Automatic Control
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Lecture – 12
First Order System

So, welcome to the lecture on transient response analysis. In this lecture we will discuss about first order system. So, we studying the transfer function and we discussed earlier that the highest power of S in the denominator of the transfer function decides the type of the system or order of the system. So, for example, if my system is if the transfer function is.

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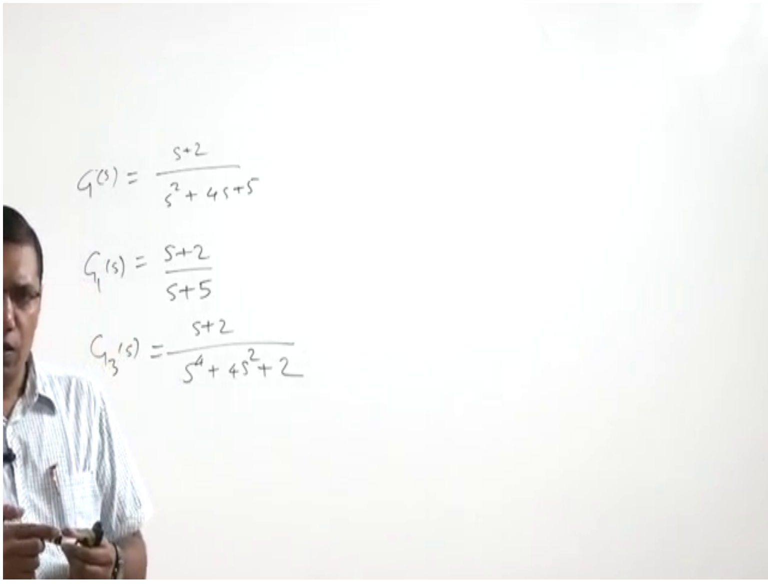

So, here the highest power of S is 2 so this is a second order system. And if it is S plus 2 upon S plus 5 it is 1.

So, this is first order system if we have so here is 4 so it is 4th order system. So, the we can from seeing this transfer function we can understand that what is the order of this system. So, when we say first order system; it means that in the denominator the highest power of S should be 1. So, this is a first order system. So, let us take a first order system
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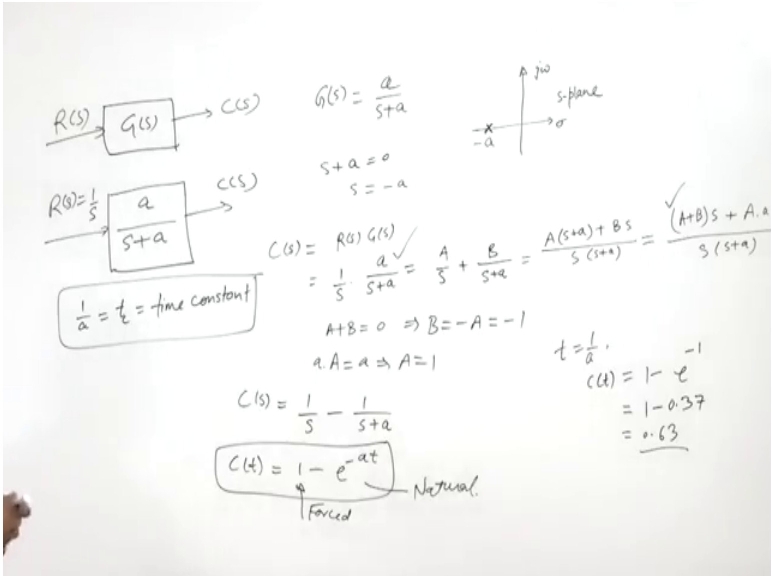

So, here G S equal to a upon S plus a; this is a general first order system. Here we can take also some not a, but K or be some other value.

But it will one effect the amplitude that will result in the forced and transient response. So, let us find the pole so here S plus a equal to 0. And so S equal to minus a. So, this is a pole of this first order system and if we plot it on the S plane ; let us say here this is minus a and this is a pole. So, we have to cross represent as a cross. So, we represent the pole of first ordered system and this is S plane. Now if this system is subjected to some unit step input. So, C S equal to R S into G S. So, R S is 1 by S into G S is a upon S plus a.

