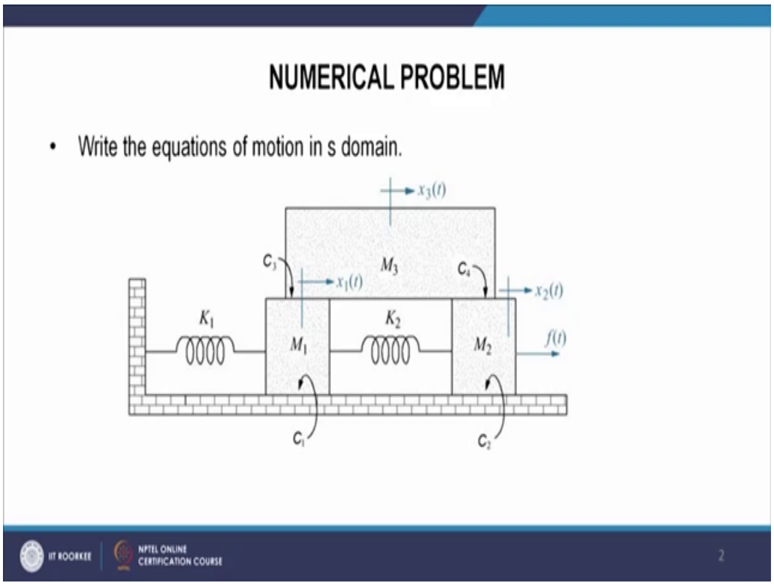
Automatic Control
Dr. Anil Kumar
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture – 10
Numerical Problems

So, welcome to lecture on mathematical modeling. In this lecture, we will discuss few numerical problems. So, in which we will use the impedance method to write the differential equations, write the equations in complex frequency domain or to obtain the transfer function. So, so far, we have written the differential equation, and then we have taken the Laplace transform, and we obtain the transfer function. We do not need to write the differential equation using the impedance method. We can directly write the transfer functions.

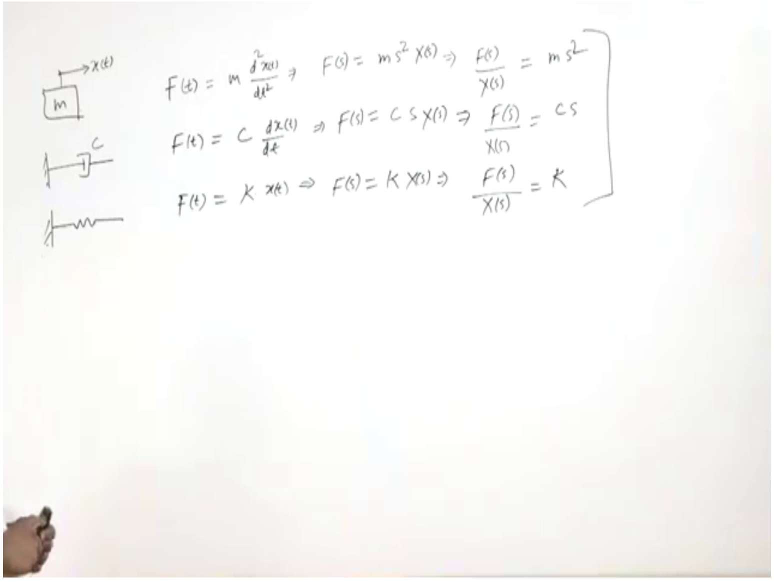
So, we will understand this with the help of few examples. So, first example we will take for a mechanical translational system.

(Refer Slide Time: 01:25)



So, we know that in mechanical translational system we have 3 components. One is mass, then there is damper, and then there is a spring.

(Refer Slide Time: 01:31)



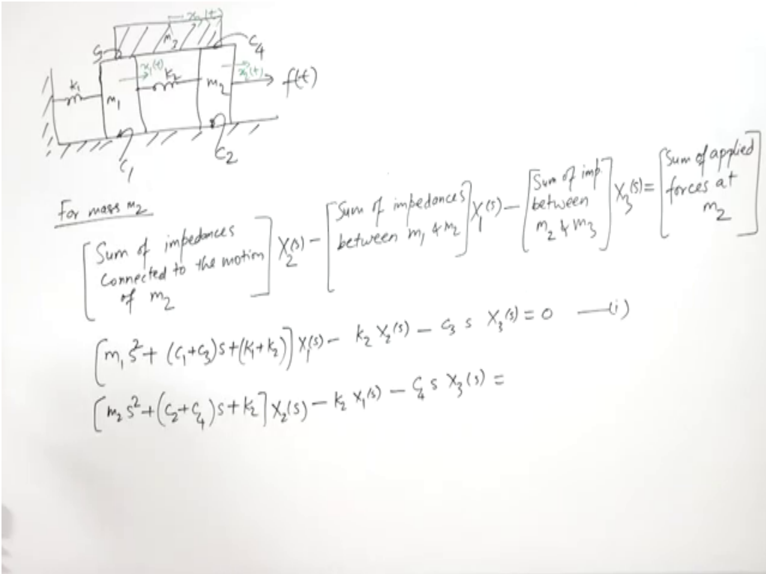
So, here we have we know that for a mass there is force F t equal to m into d square, if this is x t x t by d t square.

