Automatic Control
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Lecture - 16
Definition of Stability

So, welcome to lecture on stability and steady state error. So, in this lecture, we will discuss about stability. So, we know that there are 3 requirements for design of our control system that is, transient response stability and steady state errors. So, these are the design objectives when we must satisfy these 3 according to our requirement, when we are going to design a control system or a change a control system, existing control system.

So, we discussed transient response already in the previous weeks lecture, and now we will discuss the stability. So, first we define what is stability? So, the response of a system depends on the transfer function of the system and input of the system. So, we discussed that, the input of the system input that applies on the system governs the form of the forced response, and the second part that is the natural response is governed by the transfer function, or by the system itself.

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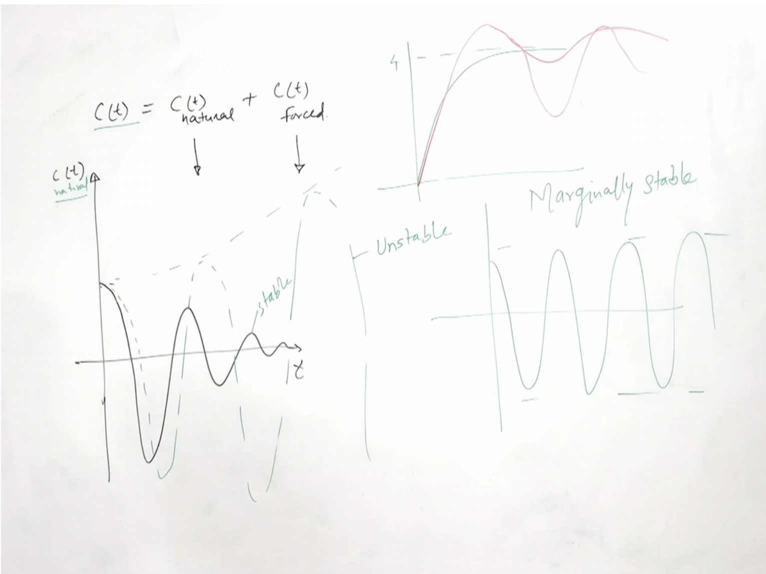

So, a the total response of a system is equal to, C t natural plus C t forced. So, here this natural response that is due to the, systems internal characteristic and this is due to the input condition. So, when the stability can be defined based on the natural response, when the natural responses of the system approach approaches 0, when the time approaches infinity. So, this C natural that should approach to 0 with the time, then we can say that the system is stable. So, for example, if we take one example here this is time, this is C t let us say, there is this natural response of a second order system, this is the natural response of the second order system and this response would approach to 0 with the time, for this undamped second order system, it is exponentially decaying amplitude.

So, it should approach to 0 with time, then it is called stable system and if this grows. So, if this response grows. So, we have that is we start from here, and it is growing with time. So, it is growing.

So, this is called unstable system and this is stable. So, this is and there is one more term that is called marginally stable. So, marginally stable system when the response neither grows nor decays, but it oscillates. So, if this is oscillating like this with constant amplitude. So, this is called marginally stable. So, based on the natural response, we can define the stability and unstability of a linear time invariant system, now based on the total response. So, total response is this is natural response, this is total response. So, when we give some input to the system some bounded input. So, if we gi give some bounded input, it should give some bounded output, then the system is stable.

When we are giving some bounded input and it is giving some unbounded output, then the system is unstable. So, when we see the total response of a system. So, when the transient part of the response grows without bound and therefore, it does not approach to a steady state value. So, because we know that we saw the elevator and we saw that the elevator response is approaching.

So, to the input value a steady state value, but suppose it is not approaching to the steady state value, but it is something some some other value, something some abs absurd value that is, not approaching to some steady state value, unbound value. So, this transient part is growing without bound, then we call it as unstable system. Now, these are from the responses, we have discussed these definitions based on the responses of the system, one from with the natural response other with the total response

Now, we know that we have already linked the response with the, poles and 0s of the system. So, transient response we have already linked to the poles, with the post location of the poles. So, we can also define the stability, with the location of the poles of the system. So, here we can see here that, stable systems have closed loop transfer functions with poles, only in the left half plane. So, here ah. Now, we discuss based on the poles. So, we know that if we have some transfer function.