So, we can solved this as A upon S plus B upon S plus a. So, here A plus B S plus A into a upon S, S plus a. Now if we compare this and this we will get A plus B equal to 0 because there is no any S term and A equal to a into a equal to a; so this means A equal to 1. And from here we get B equal to minus A that is minus 1. So, we can write here C S equal to 1 upon S minus 1 upon S plus a. Now we take the inverse Laplace transform of this. So, here we get C t equal to one minus here e power minus a t. In this part this it forced and this part is natural response .

So, the complete response comes comprised force and natural response. Now let us take at t equal to 1 by a what happens ? When we take t equal to 1 by a. So, C t equal to 1 minus e power minus a into 1 by a that is 1. So, here we can write 1 minus e power minus 1 is 0.37. So, this is 0.63. So, we see that if this response C t is equal to 0.63 when we have t equal to 1 by a and this 1 by a is called the time constant .

So, here the time constant of a step response. So, time constant is defined for; if the step response of the first order system step response writes to 63 percent of it is final value. So, here the response is 63 percent of the final value; and that is called the time constant. So, a typical response of first order system can be plotted.

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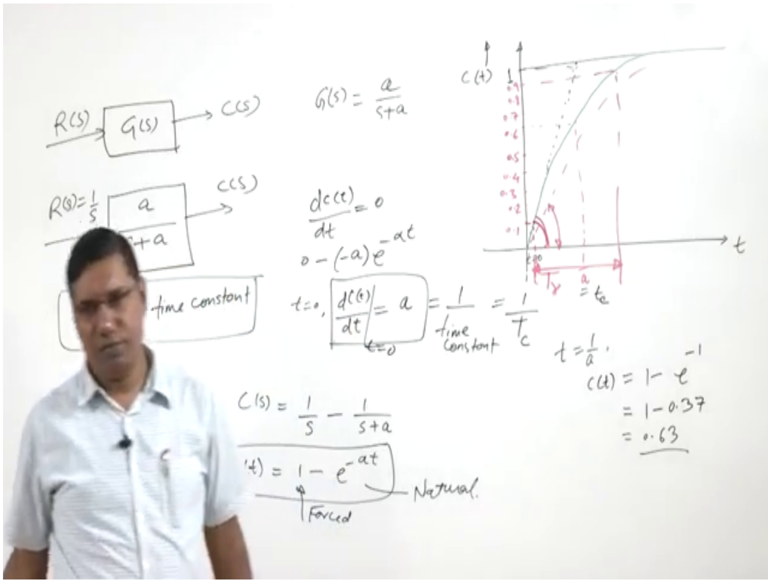

So, this is time and this is the response C t. So, if this is 1 the input and here we have this response. So, at t equal to 1 by a. So, this is let us say this is 0.5 this is 0.6, 0.7, 0.8 and 0.9 and similarly here we can 0.1 , 2 , 3 , 4.

So, 0.63 will come something here. And this is 1 by a because at t equal to 1 by a we are getting 0.63. This response as 0.63 and this we define as time constant. Now how can we find the value of time constant?. So, let us take d C t by d t equal to 0. So, if we take d C t by dt equal to 0 , what we are getting? we are getting 0 because this is constant term minus minus a into e power minus a t. So, at t equal to 0 d Ct by dt. So, this is the slope of this response d Ct by dt is showing the slope of this response C t.

So, at t equal to 0 we are getting as a. So, d Ct by dt, we are getting equal to a. So, here we have the slope of this response at t equal to so this is t equal to 0 and here the slope the slope will give us the value of a. So, we can see that initial slope equal to 1 upon time constant because this is 1 upon time constant 1 by t c. This is equal to tc. So, when we talk about the characteristic of a transient response of first order system or performance specification of transient response of first order system; time constant could be one performance specification, why because it shows the initial slope of the response ; it means it shows the speed of the response against a step input.

So, when we are in an elevator and we saw that elevators speed of is 1 parameter how fast it the response starts. So, if the slope so this is the slope, this slope could be changed so here time constant will change. So, it this shows the slope or speed of the response, there are some other parameter.

