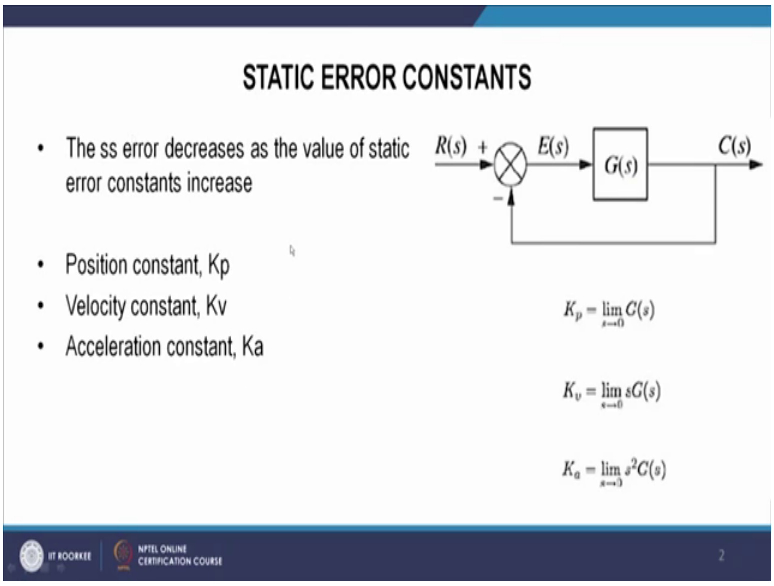

So, we have to define a static error constant. So, here we have 3 static error constants.

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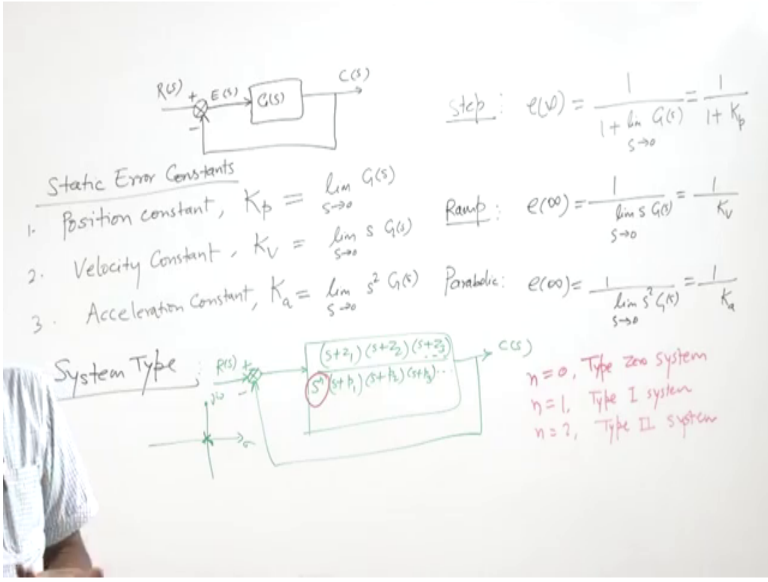

So, one is position constant, second one is velocity constant and third one is acceleration constant. So, we call this as K p. this as K v and this as K a. So, as we saw that in case of transient response of a system, we could we were able to give some specifications performance specifications or output specifications. For example, for second order under damped system we were able to give the peak time percent overshoot, damping and settling time and highs time. So, they could from there we could from there seeing the response we could say that what is the performance of the system whether it desired or not.

Now, in case of a steady state errors we have the these static error constants, that are the performance specification of the steady state errors against different inputs. So, here position constant K p is defined as, K p defined as limit s tends to 0 G s and K v is defined as limit s tends to 0 s G s and K a is defined as limit s tends to 0 s square Gs.

So, these are these 3 error constants, and the relation of this constant with the steady state error is that, the steady state error decreases if the value of state static error constraints increases. So, if this value is more the steady state error will be less. And now we can correlate these constant with the steady state error. So, we know that for a step input for a step input we had. So, here if we have this step input we have steady state error 1 by 1 plus G s and here limit s tends to 0.

