Automatic Control

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Lecture - 17
Routh-Hurwitz Criterion

So, welcome to the lecture on stability and a steady state error. So, in this lecture we will discuss about Routh Hurwitz criterion for stability. So, in the previous lecture we saw that ah; we can tell about the stability depending on the location of poles. So, we we need to solve the denominator of the closed loop transfer function to find the values of the poles. And then we can locate the poles on the s plane, to know whether these poles are to the left side or right side of the plane, and then we can tell about the stability. Only the information about the location of poles whether they are in the left side or right side is important to tell about the stability, not the exact values of the poles.

So, therefore, we know we do not need to solve the equations, characteristic equations for the poles, but only we can know that how many poles are to the left side and how many are to the right side how many are on the j omega axis. We can know about the stability. And we will use the method given by routh Hurwitz criterion for stability. So, this method yields stability information without the need to solve for the poles, as using this method we can tell how many closed loop system poles are in the left half plane, in the right half plane and on the j omega axis. So, to find this information we need to generate and interpret the routh table. So, we will develop the routh table and we will discuss with some examples how to find the stability in formation. So, let us take that we generate the Routh table.

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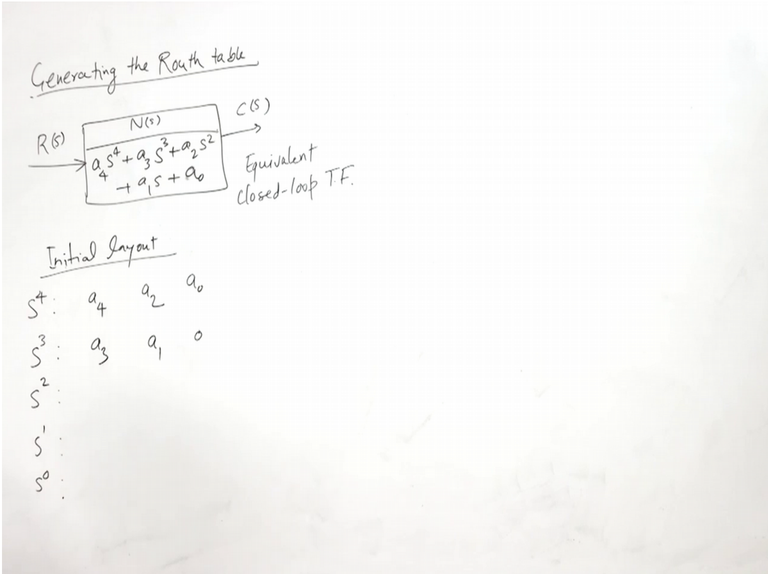

So, let us we have this system. So, this is Equivalent Closed-loop Transfer Function. This we have already defined and discussed in the previous lecture that how we can obtain the eq equivalent closed loop transfer function of a given system. So, this is our this denominator will tell about the information of the poles and their locations. So, we do not need to solve this equation a 4 S 4 plus a cube S cube plus a 2 S square plus a 1 s plus a 0 equal to 0. So, we do not need to solve to find the values of S, but we can create the routh table. So, initial layout. So, what we will do? So, to create the routh table we begin by labeling the rows with powers of S from the highest power of the denominator to S 0.

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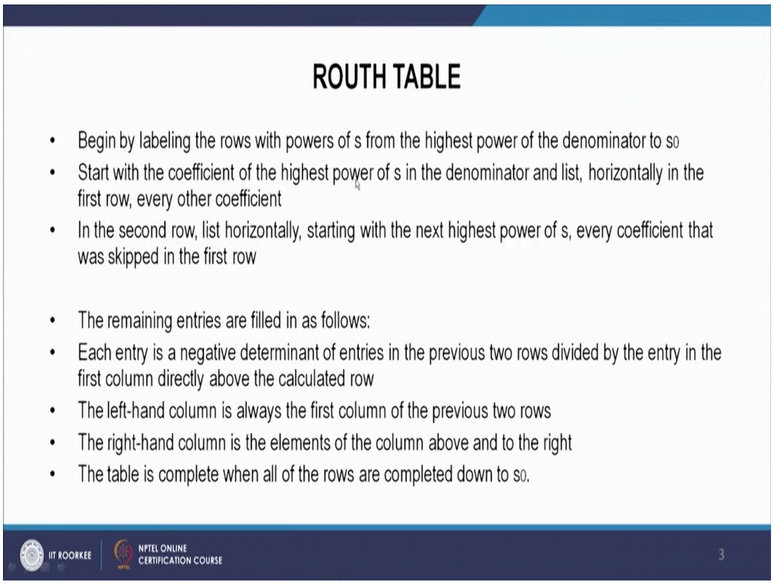

