Automatic Control
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Lecture – 18
Routh-Hurwitz Criterion - Special Cases

So, welcome to the lecture on Stability and a Steady State Error. So, we will discuss in this lecture about special cases of Routh-Hurwitz Criterion. So, we were discussing the case like when there is 0 in the first column.

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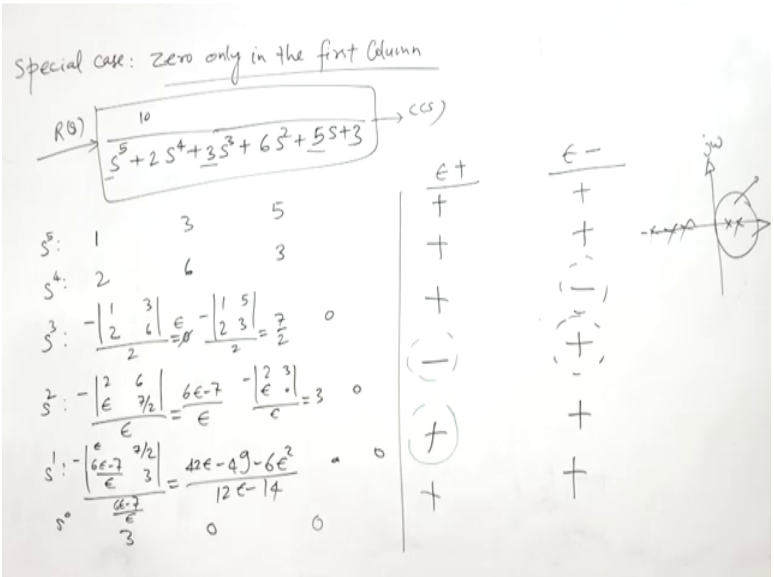

So, here 0 is only in the first column. The one method, we said that we can assign epsilon in place of 0 and then we can continue with this table, then we take whether epsilon is positive or negative; small quantity and we can see how the sign is changing. And we can get how many poles are to the right half plane and how many are to the left half plane. And we can decide about the stability. There is another method also and that method we can calculate, we can write this polynomial as the reciprocal polynomial.

So, in spite of writing s power 5, we can write as 1 by d power 5. So, we reciprocal we replace this with some reciprocal variable and we expect that this will make the polynomial, so that, the first column will not be 0. So, let us take the same example and we try to do here.

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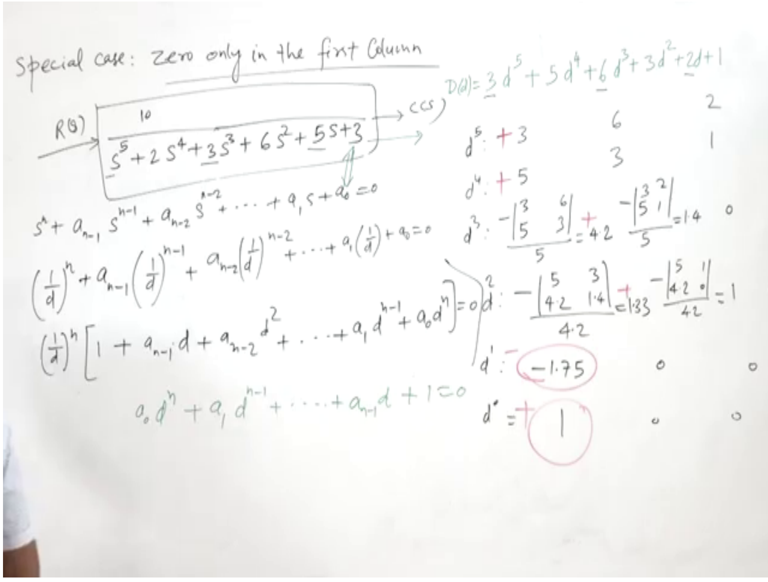

So, if we have this polynomial s power n plus a n minus 1 s n minus 1 plus a n minus 2 s n minus 2 plus plus a 1 s plus a 0 equal to 0. So, this is the original polynomial. Now we replace s with 1 by d. So, is replaced by 1 by d. So, 1 by d power n that is reciprocal of s plus a n minus 1, 1 by d n minus 1 plus a n minus 2 1 by d n minus 2 plus plus a 1 1 by d plus a 0 equal to 0. So, now, we can take 1 by d power n outside as common. So, this is 1 plus a n minus 1 and here it will be d into d plus a n minus 2 into d square plus plus a 1 into d power n minus 1 plus a 0 d power and that is equal to zero.

