Automatic Control
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Lecture – 13
Second Order System

So, welcome to the lecture, on transient response analysis. In this lecture, we will discuss about the second-order system. So, in previous lectures, we discussed about first order system, we know that the highest power of a transfer function will give us idea about the order of the system. So, if we say second order system means the highest power of S h 2.

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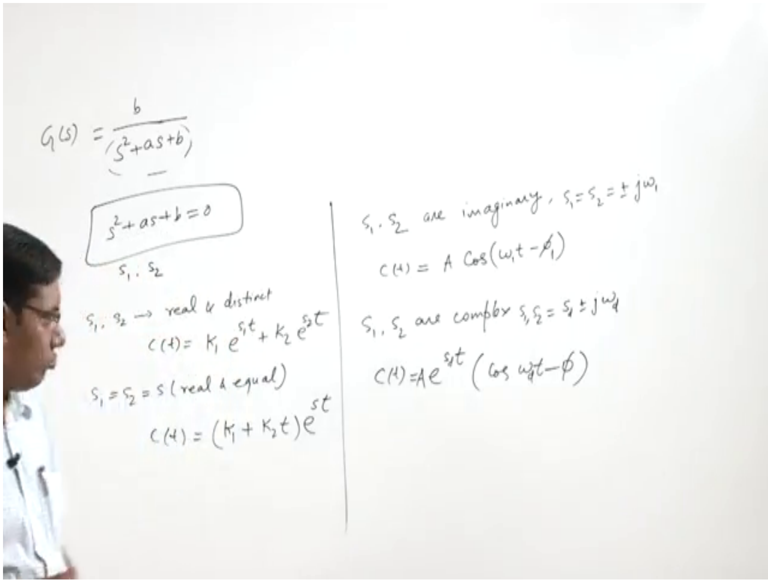

So, system such as; so, if the transfer function is like this b upon S square plus S plus b; so, this type of system can be second order system.

Now, the poles of this system, if we want to find the poles of this system, we should have S square plus a S plus b equal to 0. So, we will get two poles, let us say S 1 S 2. Now, the response of the system will depend on the values of these poles and nature of these poles. So, if these two poles, they are S 1 S 2, they are real and distinct. So, we will have response ct equal to k 1 e power s 1 t plus k 2 e power s 2 t, if they are real and equal.

So, S 1 equal to S 2 equal to S. So, they are real and equal. So, we will have ct equal to k 1 plus k 2 t e power s t or s 2 t. Similarly, if S 1 S 2, they are imaginary. So, for example, S 1 equal to S 2 equal to plus minus j omega 1. So, in this case, we will have a response c t equal to a cos omega 1 t minus phi 1. So, it is a harmonic response and when they are complex. So, S 1 S 2 or complex poles.

So, something S d plus minus j omega d. So, they are complex. So, we will have or we can also write it. This has a phase in terms of phase. So, minus 5. So, these are the four cases, because these roots of this equation can be real and distinct. So, we will get the solution, if the roots are equal and real. So, we will get this solution and if they are imaginary, we will get this harmonic response and if they are complex, we will get this type of response.

Now, we take this case. So, when the roots are imaginary.

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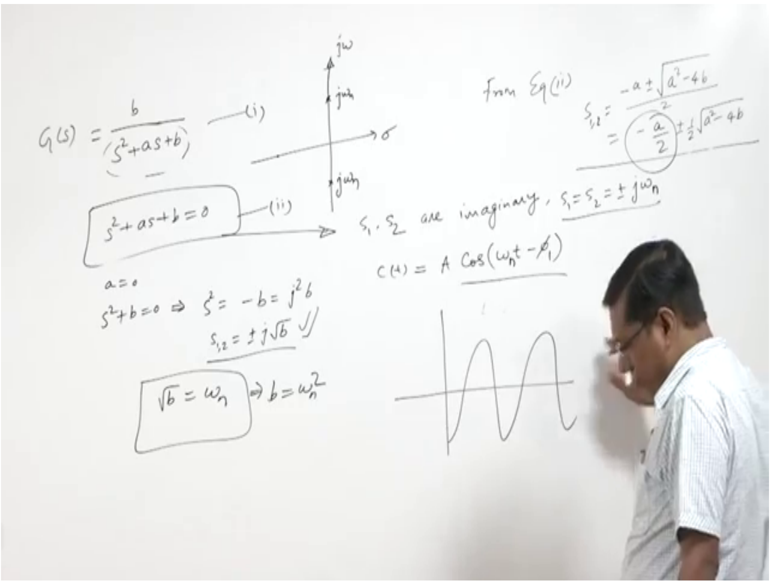

