Automatic Control
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Lecture – 11
Poles and 0s

So, welcome to the lecture on transient response analysis. Today we will discuss in this lecture about poles and 0s. So, in previous lectures, we have discussed that to design a control system, we have 3 objectives. First one is the transient response. Second is stability, and third is a steady state error. So, we saw that transient response is defined as a systems a characteristic. Because it is governed by the systems internal characteristics; transfer function, and we test the systems response with some defined input such as unity step input or impulse input and so on.

So, today we will discuss about poles and 0s, because and link these poles and 0s to the transient response characteristic of a system. So, the poles and 0s they simplify the evaluation of systems response. So, what is; let us define what is pole and what is 0. So, we have a transfer function. So, let us we have this transfer function G S.

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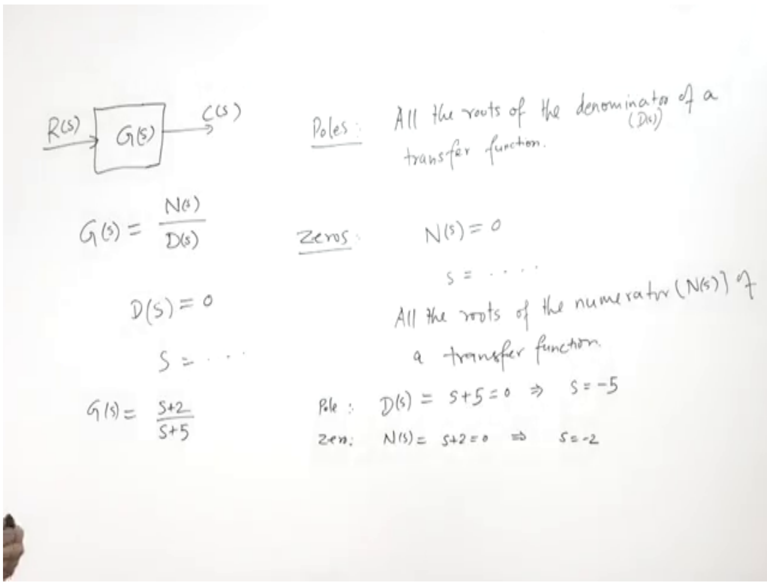

It is subjected to some input, and it is giving some output. So, this transfer function G S can be written as N S by D S. So, N S the numerator part of the transfer function. And D S is that denominator part of the transfer function. So, from the transfer function how can we get the poles and 0s of a system. So, poles or poles of a transfer function or the values of Laplace transform variable S the values of S that caused the transfer function to then be infinite. So, the values of S for which the transfer function will become infinite is the pole. Or we can say that if we can put D S equal to 0 this will make this transfer function infinite, and D S equal to 0 will give the values of S.

So, the roots of this equation D S equal to 0; that is, the values of S or the poles because, these values will make the D S equal to 0 and so, the transfer function to infinite. So now, come to the so, the poles or all the roots of the denominator, all the roots of the denominator of a transfer function. So, here denominator we say the D S. Now come to 0s. So, 0s of the transfer function or the values of Laplace transform variable S that causes the transfer function to be come 0.

So, when this transfer function; all the values that makes these transfer function 0. So, that is N S equal to 0 if n is equal to 0. This transfer function will be 0. So, when we solve this equation and we find S these values or the 0s. So, 0s are all the roots of the numerator N S of a transfer function. So, here we have defined the poles and 0s. So, let us take one example. So, let us take that G S equal to S plus 2 upon S plus 5. So, this is G S gs equal to S plus 2 upon S plus 5. So, we want to know the poles. So, to calculate the poles we should what is numerator denominator D S D S equal to S plus 5, D S equal to S plus 5 and that equal to 0 this implies that S equal to minus 5.

So, S equal to minus 5 is a pole of the system, or of the transfer function. Now we get the 0s. So, here we have only one pole now come to 0. So, to get 0 we need to put the numerator equal to 0. So, N S equal to S plus 2 equal to 0. So, this implies this S equal to minus 2. So, S equal to minus 2 is a 0 of the system or transfer function. Now let us show these poles and 0s on S plane that is complex frequency plane S or complex Laplace transform variable plane.

So, we can show it.