So, we take Laplace transform F s equal to m s square into X s with initial conditions 0. So, the impedance is defined as F s by X s that is equal to m s square. Similarly, for a damper the force damper force is C d x by d t. And when we take Laplace transform it is C s X s. So, impedance is fs by X s and that is equal to C S. For a spring force is k into x t. We take the Laplace transform. We will find F s equal to k into X s.

So, here the impedance is F s by X s, that is k. So, these are the impedances of the mechanical elements mass damper and spring. So now, we take the example. So, we take this first example first problem, write the equations of motion in s domain. So, we want to write for this system the equations of motion in complex frequency domain. Once we are able to write the equations in complex frequency domain, we can obtain the transfer function of that system.

So, here we are going to directly write the equations. If you can the impedance method. So, this system is something like this.

(Refer Slide Time: 04:19)



So, here we have a spring, and there is a mass, then there is another a spring, there is another mass. And on these there is one more mass. So, here is another mass, and these are 2 masses. This is m 1 m 2, this is k 1 k 2. This is m 3. The damping between. So, these surface it is c 1 and damping here is c 2. Here is c 3 between these 2 blocks there is damping c 3 and here is c 4.

So, we apply a force f t here. Now we assume that this system most and x 1 t x 2 t and x 3 t. So now, we have to write the equations of motion for this system using impedance method. So, we can have for mass m 1. So, sum of impedances connected to the motion of m 1 into x 1 s minus sum of impedances between m 1 and m 2 into x 2 s minus some of impedances between m 1 and m 3 into x 3 s equal to sum of applied forces at m 1.

So now some of impedance is corrected to the motion of m 1. So, this is m 1 and the m 1 is getting impedance due to this a spring this damper c 1. So, c 1 is equivalent viscous damping that is coming friction from this surface. And so, we can write it. So, this mass impedance m 1 s square, because your mass has the impedance m s square. Here we have c 1 and c 3 into s k 1 and k 2. This is the step impedance due to these 2 springs in to x 1 s minus sum of impedances between m 1 and m 2. So, between m 1 and m 2 there is impedance only due to spring k 2, that is connecting these 2 masses.

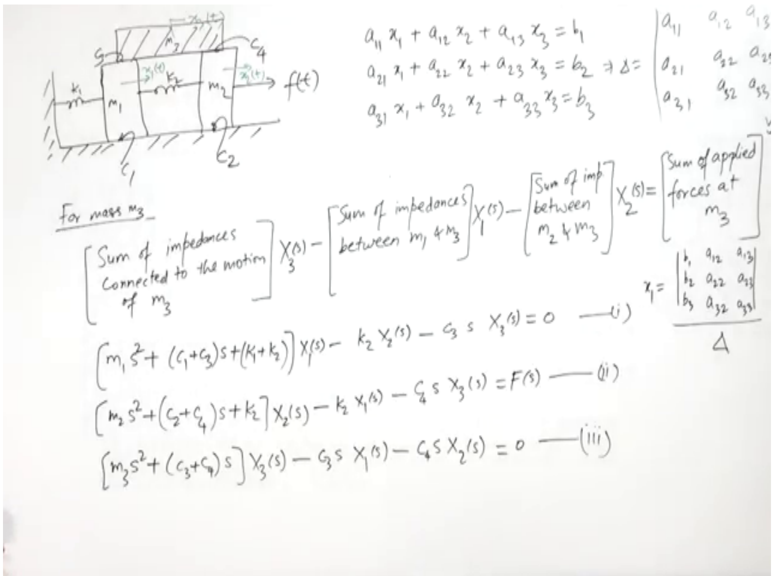
So, minus k 2 into x 2 s. Minus sum of impedance between m 1 and m 3. So, between m 1 and m 3 there is c 3. So, minus c 3 s into x 3 s equal to sum of applied forces at x m 1.