So, if we plot these roots on the S plane. So, we will have here let us say this is omega n. So, here we have j omega n and here it is j omega n. So, these are the two poles of the system and we are getting that the response is harmonic and this is called the omega n is called the natural frequency of the system. So, the natural frequency is defined as when the real part is 0 there is only imaginary part and. So, the poles lie on the imaginary axis, both the poles lie on the imaginary axis.

So, this is called the natural frequency and so, here this natural frequency. We can have when a equal to 0. We will have S square plus b equal to 0. This means S square equal to minus p equal to j square b. So, S 1 2 equal to plus minus j root b. So, we see that here root b equal to omega n if we link this equation 2. So, we get that, if we have this kind of transfer function, the constant term will give you the natural frequency of the system, when the poles will be on the post will be on the imaginary axis and the system will have oscillations with natural frequency omega n or we can say b equal to omega n square.

From equation 2 we can have S equal to minus b plus minus under root b square minus 4 into a 1 into b by 2. So, we get minus a by 2 plus minus 1 by 2, a square minus 4 b. So, we saw that here to get only imaginary part, we put a equal to 0 this part equal to 0. And so, we got plus minus b j into b j into root b, these poles and we defined natural frequency. Now, we define one more term that is, damping ratio. So, we know that the this in the pole, the real part, if the pole is on this, if it is here or in the real part we saw the effect is the exponential decaying. So, due to this part, real part there will be the effect of damping and so, the damping ratio is defined.

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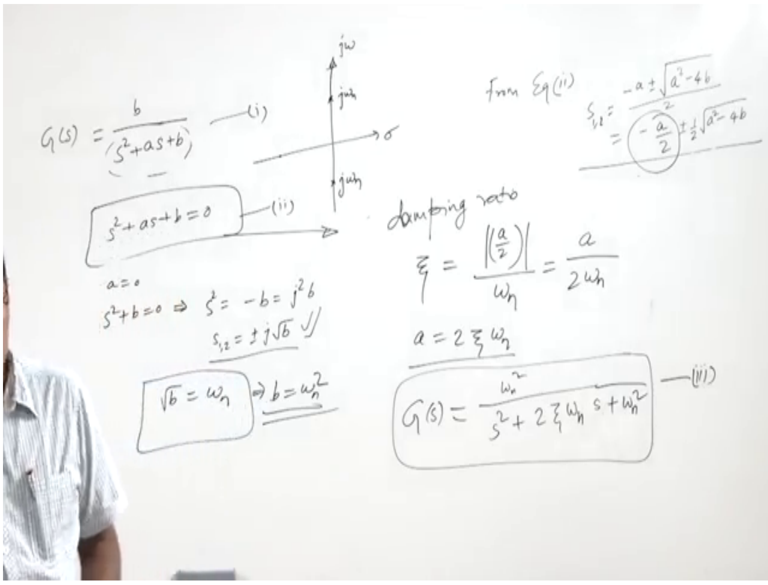

Damping ratio is defined that is d zeta damping ratio. So, damping ratio is, is defined as this real part upon the frequency. So, this real part upon the natural frequency is defined the damping ratio, and we get a equal to 2 zeta omega n. Now, we can express a and b in terms of damping ratio natural frequency in equation 1 and, so, we can convert this equation the transfer function G S equal to. So, here be equal to omega n square. So, we put omega n square upon S square plus a equal to 2 zeta omega n into S plus omega n square.