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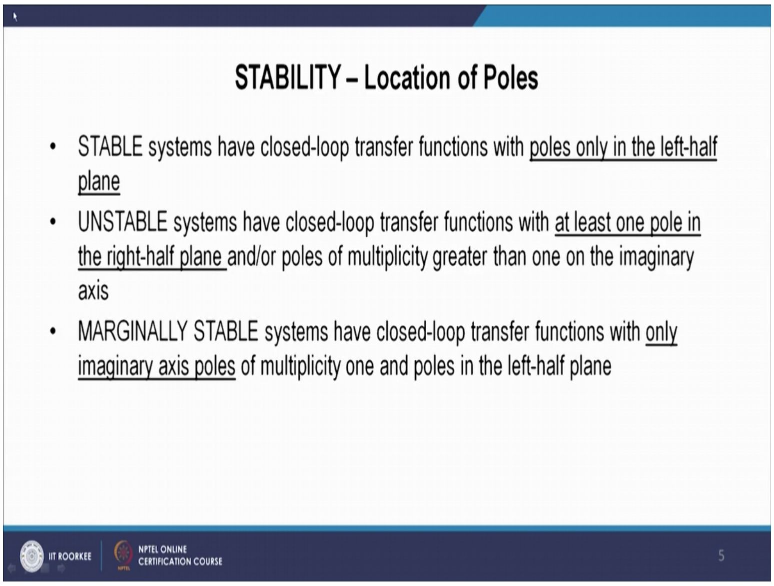

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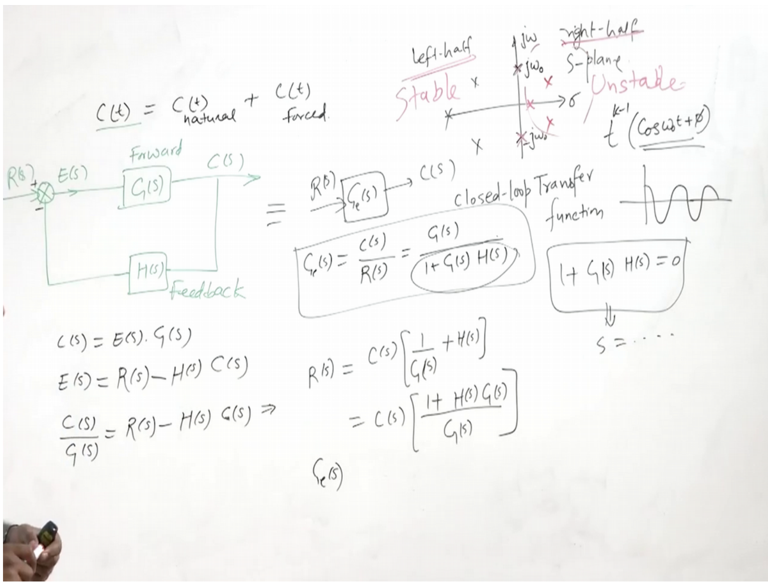

So, this is a transfer function and we can find the closed loop transfer function, because this is the forward loop forward transfer function, this is feedback, we have to find the equivalent transfer function of this system, and this we can find we can write C s equal to E s into G s, C s equal to E s into G s, and E s equal to R s. So, this is plus this is minus R s minus H s into C s.

So, we can replace this E s with C s by G s. So, here we can write C s by G s equal to R s minus H s C s. So, we want to find C s by R s. So, we want of equivalent transfer function G es, where there is input R s and output C s. So, here we can write. So, R s equal to C s, 1 by G s plus H s this is equal to C s 1 plus H s G s by G s. And so, R s by C s equal to G e s. So, here G es. So, here G es equal to C s by R s. So, output transfer function by input. So, C s by R s is equal to G s upon 1 plus G s H s.

So, this is called the closed loop transfer function or equivalent transfer function of this system. So, this is an equivalent closed loop transfer function. And so, when we base the definition of stability with the location of poles, we should consider for this system we should consider the poles obtained from G es because. So, when we say 1 plus G s H s equal to 0, and the poles that we a get from this. So, s. So, those poles must be in the left half plane of the complex plane. So, here if we have this plane s plane, that is sigma j omega that is s plane. So, these poles must be left half. So, this is left half plane and this is right half.