So, finally, we find this polynomial as, so, this polynomial is now a 0 d power n plus a 1 d power n minus 1 plus plus a n minus 1 d plus one equal to 0. Now, we make the Routh table, we create the Routh table for this polynomial, that is the in the variable is the reciprocal of the original variable s and we create the Routh table. So, now, we have this polynomial original polynomial. This is our original polynomial and so, we have to convert this into these polynomials.

So, we see that simply a how we can convert just by looking these. So, this constant term is getting the highest power of d. So, d power 5 into 3, because here this constant term a 0 is getting d power n. So, here this will be, we can write the polynomial is 3 times d power 5 plus the next term s power 1 is that is a 1 is getting d power n minus 1. So, a 1 is 5 and d power 4 plus this will get 6 into d power 3 plus 3 into d power 2 plus 2 into d plus 1.

This is the polynomial. So, this is a polynomial that is function of d. Now we have to create the root Routh table for this. So, let us generate the Routh table. So, we have d power 5. So, this, this and this, so, 3 6 and 2 then d power 4, 5, 3 and 1. Now we create the this term d cube. So, here minus 3 5, then 6 3 by 5 that is equal to minus 21 by 5 21 by 5 and that is 4.2. And then the next 1 is minus 3, 5, 2, 1 by 5 and that is 1.4 and this is 0, because that column is 0, then d square. So, we will calculate 5, 4.2, then 3 and 1.4 by 4.2 that is equal to 1.33 and this term is 5, 4.2 1 0 by 4.2 that is equal to 4 1 and this is 0, then d 1 equal to minus 1.75 0 and 0 then d 0 is equal to 1.

So, here we get that, this is positive, this is positive, this is positive, but here is also positive. Then there is a sign change and then there is another sign change, because this is negative and this is positive. So, there is one sign change here and the second sign change here. So, there are 2 2 poles on the right half plane of the system. So, 2 system poles are to the right side and rest, 3 are to the left side. And the same information we obtain with the previous method that is epsilon method.

So, therefore, these 2 methods we can use when one any one element in a row the first element is 0 in any col, in the first column there is 0 in the any row. So, now there is another special cases. So, another special case is that entire row is 0, because here we took only that the 0 is only in the first column.

Now, but if 0 is in all the column in a row, then how we should proceed. So, we take this second case.

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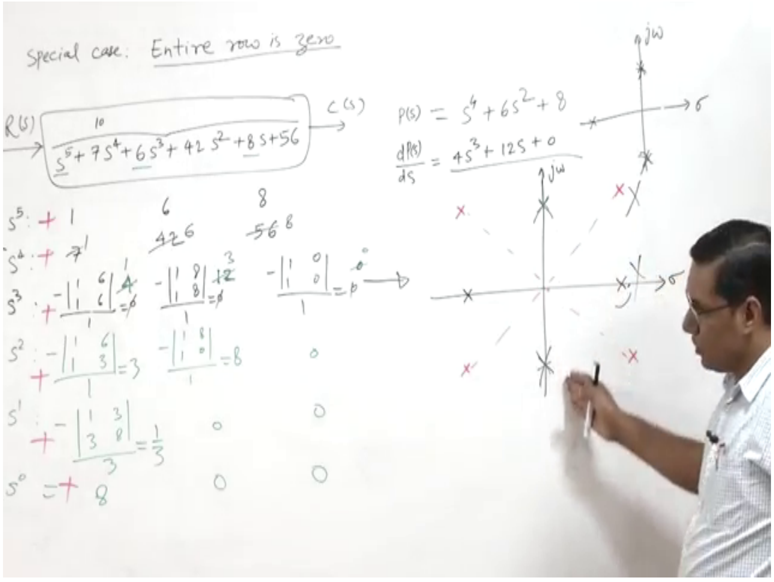

So, there is the second case when the entire row is 0. So, when entire row is 0, how we should create the Routh table? This we will discuss. So, let us take one example that is 10 by s power 5 plus 7 s power 4 plus 6 s cube plus 42 s square plus 8 s plus 56. So, this is the system the equivalent transfer function and we have to create the Routh table. So, let us make this s power 5. So, s power 5, we have. So, we have 1, then 6, then, 8 the next power 4, we have 7, then 42, then 56.

