Automatic Control

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Lecture – 19
Steady State Errors

So, welcome to the lecture on stability and steady state error. So, in the previous three lectures, we have discussed about stability in the two lectures and that is this lecture and the next lecture. We will discuss about steady state error. So, we have discussed that to design a control system, there are three objectives that is the transient response and stability and steady state error.

So, we have discussed the transient response and we have discussed a stability today we will discuss about the steady state error. So, we know that steady state error is the difference between the input and the output for a prescribed test input at highs time t tends to infinite. So, here we can say when we check about the any a check about the steady state error for a system, we use certain prescribed testing inputs. So, these test inputs could be step input they could be ramp input parabolic or impulse input and these different inputs when they are input to the system, we study the b at the output of the system against this input, and we defined the steady state error in that condition.

So, when we remember that, when the; we had the elevator response.

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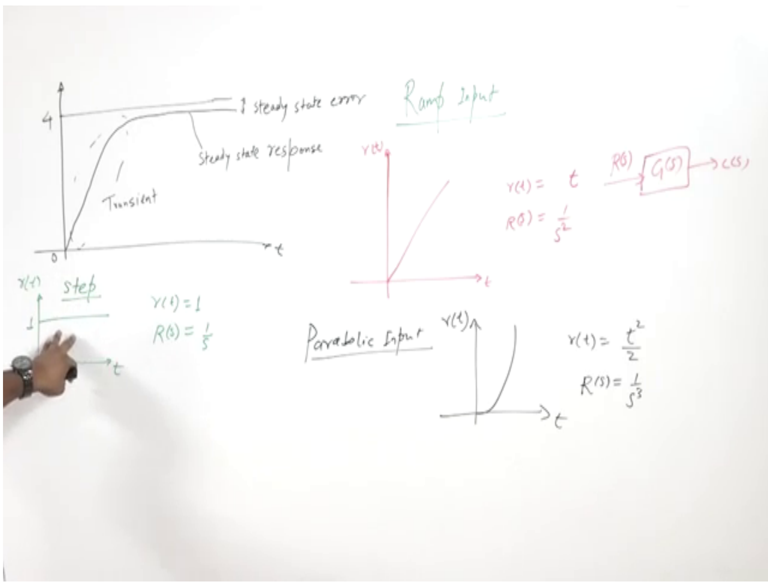

So, if we remember here we had time t we have this floor number 4. Let us say we are at 0 or ground floor, and we press this button number 4 to reach that 4th floor the elevator response will start like this and it will be like this. So, of course, this part we show this is the transient response, transient response and this is the steady state respond steady state response and this difference that is the difference between the inputs. Because our objective was to reach at the 4th floor, but the lift or elevator reaches little with little difference. So, it did not level with the floor of the 4th floor. So, this is difference is there was a steady state error. So, today we will discuss that how we can define this how we can define the test inputs, and how can we define a system or design a system; so that we can reduce the steady state error.

So, here we have the test inputs that we talked about this step input. So, we have here this is the step input, the time and here we have let us say we have the input rt with time. So, this is constant or if you give this value one that is unit step. So, if this is 1. So, this rt equal to 1 for all the time greater than 0. So, if we take Rs; R s equal to if we take the Laplace transform that is 1 by s. So, we know that this step input can be used to for the control system to monitor the position stationary position of some part or some component. So, for example, if there is some for example, this lift we press this 4 and this is a system whether it is following this position or not. So, we can use this step input for testing as a test input for this kind of system or this kind of application.

Now second input comes as the ramp input. So, here we have ramp input. So, what is ramp? So, ramp is the linearly varying input with time. So, here is time t and this is r t and the input is linearly varying. So, it is a constant slope, it is not the constant, but it has some constant slope. So, here we can say that d rt by. So, rt we can write as it is function of time t with some constant. So, here we take the Laplace transform. So, Rs. So, R s of this is s is ca 1 by s square.

So, here we will have 1 by s square this Laplace transform as R s. So, such a input. So, this kind of input can be used to monitor a system that is have constant speed. For example, if there is some satellite orbiting satellite and because it is angular speed magnitude is constant, but it is orbiting. So, we can use this input to monitor the angular speed of this input. So, this test input can monitor the some changing position with constant velocity, now there is the third kite type of input that is parabolic input.