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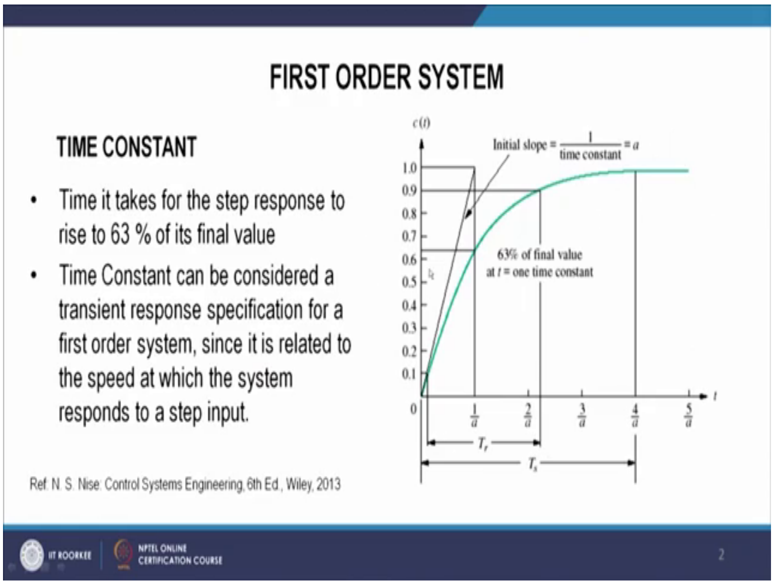

So, here in this diagram we can see the same plot that I have plotted, we can see here this response of a step input and at one-time constant there is more 62 to 63 percent of it is final value. And so, time constant is considered at one of the transient response specification for a first order system.

Now, here are 2 other terms Tr and Ts. So, another transient response specifications can be rise time and settling time; in addition to the time constant so rise time is defined as the time when the response goes from 0.1 to 0.9 of it is final value. So, when the response grows and reaches so this is 0.1 and here is 0.9. So, this time when the response grows from 10 percent to 90 percent this is called the rise time Tr.

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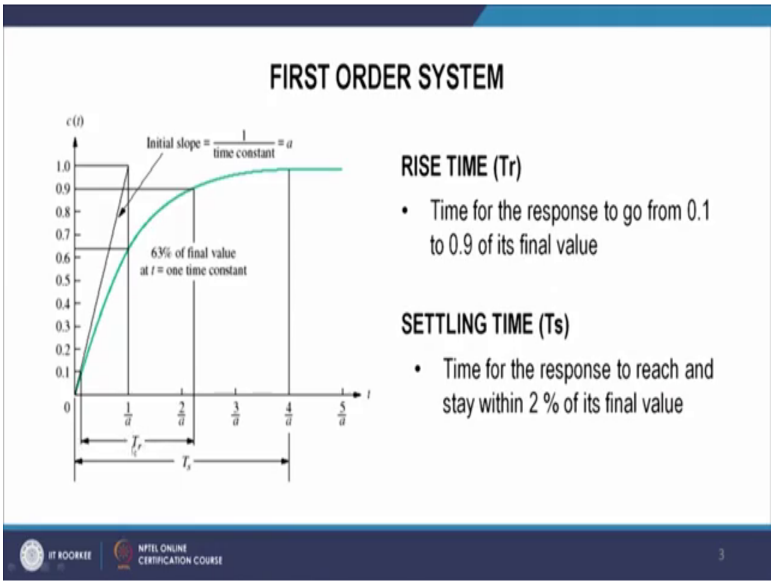

And we can see in this diagram also here we have the rise time tr from 0.1 when response is at 0.1 and then it reaches to 0.9 and this is the rise time.

So, rise time is the difference between the 2 times from 10 percent to 90 percent. Now another one is settling time so settling time is the time when the response reaches and stays within 2 percent of it is final value. So, that is settling time so the when the response recess to 98 percent; and it stays between 2 percent of it is final value. So, that is the settling time so here we see that from 0 we start and let us they say at 4 time constant this response reaches to 0.98 and it stays within 2 point 2 within 2 percent of it is final value so this is the settling time. So, we see that time constant, rise time and settling time these are the 3-transient response a specifications of first order system. And we can characterize the first order system from the response and by calculating these times of the response.