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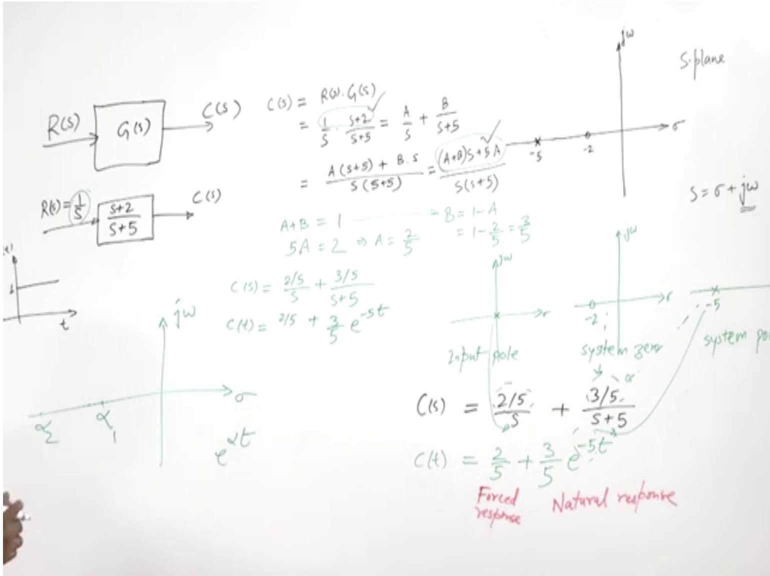

This is S plane, and here S equal to sigma plus j omega because S is a complex variable. So, it is sigma is the real part or real axis and omega is the y axis. And this is complex axis. So, here if we want to show S equal to minus 5; so, here we have only the real part, here complex imaginary part is 0. So, x equal to minus 5 we can show here, let us show here this is minus 5, then 0 that is S equal to minus 2. So, it will be somewhere here.

Now, there is representation for false and 0 different representation. So, a 0 is represented as a circle a small circle, and pole is represented as a cross. So, here we have pole S equal to minus 5 here we show as a cross, and 0 we show as a small circle. So, we have shown the poles and 0s of a system on the S plane. Now let us try to find that when this system is subjected to some step input, unit step input what will be the response, transient response and complete response, and how we can link it with the poles and 0s.

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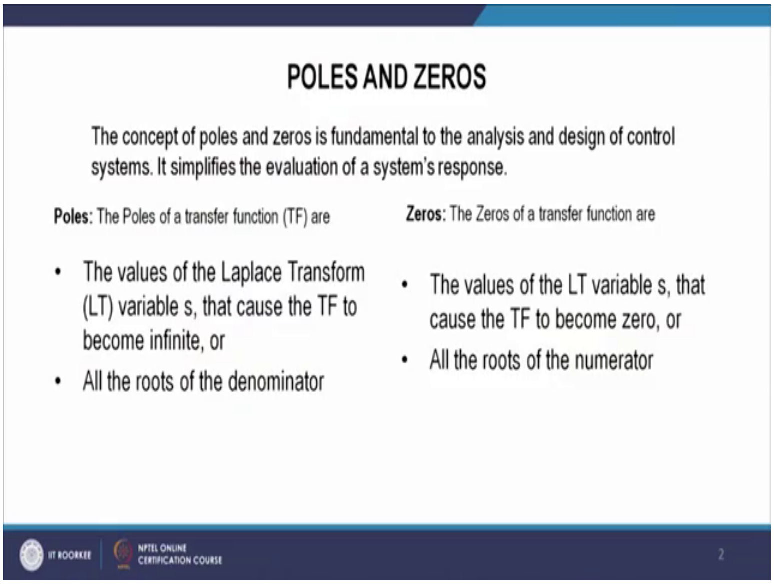

So, here we have let us have this G S. So, G S here is ok, let us this is G S. So, let us show this is S plus 2 upon S plus 5 this is R S, this is C S. So, let us R S is a unity step input something like this. And so, the Laplace transform of unity step function is 1 by S. So, here r is equal to 1 by S. Now we have to find what is the response of this system under this input. So, here we have C S equal to R S into G S. C S equal to R S into G S.

So, R S is 1 by S and G S is S plus 2 by S plus 5. So now, we will use partial fraction to break the this function. So, a by S plus B by S plus 5. So, let us we break this into 2 parts. So, a and B are the constant or coefficient. So, we can write this a S plus 5 plus B into S on S s plus 5. So, this we can write a S and B a. So, a plus B S plus 5 A upon S s plus 5.