So, all the poles must be in the left half side then, only and know any pole to the right half side then only the system is stable and if any single pole is in the right half plane then, the system is unstable; however, what happens when the pole is here, on the j omega axis or imaginary axis. So, if we have n number of po poles, because these poles will be in pair. So, if we have this is omega 0 j omega 0, this is j omega 0 with minus sign and we have, on the imaginary axis these poles. So, the response will be tk minus 1 cos omega 0 t minus 5.

So, if k equal to 1 this is 1, because k equal to 0 this is t power 0 that is one. So, we will have a harmonic response something like this; however, if we have more polls, on this j omega axis with multiplicity more than 1. So, the pair of poles if there is one pair of fole, then this is an oscillatory motion or oscillatory response, but if we have k greater than 1 more poles pair on the same axis, then we will have k equal to 2 then, this will be t times cos omega 0 t and.

So, this response when t will increase this will increase, and again the system will be unstable in this case therefore, not more than, if there is one pole here one pole pair then the system is marginally stable; however, if there are more poles, with multiplicity more than 1 greater than 1 then this system will be unstable here. So, we can see that, we can define this stability in 3 3 with 3 parameters one is the natural response, second is the total response and third is the location of poles. So, with the natural response, if the natural response grows with time then the system is, unstable. If it decays to 0 with time, then the system is stable.

If it oscillates with constant amplitude then, the system is marginally stable. With total response the transients must lead to the steady state value, tangent must decay with time and lead to the steady state value then only the system is stable, with the location of poles the all the poles must be a right-hand sorry left hand let left half plane then only the system is stable. If there is any pole to the le left half right half pla plane, then the system is unstable.

So, any pole here or here this left hal right half plane, the system is stable. So, here it is stable and here it is unstable and one pair of pole at the imaginary axis, that is marginally stable, but if the multiplicity is more than 1 mo more poles pair on this, then the system is unstable. So, we saw that closed loop transfer function from the closed loop transfer function, we must calculate the poles and not with the forward loop transfer function.

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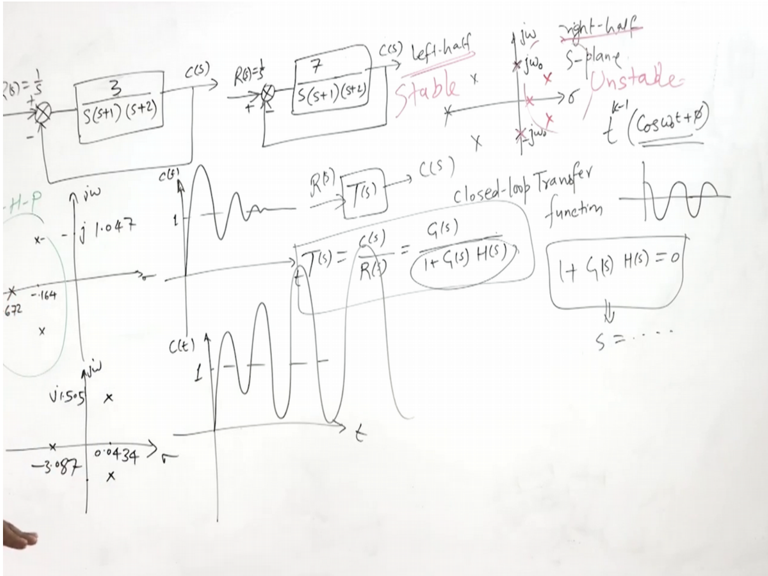

So, let us take one example, if we have this system. So, this is R s equal to 1 by s and this system is 3 by s s plus 1 and s plus 2 and this is C s. So, this system is we can find the closed loop transfer function, of this system let us call it T s. So, closed loop transfer function of the system and we then we can find the poles of the system. So, when we find the poles we will see that on this axis, the poles lie to the left half plane and this is minus 2.672 and this is 0.164 and this is j 0.047 and this is minus j 1.047, and we can see the response of this system with time, for the step input C t, let us say this is one the step input and the response will go like this.