So, m 1 is not subjected to any force. So, this is 0. So, this is equation number 1. Similarly, for per mass m 2 we can write here m 2. And then sum of impedance is connected to the motion of m 2 into x 2 s minus sum of impedances between m 1 and m 2 into x 1 s minus sum of impedances between m 2 and m 3 into x 3 s equal to sum of applied forces at m 2.

So, if we apply these for this mass. So, some of impedance is connected to the motion of m 2. So, here is k 2 c 2 and c 4. And the impedance of this mass m 2 s square. So, this is m 2 s square plus c 2 plus c 4 s plus k 2. And x 2 s x 2 s, minus sum of impedance between m 1 and m 2 is k 2. So, k 2 into x 1 s minus sum of impedance between m 2 and m 3 so, m 2 and m 3 c 4. So, it is c 4 s x 3 s that is equal to sum of applied forces at m 2.

So, here m 2 is subjected to a force F t, and here we have to take the Laplace transform of F t. So, we write F s this is equation number 2. Similarly, for mass m 3; so, here motion of m 3 into x 3.

(Refer Slide Time: 12:44)



Minus sum of impedances between m 1 and m 3 into x 1 s. Minus sum of impedances between m 2 and m m 3 and x 2 s and some of applied forces at m 3. So, your sample sum of impedances connected to the motion of m 3. So, they are the m 3 s square plus c 3 plus c 4 s x 3 minus some of impedance between x 1 and between m 1 and m 3. So, between mm m 3 c 3. So, c 3 s into x 1 s minus here between m 2 and m 3 c 4 s into x 2 s. And that is equal to there is no any force applied here.

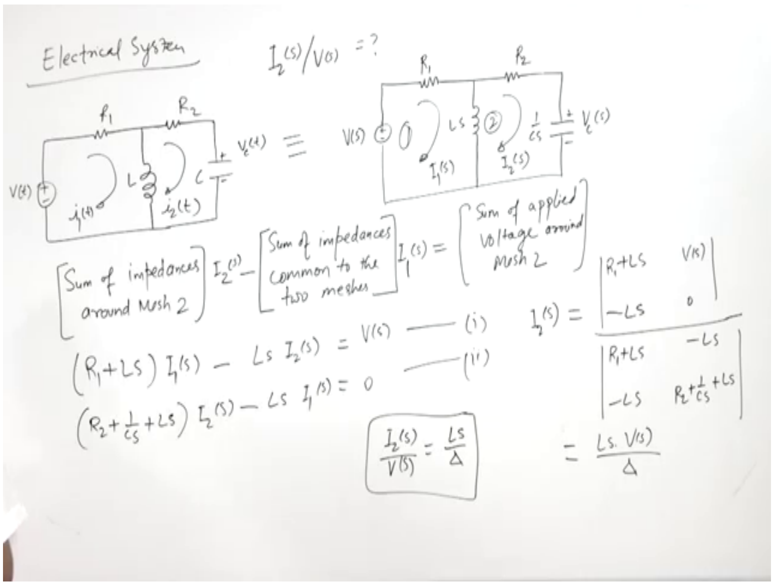
So, it is equal to 0. This is equation number 3. So, we can see that using the impedance method we are able to write the equation of motion in s domain, complex frequency domain directly, and very easily and these 3 equations are like this form. So, a 1 1 x 1 plus a 1 2 x 2 plus a 1 3 x 3 equal to b 1. And this is a 2 1 x 1 plus a 2 2 x 2 plus a 2 3 x 3 equal to b 2. And here a 3 1 x 1 plus a 3 3 x 2 plus a 3 3 x 3 equal to b 3.

So, these equations, because here x 1 x 2 and x 3 or these 3 variables. And these terms are b 1 b 2 b 3 so, 0 F s n 0. Now we can form a matrix delta of a 1 1, a 1 2, a 1 3, a 2 1, a 2 2, a 2 3, a 3 1, a 3 2, a 3 3, this is delta. So, we can use the cramer's rule to find the transfer function. Whether we are interested to find transfer function; that is, x 3 s upon F s or x 2 s upon F s or x 1 s upon F s. So, we can find this transfer function.

So, for example, if we want to find here this x 1; so, x 1 equal to this first column we have to replace with the b 1, b 2, b 3. B 1, b 2, b 3, and these terms will be as such a 1 2, a 2 2, and a 3 2. A 1 2, a 2 3, a 3 3 on delta. So, we can find here x 1 equal to this determinant upon this determinant, and we will get the; so, we can get x 1 s and here is b 1, b 2, b 3. So, there will be F s also coming from this. And so, we can find x 1 s by F s.