Now, in case of first order system we see that there is no any overshoot. So, the first order system is characterized with also with the absence of overshoot. So, this value listen this final value the response does not overshoot does not go above this value, it is always below this final value or input or a steady state value. So, the first order system is characterized with the absence of overshoot; in the response. Now we have one more point that we want to discuss that suppose we have a response of a first order system from some experiment. And from that response we want to make the transfer function of that system.

So, how can we do? So, how can we make the transfer function of a system by the response of the system? So, let us discuss so let us assume first that we have general transfer function of first order; that is GS equal to K upon S plus a.

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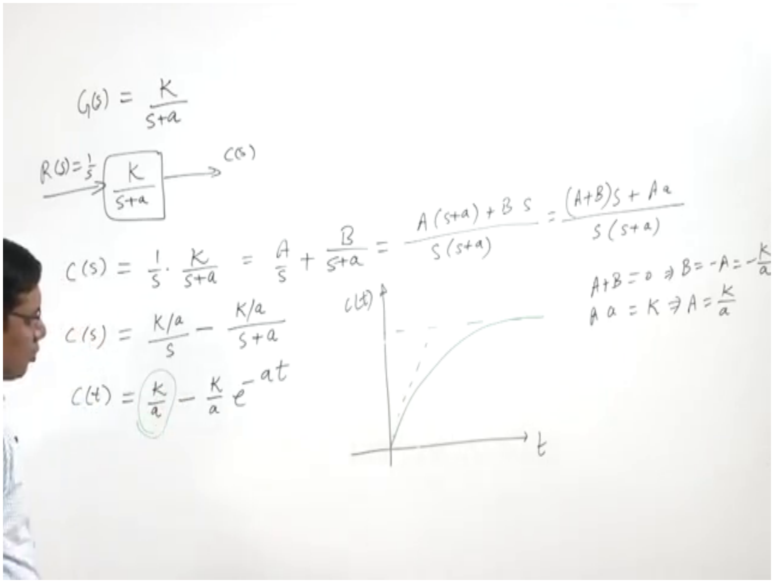

And we have this step input RS that is equal to 1 by S and this is CS. So, here C S equal to 1 by S into K by S plus a. This is most general transfer function of a first order system. So, this we can write K by a upon S plus so here we have A by S plus B by S plus a.

So, we can write A S plus A plus B S by S S plus a. So, A plus B S plus A a by S S plus a. So, here A plus B equal to 0 and A a equal to K. So, we find here A equal to K by a and B equal to minus A that is equal to minus K by a. So, here a equal to K by a upon S minus K by a upon S plus a. So, when we can have C t equal to K by a minus K by a e power minus a t. So, if we want to make the transfer function of this parts first order system ; we need to know K and a from the response and this is the response.

So, suppose if we have here some response; first order response from this response by comparing this equation to the response we have to find the K by K and a values. So, we know that the time constant that is 1 by a, we can find from the initial slope. At t equal to 0 we calculate the slope. And from the a steady state value that is the we can connect with the first part of the response. So, from these 2 we can find K and a. So, let us take one example so let us take that this is 0 here it is 0.72 this is a particular response to the 0.8.

So, let us see here we have to find the transfer function from the response of a first order system.