And start with the coefficient of the highest power of s in the denominator and list horizontally in the first row every other coefficient. Then in the second row we list horizontally starting with the next highest power of s, every coefficient that was skipped in the first row.

So, we begin by leveling the rows with powers of s from the highest power of the denominator to s 0. So, we have highest power here is s power 4 and the coefficient of s 4 is a 4. Then we skip one we go to a 2, then we skip we go to a 0. So, we label like this these coefficients. So, we start with the coefficient of the highest power of s in the denominator and least horizontally in the first row every other coefficient. So, here every other coefficient from a 4 then a 2 then a 0. Then we start with the second highest power that is s cube and s cube. So, coefficient a 3 and then every other coefficient so a 1. And then here is nothing so we put 0 here. So now, other term s 2 s 1 s 0, this we have to find this table these terms of this table. So, here we will we can see that to find h 2.

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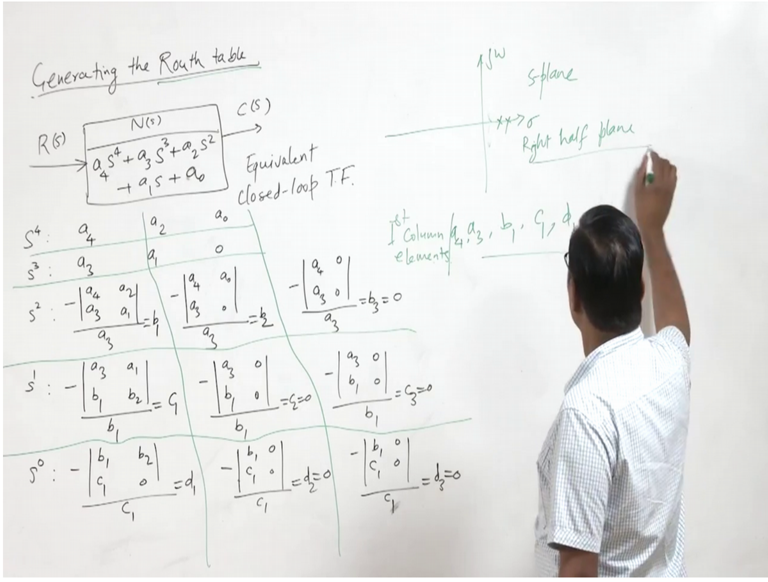

So, let us here s 4 a 4 a 2 a 0, then s cube a 3 a 1 and 0 now s 2 s square. So, these to get this term we will take minus and determinant. And the first column we will be the coming from the the top to first 2 rows the first column element that is a 4 a 3, and the second column will be the elements to the next column that is a to a 1 and this will be divided by this element here a 3.

So, each entry is a negative determinant of entries in the previous 2 rows, divided by the entry in the first column directly above the calculated row. So, here this column and the next this column by this element a 3. And let us say this is b 1. Now for this second element we will have minus this will be fixed a 4 a 3 this first column will be fixed in this row, and then this element will be the element to the next column of this column.