So, this we can take common 7 to 7 one, 6 8 we can write. Now, come to s cube. So, this is minus 1 1, then 6 6 this is 1 and so, 6 minus 6 that is 0, then here minus 1 1, then 8 8 by a 1 that is equal to 0, here minus 1 1, then 0 0 by 1 equal to 0. So, we see that this complete row a is 0. So, how to proceed in this case? So, when there is a rules complete row 0, it gives a signal that assign that there is an even polynomial present in the in this expression. So, we will write p s, we go one step back and we take here s power 4 plus 6 s square plus 8. So, there is this even polynomial present in here and so, we will do d p s by d s. So that, we will get 4 s cube plus 12 s plus zero.

So, now, we take the coefficients from here. So, s cube coefficient is 4. So, we write 4, here we have 12 and here is 0, so, here this 4, 12 and 0. So, we get from here. Now, we can chance we can take common 4. So, 1, 3 and here is 0. Now, we proceed for the next term s 2. So, here, s cube now s 2. So, minus 1 1 6 3 by 1, so, that is 3, then minus 1 1, then here 8 0, so, 0 minus 8, so, 8. And then this is 0, because the column is 0, then s 1.

So, s 1, we will get minus 1 3, 3 8 by 3. So, 8 minus 9, so, 1 by 3 we get here and then here this is 0. So, it will be 0 0 and s 0 that is we get 8 0 0.

So, when there is rows 0 as it was here, it tells that there is an even polynomial. So, this is even polynomial and the roots of the even polynomial are symmetrical about the origin. So, there could be symmetry about the origin several cases of symmetry. So, if this is sigma j omega axis. So, if we have our roots can be symmetrical, if they are on the real axis here. So, this is these 2 roots or symmetrical a about the origin. Also the roots can be symmetrical, when they are on the j omega axis. So, they are symmetrical about the origin, then another symmetry is possible when quadrantal roots. So, if the roots are like this, so, 4 roots. So, they are symmetrical about the origin.

So, here it is possible that the out of this, because when this even polynomial, we find the 4 roots will be even polynomial roots will be symmetrical about the origin. So, these roots can be either like this. So, 2 roots and 2 so, 2 pairs of roots like this. So, this is symmetrical, then another 2 pairs that is symmetrical about this origin or like this, the 4 roots are like this. So, they are also symmetrical. Now, we must know see that (Refer Time: 18:41) in these case, we had this plus sign, plus sign, here it is plus, here plus plus and plus. So, we see that one root is, so, we take we plot this here sigma j omega. So, one root is to the left side. Now, the other 4 roots are roots of the polynomial. So, these roots should be symmetrical about the origin.

Now, we see that there is no sign change from s 4 to s 0; means, these roots are cannot be to the right hand side right half plane. So, this symmetry is not possible, because in this case the roots come to the right half plane, these symmetry is not possible, because again the roots come to the right half plane. Then only this option is possible that all these 4 roots are on the j omega axis. So, one po one root is on the a left half plane and other 4 are on the j omega axis. So, the system here, we can see that is in this condition. So, if we do not have the ge even polynomial, we will not have the roots on the j omega axis. Now, we take one more example.