And; so, here we can write that is equal to 1 by 1 plus K p, similarly for ramp input we had e infinite equal to 1 by 1 plus limit s tends to 0 s G s and here we can write 1 plus 1 by 1 plus K v. And in case of parabolic input we had e infinite equal to 1 by 1 plus limit s square G s tends to 0 and so, this is 1 by 1 plus K a sorry here is some mistake.

So, here is this because we had this. So, we can directly get 1 by K v and 1 by K a for these 3 inputs. So, we can see that if we have K p is more this error will be less K v is more this error is less; k is more the steady state error is less. So, here one more term we should discuss is the system type. So, here the system type is if we have we said that G s should be in the form of s plus z 1 s plus z 2 s plus z 3 and so on upon s plus p 1, s plus p 2 and s plus p 3 and so on into s power n and this we can have R s and here C s and here ze is the unity feedback.

So, in these case this n [noise] s n. So, if n equal to 0, we have type zero system. So, system type is 0 if n equal to 1 type 1 system and n equal to 2 type 2 system and. So, on now when we say this type 1 type 2 system when n. So, what does this n means? That when n equal to 0 this is 1, n equal to 1 means this is s and s means if we plot the s plane. So, sigma j omega plane. So, there is 1 pole at 0. So, means that if we have 1 pole at origin we call it type one system, if there are 2 poles at origin we call it type 2. So, system and so on. So, we can relate because we know that this n will end will govern the steady state error values whether they are 0 finite or infinite. So, in other terms we can also talk about system type. If we have type one system means there is at least 1 pole at the origin and if there is type 1 system it means we know that for a step input if this is type 1 system we will get 0 steady state error.

And if it is type 2 system we will get 0 steady state error for ramp input and so on. So, now, let us see this table for release relation what we discussed we can tabulate here this is steady static state error and static error constant so, we see that a step input, ramp input and parabolic input.

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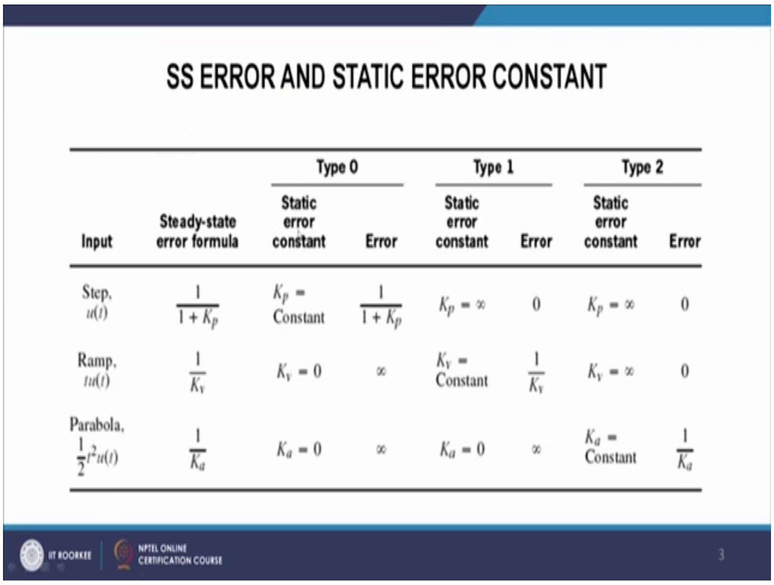
SS ERROR AND STATIC ERROR CONSTANT

		Type 0		Type 1		Type 2	
Input	Steady-state error formula	Static error constant	Error	Static error constant	Error	Static error constant	Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p =$ Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a =$ Constant	$\frac{1}{K_a}$

So, it is steady state error formula is 1 by 1 plus K p that I have already given 1 by K v and 1 by k now if this is type 0 system. So, this means n equal to 0. So, we know that when n equal to 0. So, for n equal to 0 this G s will be if s we put s all s 0 and this is not here or 1. So, we will get some finite value. So, we will get this finite. So, K p will be some finite in constant value. So, therefore, it is written K p equal to constant, here the steady state error that is 1 by 1 plus K p. So, steady state error defined as 1 by 1 plus K p. Now here for type 0 system K v equal to 0; because if it is type 0 systems.