So, a 0 0 upon a 3. So, we see that this column is fixed and this first element will only come to the de will divide these elements of this row and let us say this is b 2. Then this third element minus a 4 a 3 then here the next one if nothing we assume 0 0 by a 3 and that is equal to b 3 and this is 0. So, we have got the next row of this table, now the s 1 now we will get again apply the same rule and we will get minus. Now the next that the previous 2 rows column is a 3 b 1 a 3 b 1 and the next one is a 1 b 2 divided by b 1 and let us say this is c 1 equal to c 1. Now this element is minus a 3 b 1 then next element

so, here we have 0 0 by b 1. And that is equal to c 2 and that is equal to 0. Then this 1 minus a 3 b 1 and then 0 0 by b 1 that is equal to c 3 equal to 0. Then come to s 0. So, minus here this column b 1 c 1 and then the next column b 2 and 0 by c 1 and let us say this is d 1 then minus b 1 c 1 and then the next column 0 0 by c 1 that is equal to d 2 equal to 0. And this also minus b 1 c 1 0 0 by c 2 c 1 equal to d 3 equal to 0. So, thus we have created this this is called Routh table. So, we have created Routh table for this closed loop transfer function. So, you can see here a 4 a 2 a 0 this second row, this is third row and here is the 4th row and this is the fifth row.

And similarly, here is this column first column then second column and third column. Now from this table how we will know that how many poles are to the left side or how many poles are to the right side. So, suppose this is our s plane. So, we have to watch on the first column so first column elements. So, a 3 b 1 c 1 and d 1 these are the first column element. So, we have to notice that whether these elements are changing their sign or not. If there is one sign change means one pole is to the right half plane, if there is 2 sign changes then 2 poles are to the rife right half plane. So, suppose here it so also a 4 here a 4, a 4 a 3 b 1 c 1 d 1 suppose here a 4 a 3 are plus positive, but b 1 is negative then one pole is to the and then again this is positive means 2 poles either this way or this it could be to the right half plane.

And so, even one sign change will tell that one pole is to the right half plane. And so, the system is unstable. So, the table is complete when all of the rows are completed down to s 0.

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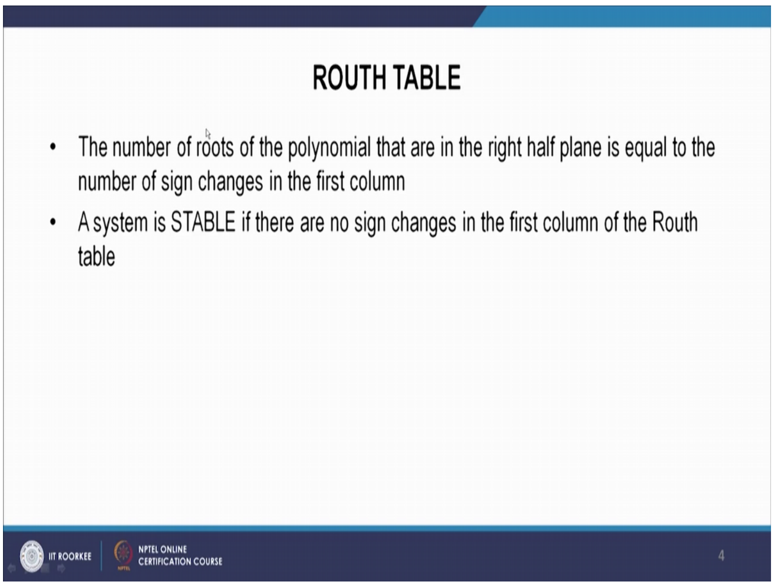

Now the number of roots of the polynomial that are in the right half plane is equal to the number of sign changes in the first column. So, number of roots or poles in the right plane is equal to number of sign changes in the first column and a system is STABLE if there are no sign changes in the first column of the routh table. So, if there is no sign changes in this first column then the system is stable.