So, the parabolic input we have parabolic input. So, we have here r t and this is something like parabola here and here this r t is let us say this is a parabola half t square and so, its Laplace transform is 1 by s cube. So, this type of test input can be used to monitor the accelerating objects or some object that is accelerating and we can use this test input to monitor the acceleration of that object. For example, we launch some missiles and because missiles are accelerating objects they have to accelerate to hit the target. So, we can use this type of input to monitor the acceleration or accelerating objects. So, we see the important of these three inputs. So, step input to monitor the stationary position, this the position changing with constant velocity and this position changing with constant acceleration.

So, we will study the steady state error against these three type of inputs three types of test inputs.

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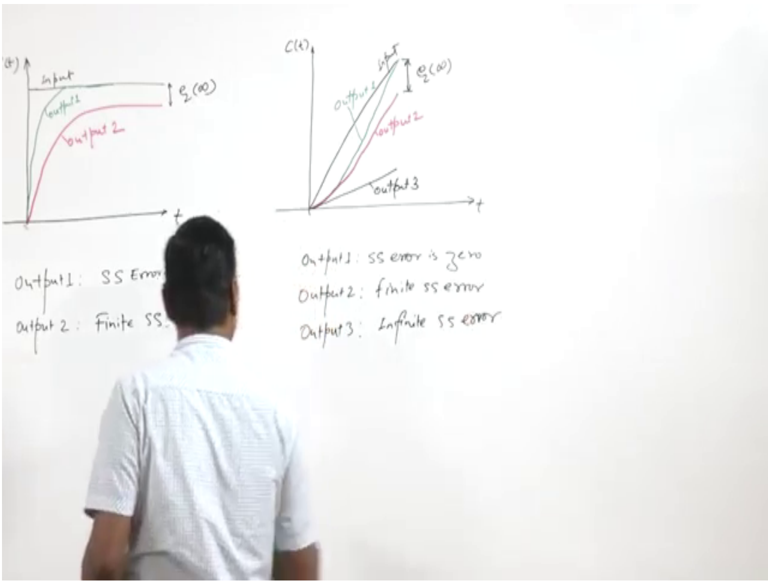

Now, let us see some output against these inputs and how we can define the steady state error against these different types of inputs. So, here; so let us me give some input here, this is input or desired output, that is a step this is a step input, and we are getting the response of our system. So, let us say we are getting something. So, this is output 1, this is output and let us says output 1. Now we have another output; output 2. So, this is output 2. So, here we can see that this output 1 the steady it is starting following this after the transients are decade it is following the input perfectly.

So, here the steady state error is 0. So, in this case output 1. So, steady state error is 0 because it is matching to the input. Now, if we see output 2 and this difference we call it e 2 infinite. So, when time tends to infinite the difference is constant. So, output 2 there is some finite steady state error here. So, for output 2 there is some finite steady state error. Now let us see the ramp input. So, here t this is c t and we have some ramp as input. So, this is our input. Now here. So, this is output 1 this is output 2 and this is output 3.

So, here we see that this output 1 is finally, going and meeting or to the input. So, here output 1 has 0 steady state error output 2 has. So, this is finite steady state error let us call it e 2 infinite. So, finite steady state error and this is parallel to this. So, slope is same, here also the slope is from here it is same now to see the output 3. Its slope is different and the th the it is not tending towards the input ands this they are more diverging. So, here this is infinite steady state, this is this can be called as infinite steady state error.

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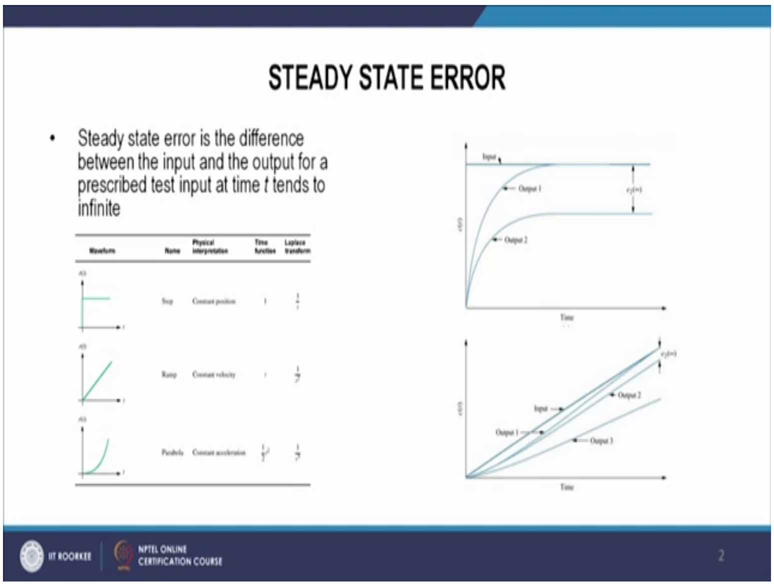

So, now let us obtain. So, these things we can see here in this diagram, what I discussed we can see here the same thing that I explained.