Let us say equation number 3. So, this is another representation of the second order differential equation, when, where we have changed that parameters a and b in terms of damping ratio and natural frequency and this representation is more important, because based on the damping ratio, we can divide the system. The second order system into four different cases that we discussed, based on the roots. So, based on this we can, the response of the system will also change. So, we have two form.

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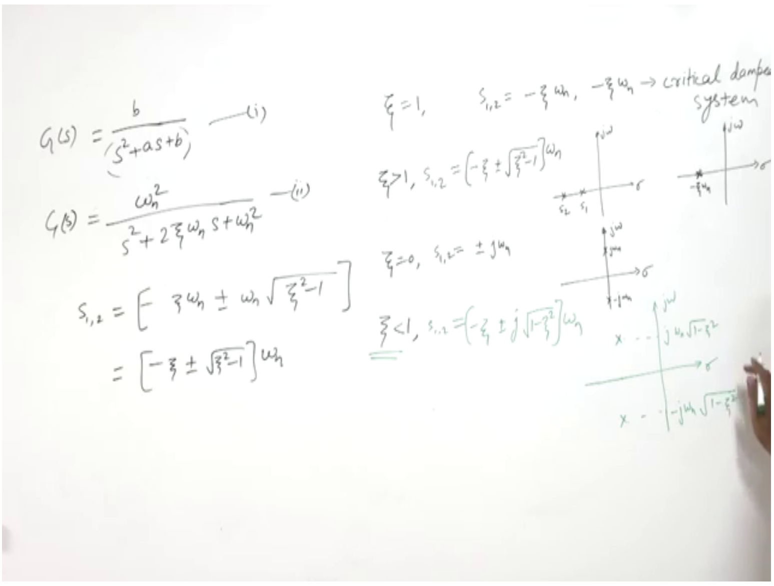

The second form G S equal to omega n square S square plus 2 zeta omega nS plus omega n square. This is equation number, let us say 2. So, here S 1 2 we can write as minus 2 zeta omega n plus minus omega n root zeta square minus 1. So, we can write is. So, these are the two roots for this equation number 2. This transfer function and now, we can see this case inside. So, if zeta equal to 1 will have S 1 2 equal to. So, if zeta equal to 1, this is 0. So, we will have minus zeta omega n minus zeta omega n and minus zeta omega n. So, the roots are equal and real. So, this is, this system is called critical damped system and when we want to present the roots of this system we have these roots or real axis and the same here.

So, minus zeta omega n, when we have zeta it is greater than 1, we have 2 roots S 1 2 equal to minus zeta plus minus root zeta square minus 1 omega n. So, here the roots are 1 with plus another with minus. So, the roots are real and distinct, because zeta is greater than 1. So, this under root is, part is greater than 1 and greater than 0. So, this is real and distinct. So, if we want to present this, we can present something like here. So, first root S 1 S 2, we can represent. Now, third case is when zeta is equal to 0. So, when there is no damping. So, in this case we will have S 1 2 equal to plus minus j omega n. So, this is a case of imaginary roots. So, here j omega n and here minus j omega n .

So, this is undamped system no damping and this defines natural frequency of the system. Now, if zeta is less than 1. So, if zeta is less than 1, we will have here, S 1 2 is equal to minus zeta plus minus. So, here zeta is less than 1. So, to make this quant, under root quantity positive. Let us, we put 1 minus zeta square and we take here j under root 1 minus zeta square. So, j is under root minus 1 and here omega n. So, we see that these roots when zeta is less than 1, these roots are complex pair and we can represent these roots here, sigma j omega and the constant term is minus zeta omega n and we represent this here on the S plane.

So, this is minus zeta omega n and this is plus j omega n root 1 minus zeta square and this is minus j omega n root 1 minus zeta square. So, we have represented the four conditions of damping gives the four different conditions of the roots and we know that these four conditions of roots, where the roots are real distinct equal imaginary, or complex we saw that the response of the second order system varies and we can see here.