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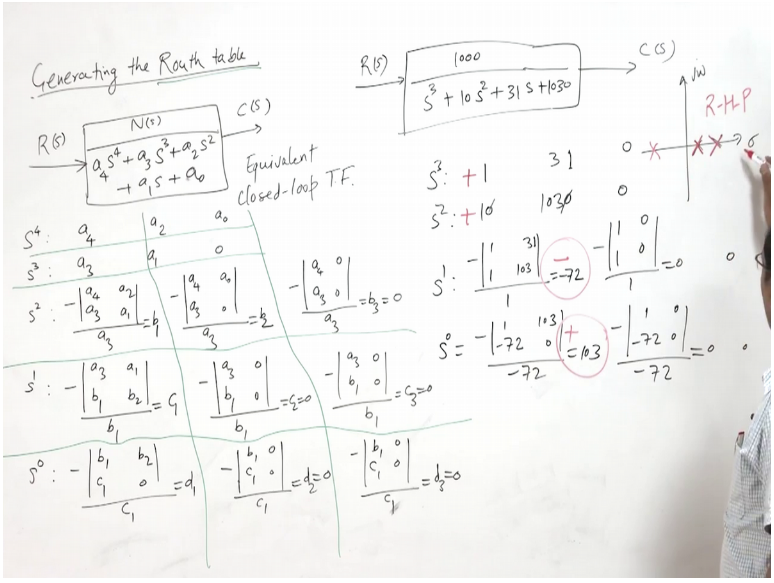

So, let us take one example. So, we have this equivalent closed loop transfer function, now we have to generate this routh table. So, highest power is s cube and the coefficient is 1 then we skip these 31 and then we have 0 then s square we have 10, then we have 1 0 3 0 and then 0 now we generate for s 1. Now in this 1 row we can cancel out this if there is some common element we can take it out. So, we can there is 0 so 1 and 1 0 3 we can write. Now we create this table so this element minus this column 1 1 and then the next 1 3 1 0 3 by 1. And this is equal to sorry this is 31 we have 31 these. And so, we will get minus 72 this element.

now the second element minus 1 1 and 0 0 by 1 that is equal to 0. And the third element will also we will get 0 because this column will be 0. Now come to a 0, so again minus here is one this is minus 72 and then the next one 1 0 3 and then 0 by minus 72 equal to 1 0 3. Now this 1 minus 1 minus 72 and 0 0 by minus 72 that is equal to 0 and this is also 0. So, we can notice here that we have to sign changes here it is plus plus, but here it is minus. So, there is one sign change here then again it is plus. So, second sign change here. So, there are 2 sign changes and 2 sign changes means we have 2 poles to the right half plane. So, when we plot this hole, we will find that this is sigma j omega. So, 2 poles are here to the either this way or this way whatever is applicable we do not know this information, but we know that the 2 poles are to the right half plane.

and because this is power 3 so one pole is to the left half plane. So, because 2 poles are to the right half plane the system is unstable. So, the we saw that how we can using the routh table we can generate routh table for a system, by finding the equivalent closed loop transfer function. Without calculating the without calculating the position of poles or without solving for poles we can tell about the stability or unstability of the system. So now, te we will considered some special cases. So, there could be special cases because here we are using the first column terms as the for division in the subsequent rows.

and suppose these terms are 0, a 3 a 0 or we get b 1 and b 1 is 0 then, how we will proceed? How we will be able to make the routh table? So, we there are some special cases. And so, if 0 is only in the first column. So, if we find 0 in the first co column somewhere it will be difficult to compute further next next rows. So, how should we deal with this kind of cases. So, we will discuss some special cases.