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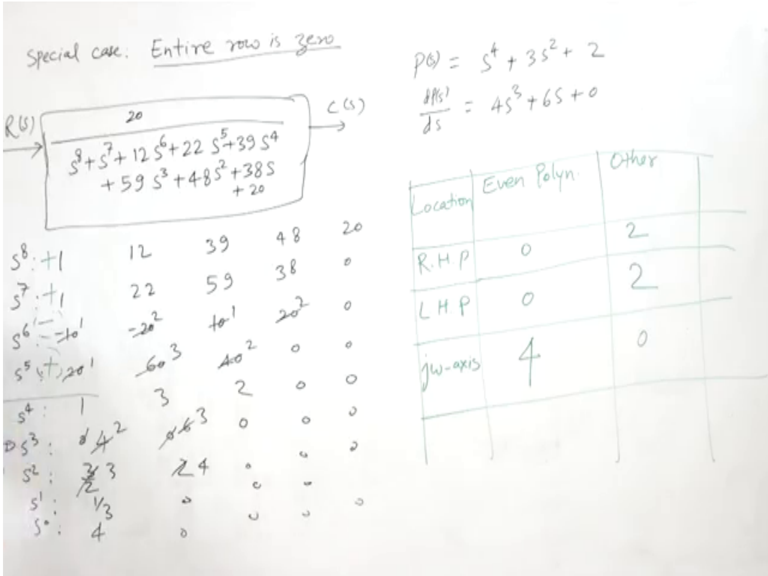

You so, this is the system, s 8 plus s 7 plus 12 s 6 plus 22 s 5 plus 39 s 4 plus 59 s cube plus 48 s square plus 38 s plus 20.

Now, we create the table. So, we have s 8, we have here 1, then 12, then 39, then 48 and then 20, then s power 7. So, we will have 1, 22, then 59, then 38, then 0. Now, we find s power 6. So, we will follow the usual method. So, we get minus 10 minus 20, 10, 20 and 0. So, we take 10 out. So, it is minus 1, minus 2, 1, 2 and this is0. So, s power 5 if we calculate. So, as power 5 we get 20, 60, 40, 0, 0. Can we take 20, so, 1, 3, 2, 0, 0 then s power 4, so, 1, 3, 2, 0, 0. Now s power 3, we will get zeros. So, all the this the s 3 this row is 0. So, we have even polynomial starting from s power 4. So, here is power 4 equal to 1. So, s sorry here, we will have the even polynomial p s equal to s power 4 plus 3 s square plus 2.

So, d p s by d s, that is equal to 4 s cube plus 6 s plus 0. So, here we can write 4 and here we can write 6 and 0. So, here we can again write 2 and 3, because we can take 2 outside, then s 2 we can write s 3 by 2, 4, 3 by 2 and 2 and 0, 0, 0. So, here we can write this 3 and 4, we take 1 by 2 outside, then s 1 is 1 by 3, 0, 0, 0, 0 and s 0 is 4, 0, 0, 0, 0. So, here we can now notice that here we are getting. So, from here onwards, this is even polynomial and here we see that this is plus, plus, here it is minus and then plus.

So, now if we make the roots, so, here location of the roots, this is even polynomial and other and here is. So, we see that location right half plane, left half plane and j omega axis and in the even. So, here we see that these 4 roots are from even polynomial, 4 roots are of even polynomial and 4 roots are other. So, we see that, out of these 4 there are 2 sign changes. So, 2 will be to the right half and 2 will be the left. So, in this other, 2 will be to the right half planer and 2 will be to the left half plane and even polynomial roots will be 0. So, no any even polynomial root to the (Refer Time: 27:16) right half plane or left half plane. And on the j omega axis raised 4, because here there is no sign change.

So, if there is no sign change, all the roots will be on the j omega axis. So, these 4 roots will be on the j omega axis and total here will be 8 roots 2 plus 2 plus 4. So, no sign change, no left half plane roots. So, no right half plane roots and so, all these 4 are on the j omega axis. So, this is how we understood the that, how to create the Routh table and how to know the location of the pole, where they are located in the right half plane or left half plane or j omega axis. And this will gives us the stability criteria.

If any pu single pole is to the right half plane, then the system is unstable and if the poles are to the left half plane, all the poles then the system is stable. When the poles are on the j omega axis, the system is marginally stable. So, here we took some examples that is from the reference book Norman S Nice control systems engineering and so, here I would like to stop and see you in the next lecture.

Thank you.