Now, we compare this and this. So, when we compare these because this is S s plus 5, and the numerator here we find this is S plus 2. So, a plus B equal to 2 1. Because the coefficient of S is here 1. And 5 A equal to 2 for this constant term 5 A equal to 2. So, here we get a equal to 2 upon 5 and from this equation we get B equal to 1 minus A and that is 1 minus 2 by 5 equal to 3 by 5. So, we can write C S equal to A by S. So, S 2 by 5 2 by 5 by S plus B by S plus 5.

So, B is 3 by 5 by S plus 5. So, this is the C S. Now if we want to we have come from ah now we are in the S domain. We have to come to the time domain to find the response with the respect to time. So, we take the inverse Laplace transform of this. So, we will get C t. So, we take C S with C t and 2 by 5. So, universal applause of 1 by S is unit unity function. So, 2 by 5 plus here 3 by 5 and here S plus 5. So, here with S there is minus minus 5. So, it is e power minus 5 t. So now, here we can make some conclusion based on this.

So, here we have. So, input it 1 by S. So, R S is 1 by S. So, here we have one pole that is S equal to 0. So, here S equal to 0 is one pole of the input, then here we have at minus 2 one system 0. So, this is input pole this is system 0, and this is minus 5 that is system pole. And we see that here C S equal to 2 by 5 by S plus 3 by 5 by S plus 5. And here C t equal to 2 by 5 plus 3 by 5 e power minus 5 t.

We can see this is the constant term. So, this is the steady state value, and this is exponential minus 5 t. So, that is the decaying exponentially decaying value with the time. So, this is the forced response. And this part is natural response. We see that this value 2 by 5 is coming here and there is this calculation to get this 2 by 5 because this pole is coming here 1 by S. So, these values they are depending on the values of 0 and pole. And this value is depending on the input pole. And this term minus 5 that is coming from here. It is depending on this system pole.

So, we see that the amplitude of the natural response value depends on the 0 and pole, while the form that it is exponential value is depending on the system pole. Whereas, this forced part depends on the input of the system, as well as these 0s also because this value is coming by this calculation, which include the 0 and pole. So, we can from here we can see the role of the system pole system 0, and the input pole on the transient response as well as force response.

So, complete response of this system. So, what we see that a pole of the input function generates the form of the force response. So, because the input was unit a step input 1 by S. It generated a response form of 2 by 5 by s, or we can say input was unity and it gave output that is. So, output is here t C t and it is something. So, this was the input and this is output 2 by 5. And this was the input one. But the form is similar, because this is also unity function here it is also that is step function this is output is also step function. So, the form of these output forced response is governed by the pole of the input function. And pole of the input function depends on the form of the input.

Now, come to this part. A pole of the transfer function generates the form of the natural response. So, the form of the natural response is exponential here. And that is generated due to the pole of the system or the transfer function. So, due to this system pole these form is here. If this system 4 pole is here, then this exponential part will not be there. So, we can see that or if we move this pole, we can see that the form will be exponential, but it will decay in the it is it is speed of the exponential decay.

So, we conclude that the pole of the transfer function generates the form of the natural response. We see that the amplitude these amplitudes are coming from this calculation where it involve both the 0 and pole. So, the amplitude is depending the 0 and pole generate the amplitude of both the force and natural response. So, here we can one more point we can note that. So, if this is my system pole, here at alpha on this axis on the real axis. So, if the pole is on the real axis, alpha it will generate a form e power alpha t. It will generate exponential response of the form e power alpha t, and where alpha is the location of the pole on the real axis.

Moreover the this alpha will be further it could be far. So, if it is far this value will decay faster. So, the trans transient response will decay to 0 more fast than less value. So, if we this pole is moving. So, here this is alpha one here is alpha 2. So, e power alpha 2 will decay faster than e power alpha 1