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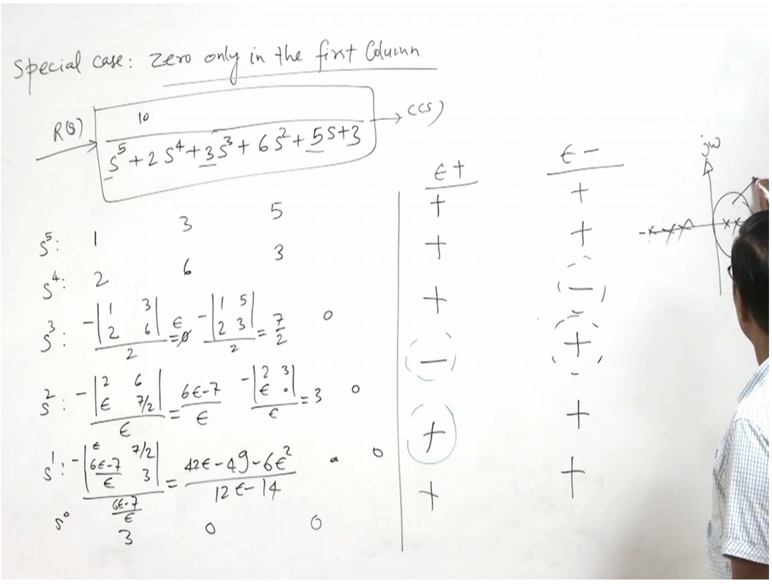

when 0 only in the first column. So, here we assume that 0 is only in the first column, and not all complete row is 0 only a at least 1 element is non0 in the row, and first column element is 0. So, when this happens so, let us take this example like t is equal to

10 upon s power 5 plus 2 s 4 plus 3 s cube plus 6 s square 5 is plus 3. So, we have this is our transfer function equivalent transfer function. Now let us try make the routh table. So, s power 5 we have one then next one is 3. So, one then 3 then 5 one 3 5, then s power 4 2 6 and 3 then s power 3. So, we will take minus this column 1 2 then the next column 3 6 by 2. So, when we compute this we will get 6 minus 6 equal to 0. And this 1 minus 1 2 5 3 by 2 equal to 3 minus 10 minus 7 by 2 and this one will be 0 because this column is 0 so this will be 0..

So, here we got 0 now if we want to compute the this term s 2. So, this 0 will come to the in the denominator and we will not be able to make this row because this that is undefined. So, there is an one method that we replace 0 with epsilon, where epsilon is some very small quantity, we assign here in this place of 0 and we carry on the making this row making this table and then in the last we approach this epsilon as plus 0 r minus 0. And we will see whether what is the sign change. So, let us give this as epsilon and now we calculate this table. So, this is minus we have 2 and epsilon and then these 6 and minus 7 by 2 by epsilon.

So, we will get. So, minus 14 by 2 minus 6 epsilon. sorry minus 7 minus 6 epsilon. So, this is 3 minus tens minus 7. So, this is plus. So, here is plus 2 7 minus 6 epsilons. So, this is minus so 6 epsilon minus 7 by epsilon and this one is minus 2 epsilon and then 3 0 by epsilon that is equal to 0 minus 3 epsilon by epsilon. So, mi this is 3 and this is 0 then we compute s 1. So, when we compute s 1 that is equal to minus epsilon and 6 s epsilon minus 7 by epsilon. And then that is 7 by 2 and 3 by 6 epsilon minus 7 by epsilon. And so, this we solve this we will get 42 epsilon minus 49 minus 6 epsilon square by 12 epsilon minus 14. And the other terms will be 0 when we compute 0 then the s 0 will be here 3 0 and 0.

So now we let us take the sign change. So, here epsilon is let us say that it is apprating approaching to 0 from positive. So, this will be this is plus this one is plus 2 is plus here if epsilon is positive it is plus of course, small quantity and this is positive. And so, this is minus 7 so this is minus negative and here this is positive this is they will be positive. So, here minus and here is minus. So, it is positive so, minus 49 by minus 14 that is positive and this is positive.

So, we see that here there is one sign change here and then other signs in 2 sign changes. Now let us consider epsilon as negative. So, this is positive positive then here it is negative then here negative minus negative by negative so it is positive. And again, the negative, so it is negative and this is negative. So, this is positive and this is positive. So, again here we have 2 sign changes one here other is here. So, here out of 5 poles we have 2 poles to the right half plane and rest 3 poles are here in the left half plane. So, because 2 poles are in the right half plane the system is unstable. So, I would like to stop here and we continue in the next lecture.

Thank you.