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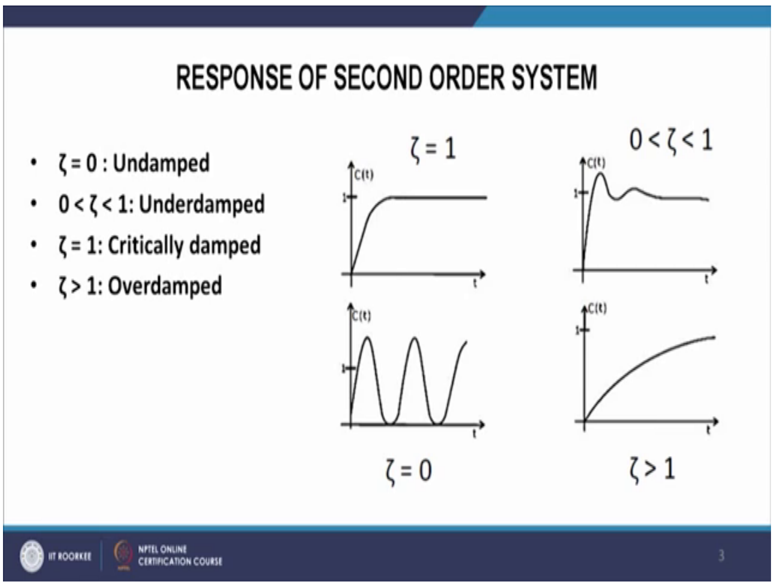

The response of these systems. So, here is zeta equal to 1. We can see critical damped system. So, zeta equal to 1, the first one we have the equal roots and real equal and real roots. So, here the response of the system we can see, this is for the critical damped system. Now, for zeta greater than 1. So, zeta greater than 1, here we can see zeta greater than 1 that is. So, this system zeta greater than 1. We called over damped system. So, this zeta greater than 1 is over damped system and we can see the response of an over damped system. Now, zeta equal to 0, here is the zeta equal to 0. So, this is a response against a step input. So, we are, our step input is ah, we have unity, a step input and we are looking the response of the system against this input. So, here we have undamped system, we can see there are oscillations for under undamped system, because there is no damping, the amplitude will remain the same and it will not change with the time.

So, this is undamped system and the frequency of this undamped system is called the natural frequency and that we can see, how the natural frequency is represented on the S plane, the complex plane. Now, we come to the under damped system, where the damping is less than 1. So, 0 is less than damping ratio, is less than 1 for under damped system. Here, we can see the values overshoot here and then they come down and after several oscillations, it reaches to the final value and we saw that when the brutes or complex, we had two parts; one was exponential part, and the second was the cosine or harmonic part and therefore, here in the under damped system, we have one oscillations and the same time the amplitude is going to decay with the time. So, now let us have one example. So, if we have GS equal to 36 upon S square plus 4.2 S plus 36.

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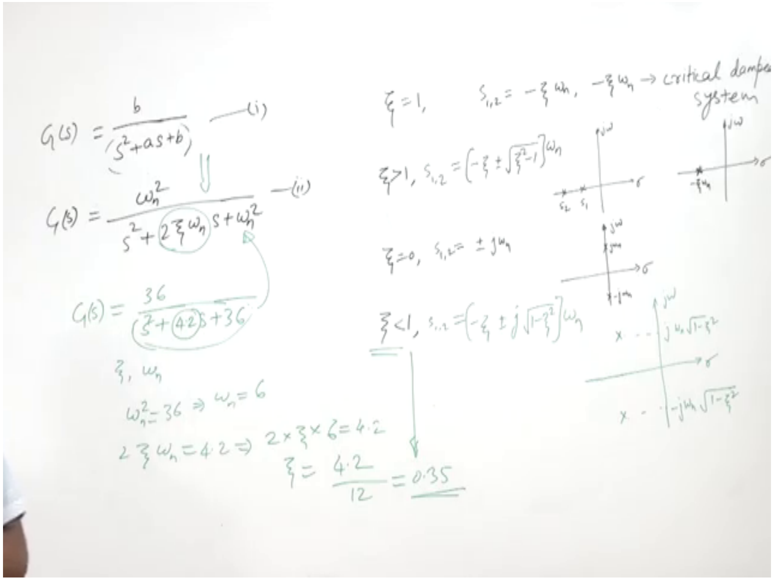