Now, let us find that how we can define the steady state error in Laplace variable form that is s form in in terms of s, because we are dealing with the transfer function and we should know how to compute the steady state error in terms of eh if the function is in s domain. So, we define the steady state error.

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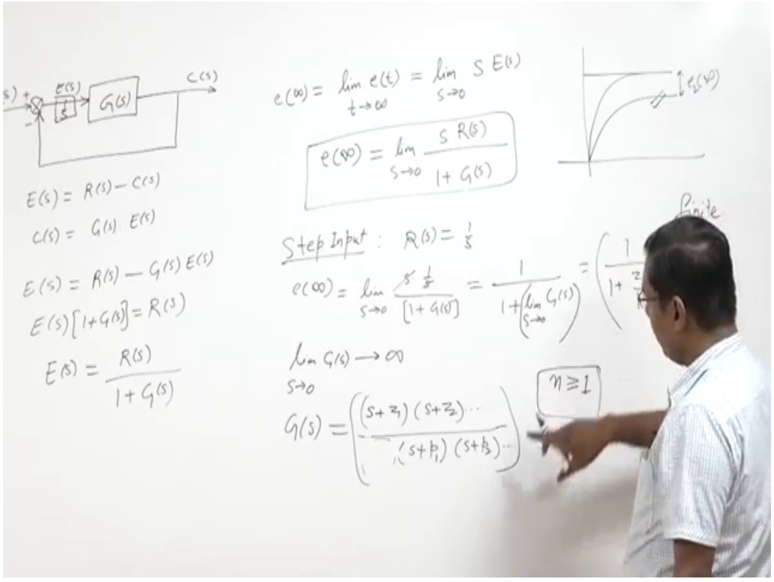

So, we have some general transfer function that we are discussing is here is G s, there is input R s and here is we are getting C s the output and we give feedback. Let us say we call unity feedback here and we give here this and here we get the E s. So, this is plus this is minus. So, this is unity feedback system and here we can write E s equal to R s minus C s and C s equal to G s into E s. So, we can write this as C s by G s equal to R s minus C s.

So, from here. So, E s equal to R s minus C s we can add G s into E s. So, we can write E s 1 plus G s equal to R s and so, E s equal to R s 1 1 plus G s. So, this is the error function that we can find. So, the final value of the error. So, e infinite equal to limit e t when t tends to infinity this is the definition of the error that when time tends to infinite the difference between the input and output is called the steady state error and this can be written as limit s E s tends to 0 and therefore, here this error we can define equal to limit. So, S into R s because we can replace this s R s upon 1 plus G s and this S tends to 0.

So, fo now here we can compute the error steady state error, with this we know input we know the transfer function, we can calculate the steady state error. Now here we should know that when we talk about a steady state error. So, steady state error is only valid when the system is stable, because if the system is not stable the response will not reach to steady state response and there is no meaning to talk about the steady state error. So, therefore, for any system we must first check whether it is stable or not, then only we should calculate the steady state error. So, now, let us see if there is a step input, if we have a step input. So, it means the R s is equal to 1 by s. So, we can calculate the steady state error infinite that is equal to limit s tends to 0 s into 1 by s by one plus G s and so, this is here cancel. So, 1 by 1 plus G s limit s tends to 0, but this limit will reach to the function; that contain the s or the funct G s is the function of s.

So, if you want zero a steady state error because our objective is that there should be 0 steady state error. So, there is should not be any finite a steady state error. So, if this term tends to infinity this part tends to infinity when s tends to 0. So, this is infinity then this term will be tending towards 0 rights. So, to find this zero steady state error limit, s tends to 0 G s. So, which tends to infinity and therefore, G s must take this form s plus z 1 s plus z 2 into s power n, s plus p 1, s plus p 2 into some other.