So, this K v will be 0 and so, error will be 1 by K v that is in finite, k will be 0 that is infinite type 1 system. So, there is 1 pole. So, we know that if there is 1 pole, this K p will be infinite and steady state error will be 0 and for ramp type 1 system, ramp input the K v equal to constant some finite value and so, the error will be also finite value. And here k will be 0 and error will be infinite. For type 2 system we will have K p and K v both are infinite and so, the relevant errors are 0; however, for k this is constant and finite. So, the steady state error is some finite value.

So, this table is we can we have got from the this discussion. So, one thing we should also discuss from this table. If we say that K v equal to 1000. So, this K v equal to 1000 will tell several aspects about the system and its response or a steady state error. So, K v equal to 1000 means we know that this is a finite and constant value.

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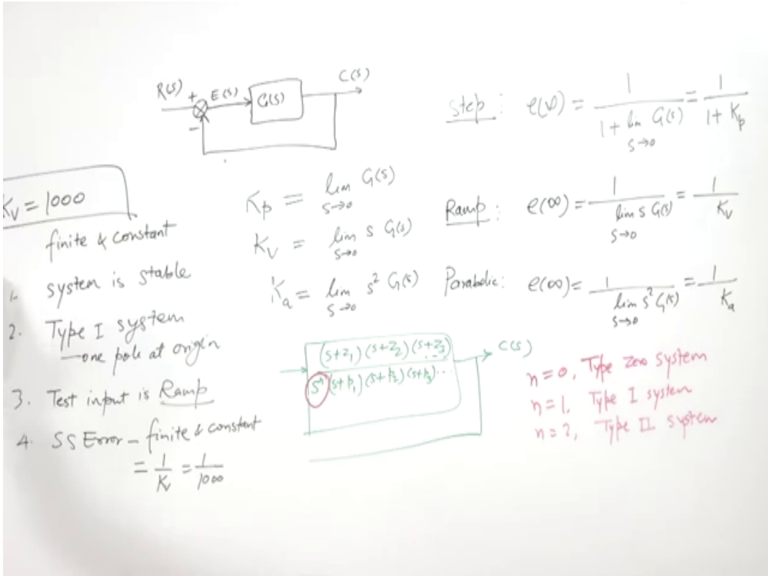

Now, see this table where is K v constant. So, K v is constant here in type 1 this static error constant K v is constant. So, it means that the system first thing the K v is existing for the system, it means the system is stable. So, first point we should note that, the system is stable. Now it is type 1 system because only for K v you can see here that K v is constant only for type 1 system. So, the system type is it is type 1 system. So, means there is 1 at 1 pole at origin. So, type 1 system means 1 pole at origin. Then third point we should note that ram is the test input. So, here for the ramp input this K v is constant the constant value of K v we are getting for the ramp input. So, this is text input is ramp, test input is ramp and the fourth point we will get about steady state error.

So, here we can see that the steady state error is finite and constant that that is 1 by K v. So, steady state error is finite and constant and that is equal to 1 by k v. So, that is 1 by 1000. So, this is the steady state error. So, you see that just by knowing these this K v equal to 1000 for a system, we can know these 4 points for that system. Now we have what we have discussed? We have discussed for a unity feedback system, but if the system is not a unity feedback system but there is another feedback transfer function H s.

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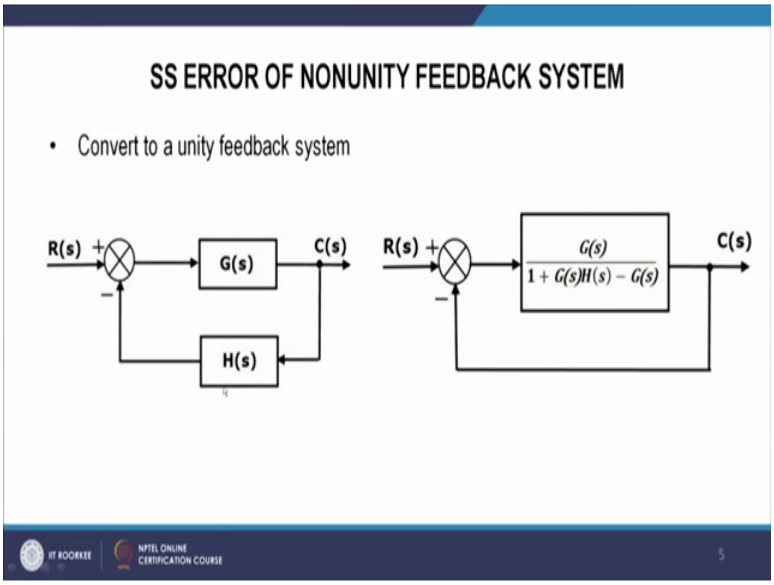