We want to know the damping, and natural frequency of this system. So, we know that we can represent, this is our system, we have to compare this part with this part and we will find the damping and natural frequency. So, here we see that here, 36 equal to omega n square. So, omega n square equal to 36. So, this implies that omega n is 6. Now, this 4.2 is equal to 2 zeta omega n, because this is the coefficient of S and here is the coefficient of S is 4.2. So, 2 zeta omega n equal to 4.2.

So, here we write 2 into zeta, into omega n, we have already got 6 equal to 4.2. So, zeta we can get 4.2 upon 12 and. So, we can get 0.3 pi over 5. So, we can see that damping is 0.35 and therefore; it is zeta less than 1. So, this is an under damped system and the natural frequency of the system is 6, omega n is 6. Now, let us take some more examples.

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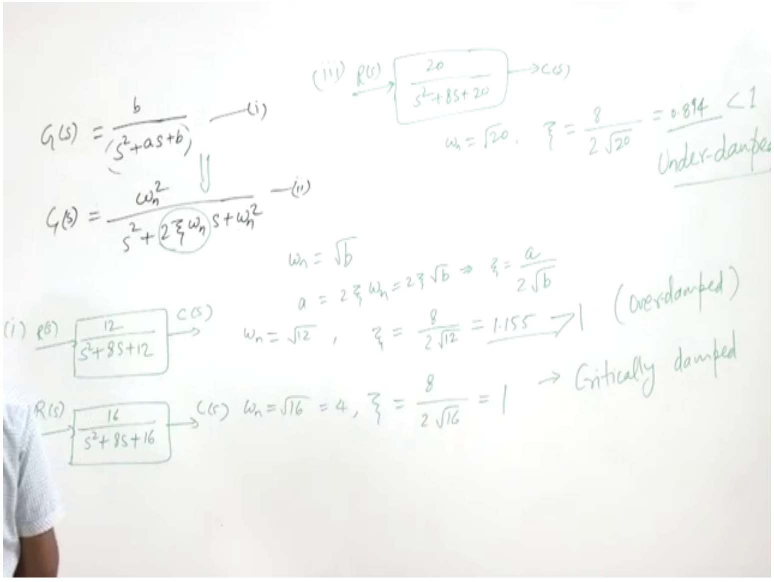

So, we have these three systems ; one is 12 by S square plus 8 S plus 12, 16 by S square plus 8 S plus 16, and 20 by S square plus 8 S plus 20.

Now, we have to find the damping and natural frequency of this system. So, here, either we compare with this or we can compare also with this, and we can relate these parameters a and b with the omega n and zeta, because we know that omega n equal to root b, because we defined omega n equal to root b and we defined a equal to 2 zeta root b 2 root b here, a upon 2 root b. So, here in case 1, we have omega n equal to root b. So, root 12 and we can find damping equal to a by root 2, root b a is 8 upon 2 root b.

So, b is 12 and this we can get damping 1.15 for the second case also we can get omega n equal to root 16 and damping equal to 8 by 2 root 16 equal to 1 and for the third case; we will have omega n equal to root 20 and damping equal to 8 by 2 root 20, that is 0.894. So, we see that here damping it greater than 1. So, this is over damped system here, this is damping equal to .

So, this is critically damped system and here damping is less than 1. So, this is under damped system. So, in this lecture we saw. So, these examples were taken from the book of Norman S Nise, control system engineering. So, we saw in this, how we can define a second order system and the second order systems are expressed more in terms of natural frequency and damping ratio and how can we find these parameters from the transfer function of a system and we can locate the poles, different poles of this system and we can characterize the response of the system ah, for the second order. So, here ah, I stop and let us continue this second order system in the next lecture.

Thank you.