So, you see here G s is in the form. So, these are the zeroes because this is G s is a transfer function. So, it could have some zeros and these are the poles, but there should be one more this term S n here, because when s tends s will tends to 0 this term there this if there is s even single s, it will make this term to infinite and when this will be infinite this error will be tending to 0. So, therefore, n must be greater or equal to 1. If we want in system that against step input it should have zero steady state error. The G s should be in this form, and a would be at least 1 n at least n should be equal to 1. So, there should be at least o1 s.

So, this 1 by s is it means this is the integration; this is the Laplace transform of some integration. So, there should be one pure integrator in the circuit or in the forward loop. So, if there is one pure integrator here 1 by s, then this transfer function is one by s into this G s and so, the steady state error will cha will tend to 0. Now if n equal to 0 suppose here if n equal to 0 then we will get no any s term and so, this term G s will be here and so, we will get z 1 into z 2 into po zeros into poles and so, these terms will be 1 by 1 plus z 1, into z 2, into z 3 by p 1 into p 2 into p 3 and so, this is finite term and this is finite. So, 1 plus this finite is finite and this complete term is finite and so, we will get a steady state error as finite. Because we remember that we when against the step input we had two type of output 1 was this and 1 was that. So, this error e to infinite this was finite error and this was 0 steady state error.

So, we will get this output with finite error and when we have at least o1 s here we will get this one. Now come to the ramp input. So, this was step now if we have ramp input, suppose here we have ramp input.

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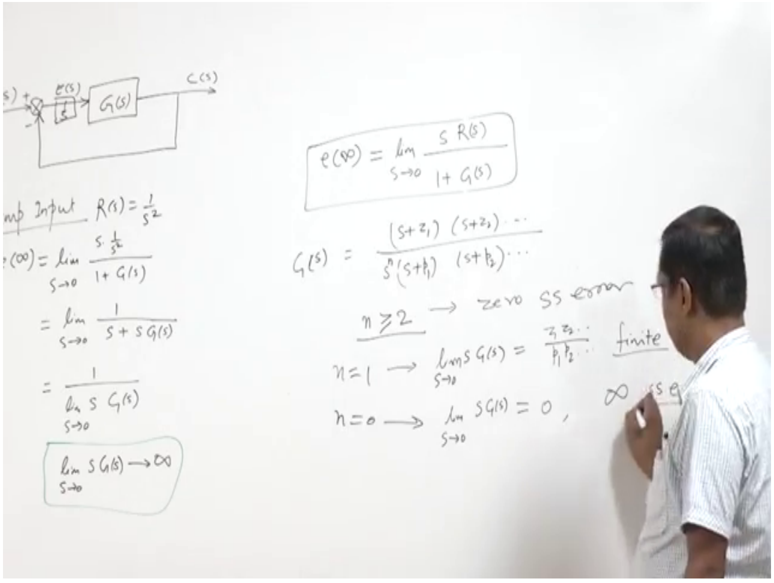

So, for this ramp input we have R s we know that R s equal to 1 by s square. So, we get e infinite equal to limit s tends to ze a z0, s into 1 by s square upon 1 plus G s. So, here 1 s will cancel out this square term. So, we will get limit s tends to 0 1 by s plus s G s and equal to 1 by s G s limit s stands to 0. Where this will be 0 and so, we will shift this limit to s e s. Now to have zero steady state error this limit s tends to 0 s G s tends to infinite and therefore,. So, it should be this and therefore, here the G s must be in the form s plus z 1, s plus z 2, s plus p 1, s plus p 2.

Now, here should be some power of s. So, s power n; now we know that n should be greater or equal to t2 here because if n equal to 1. So, there is only 1 s this s will be cancel out with this s and we will get a finite. So, if n equal to 1; so n greater than t2 we will get zero steady state error, if n equal to 1 we will get. So, n equal to 1 this s will cancel out with this s we will get here. So, p 1, p 2, p 3 upon z 1, z 2, z 3 this steady state error we will get. So, this limit s tends to 0 is s G s will be equal to z 1, z 2, z 3 upon p 1, p 2, p 3 and then when we put we will get a steady state error as p 1, p 2, p 3 upon z 1 z 2. So, this is a finite steady state error, and if n equal to 0. So, n equal to 0 means we will get this term s into G s. So, limit s tends to 0, s into G s and n equal to 0 means here is only this term and this term multiplied by s. So, this will be 0 and when will put this s 0 we will get infinite. So, here infinite ss error we will get.

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Thank you.