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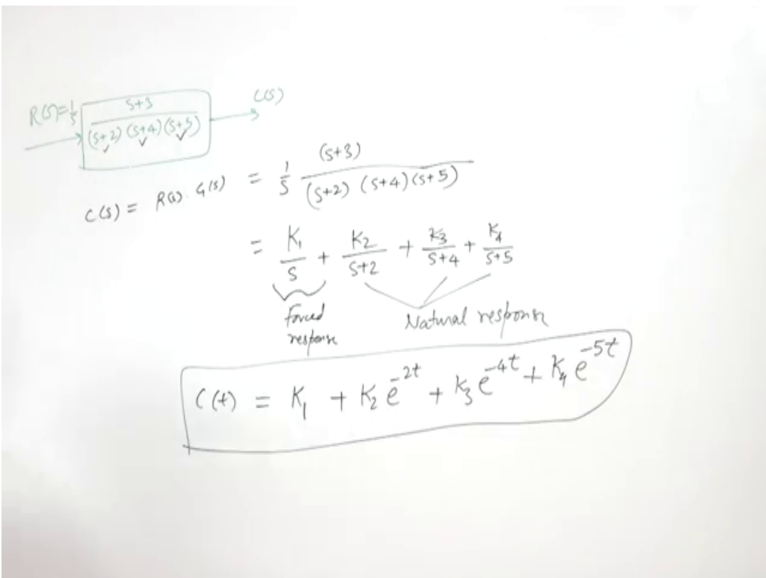

So, suppose this is the my input R S that is equal to 1 by C S. Let us say, this is unit a step input. Now I want that what will be the response of this system.

So, here C S equal to R S into G S. So, that will be equal to 1 by S; S plus 3 S plus 2 S plus 4 S plus 5. And we can write this as K 1 by S plus K 2 by S plus 2 plus K 3 by S plus 4 plus K 4 by S plus 5. So, by using the rule of method of partial fraction we can write this in this form and we can find the K 1 K 2 K 3 and K 4 these coefficients. And so, this is the forced response, and this is these 3 are the natural response. You can see these poles are this pole these 2 3 values minus 2 minus 4 minus 5. And these poles are giving the natural response and so, if we do C t.

So, we can write K 1 plus K 2 e power minus 2 t plus K 3 e power minus 4 t plus K 4 e power minus 5 t. So, here we can find the response of the system. So now, we see that when there is a system, we can go in the S domain. We can write this output. Then we can go if we find these coefficients we come back to time domain by taking the inverse Laplace transform.

So now here today; what we discussed about poles and 0s? So we summarized that the poles are the roots of the denominator of a transfer function. And 0s are the roots of the numerator of the transfer function. And we saw that the poles of the input governs the force response the form of the force response and poles of the system or transfer function governs the form of the natural response.

And the values that is amplitude of these responses are governed by poles and 0s. And we saw that if there is some pole on the real axis, the natural response is in the form of exponential alpha t; where alpha is the location of the pole on the real axis. And we see that if the pole is on the negative axis much farther from the on the axis then the decay will be faster. And so, the transients response will decay faster. So, these examples that we took we took from the book of Nise Norman as control systems engineering.

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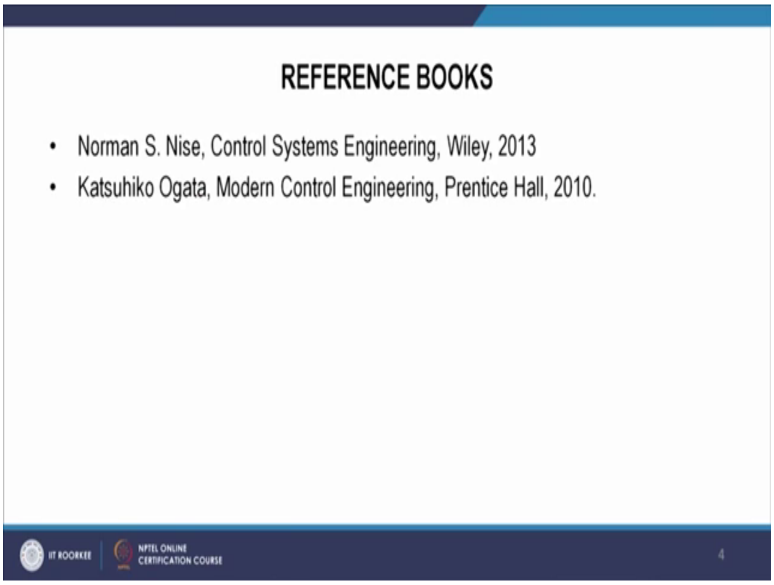

So, I thank you for this lecture, and see you in the next lecture.