So, how we will get the steady state error are the formulas that we applied here; how can we use. So, we can know that.

So, if we have this condition we have also non unity feedback system.

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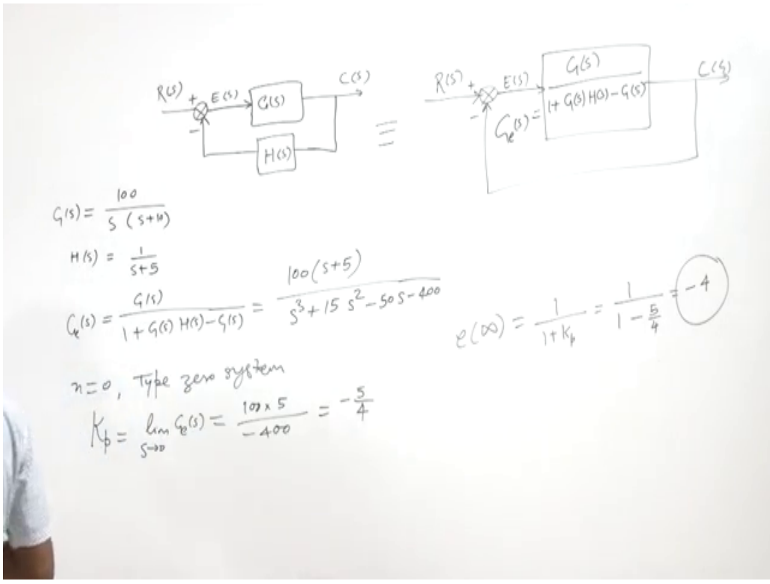

So, we have to convert first this system to an equivalent unity feedback system. So, we have these, these and this is unity feedback.

So, this is R s plus minus E s g C s. Now we have to convert these 2 and this we can convert using the block diagram algebra that I already discussed hm. So, this when we can obtain this G s upon 1 plus Gs Hs minus G s. So, we can convert this to this form and then we can apply all the formulas that we derived for unity feedback system. So, let us take 1 example. So, we have system where G a equal to hundred by s plus 10 and H s equal to 1 by s plus 5.

. So, we can get let us say this is equal to G e s equivalent transfer function. So, G e s we will get by that formula G s by 1 plus Gs Hs minus G s and when we solve this, we will get 100 s plus 5 upon s cube plus 15 s square minus 50 s minus 400. Now we know that here is no any in this transfer function, because now this is equivalent to G s and here we see that there is no any pole at origin. So, here n equal to 0. So, this is type 1 system type 0 system and for a type 0 system we should have the see the error with respect to the step input and. So, K p equal to limit s tends to 0 G es. So, when we find these we will get hundred into 5 this is 0 we will get minus 400 and so, we will get minus 5 by 4 and so, we can we will get e error equal to 1 by 1 plus K p and 1 by 1 minus 5 by 4. So, we will get minus 4. So, so we see that here we have a constant and finite this steady state error.

So, we saw that here output step is larger this. So, we get the unity step. So, unity, but here we get 4 times this output. So, this example was taken from some reference book Norman's nise control systems engineering. So, in this lecture we discussed about a steady state error and static error constraints and how we can get the steady state error for the test inputs different type of test inputs and we can design the system by taking the poles at origin for as per our requirement of the steady state error. So, I thank you for attending this lecture and see you in the ne next lecture.

Thanks.