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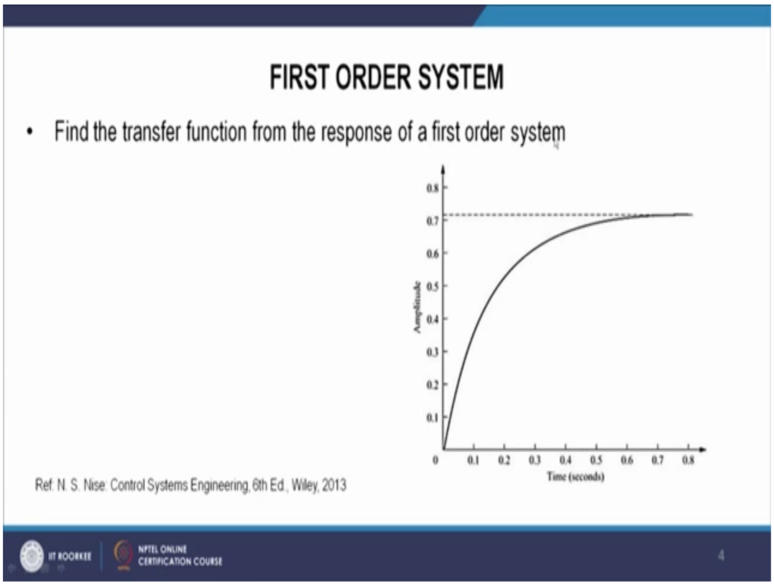

And this is and response this is a response you can see time varying from 0.1 to 0.8 seconds; and the response is varying from 0 to here 0.7 to the steady state value. So, from this we have to find we have to measure so 0.72 is the steady state value or final value.

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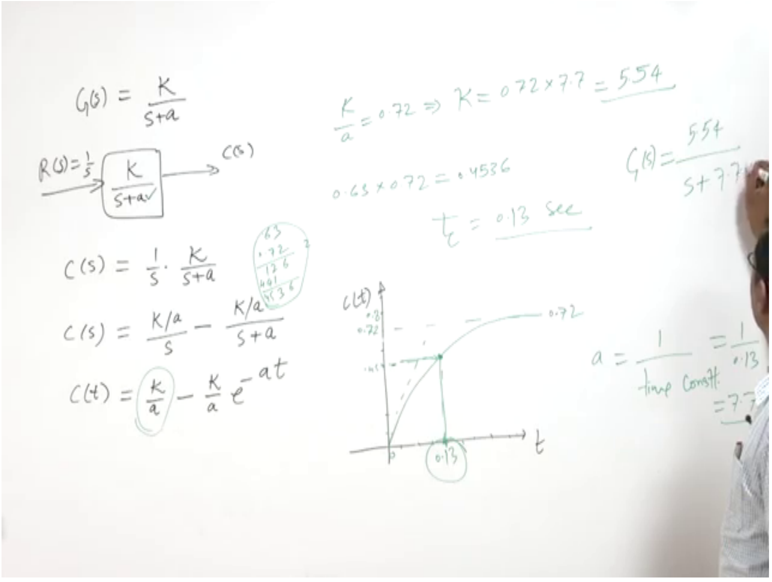

So, 0.72 is the final value and time constant we want to find. So, time constant is the time when the response reaches to 63 percent of the final value. So, the 63 percent of the final value.

We can find. So, we can see here. So, it is about 0.45 so when we have something about 0.45 and here we have some time. So, the time about this 0.45 will be the time constant because 63 percent value will be of the final value that is. So, from here we will take the time at which this value is reached for this response and that is a time constant, so we get that tc equal to 0.13 second because as this value this we are getting this value.

So, this is a time constant. So, to get the a; a is 1 upon time constant because a is this initial slope of this response curve. So, that is 1 by time constant 1 by 0.13. And we can get 1 by 0.13 7.7. Now we have got one parameter of this transfer function, a now we have to get K. So, K by a is the forced response value and that is equal to 0.72. So, K by a equal to 0.72 so K is 0.72 into a that is 7.7 and when we multiply this we can get 5.54. So, the transfer function GS we can write here K K 5.54 upon S plus a so a is 7.7.

So, so we saw that, the first ordered systems they have the transfer function and in the denominator of the transfer function; the highest power is of S is 1 so that is this is over the how we, we can identify the first order system from the transfer function. Now come to the response. So, we see that the first order system they do not have any overshoot ; and their response is exponentially growing or exponentially decaying as the case may be. So, when it is exponentially growing to the input or a step from value, when the response researches to 63 percent of the final value that is called the time constant and the rise time is the time when the response reaches from 10 percent to 90 percent of the final value whereas, the third transient response character characteristic or a specification is the settling time when the response reaches and it stays within 2 percent of the final value so that is the settling time. So now, we stop here.

So, thank you for this lecture, see you in the next lecture.