So, we can see that here the poles or to the left half plane, all these because there will be 3 poles. So, these poles are to the left half plane and the response is, the total response is going to the steady state value that is 1 and system is stable

Now, let us take another system, we have this system. So, here we change this system and let us we have here 7 put 7 here. So, let us make another system here. So, we have 7 by s s plus 1 s plus 2 and we are interested in this, R s equal to 1 by s plus minus and this is C s. So, for this system if we calculate the poles, T s here we should see that H s is 1, because here is no H s means this is unit unitry feedback system. So, H s is 1. So, this we can calculate G s G s by 1 plus G s. And so, here sigma and j omega, if we plot the poles here, we have 2 poles complex poles here 0.0434 with plus side the right side and one pole is here minus 3.087 and this is j 1.0 1.505 1.505 and this is minus 1.505.

So, we can see that these complex poles there are 2 poles to the right-hand side, as a ri or right half plane. So, therefore, we would expect that the system should be unstable. So, this is time this is C t and this is 1 and we will see that the response will grow like this. So, the response is growing. And so, so we can see from the response, when the transients parts is growing and not approaching to the steady state value, all the poles are to the right half plane, the system is unstable. Now, if we have we take one more example, suppose we have another system, So, s s plus 4 s plus 6 s plus 8 s plus 10.

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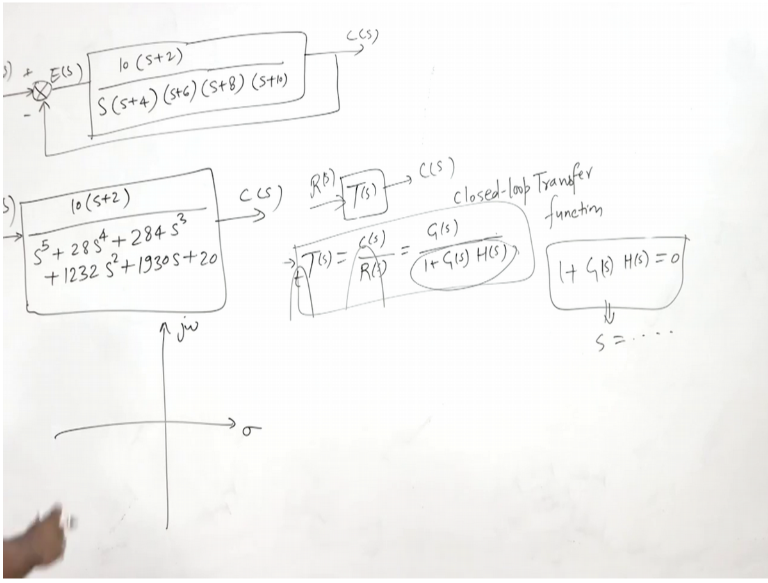

So, this is R s here is E s this is G s and H s is 1, if we compare to this. So, we have to first convert if we want to check whether the system is stable or not, we should calculate the poles. So, we should convert this system into an equivalent system, and we can call that closed loop transfer function T s. So, this should be a system like this here is C s R s and when we calculate this, we will find 10 s plus 2 upon s power 5 plus 28 s power 4 plus 284 s cube plus 1232 s square plus 1930 s plus 20.

So, this is t s. So, you we can see that, to find we have to calculate the s and then we we have to see whether it is located on the sigma j omega axis, that is s plane we have to locate it and then, we can tell whether the system is stable or not. So, we have to need to solve this polynomial ok, this denominator expression, we have to solve and find the values of the poles , one thing is clear that, we to to get the information of the stability, we do not need the exact position of the poles, or exact value of the location of the poles, but we only need to know that, whether the poles are to the left half plane or in the right half plane or on the j omega axis.

So, only this information is sufficient, to know the stability of the system and only this information is not is sufficient for the design of the system, once we are goi doing the analysis, then we may need to know the more details, but in the design stage we only need whether the system that I am designing is a stable or not, and for this I only need how many poles are to the right side? Or how many poles are to the left side? Or how many poles are on the j omega axis?

Therefore, we can avoid to calculate the values of the poles, but there are some methods that we can use to find that, how many poles are to the left side or right side or on the imaginary axis? And with that information, we can tell about the system size stability and that we will we college route Harvey's criterion for stability and that we will discuss in the next lecture. So, therefore, I stop here and let us see you in the next lecture.