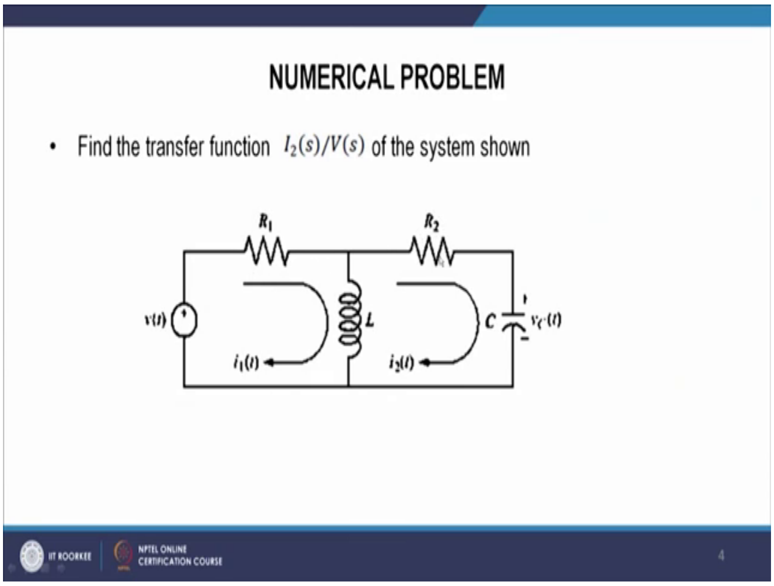
So, the transfer functions. So, although these transfer functions, we can find using these 3 equations. Now we take another example that is based on some electrical circuit. So, we have another problem based on electrical circuit.

(Refer Slide Time: 18:20)



So, we have this example. So, we can see here this example. We have this voltage source.

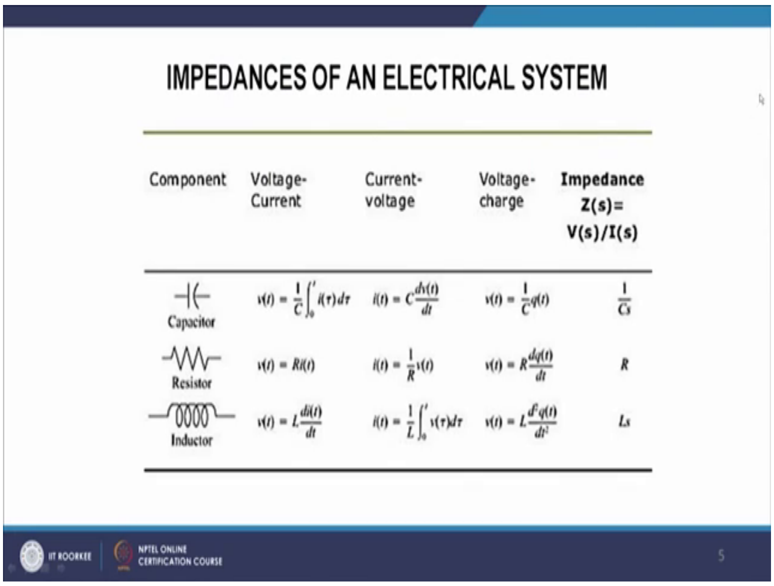
(Refer Slide Time: 19:18)



Resistance R 1 R 2 inductor L and capture c. And there are each current I 1 t in this loop first loop. And I 2 2 t in the second loop and here is the output voltage we are reading at the capacitor that is v c t.

So, we have to find the transfer function I 2 s by v s. So, I 2 s by v s we have to find. This is the transfer function we have to find. So, we already know that the for an electrical system.

(Refer Slide Time: 20:00)



The capacitor has impedance 1 by c s, register has impedance R and inductor has impedance L s. So, we can convert the circuit to an equivalent impedance circuit. So, let us convert these. So, here is v s, this is R 1 R 2. This is L s, this is 1 by c s, and here is v c s. Here is I one s, and this is I 2 s.

So now we will apply the again this impedance law to find the transfer function. So, first we write the equation of motion in s domain. So, here we have some of; so, for mesh one some of impedance around mesh one. So, this is mesh one and mesh 2. Let us say this is mesh one and this is mesh 2 into I one s minus sum of impedances common to the 2 meshes into I 2 s equal to sum of applied voltage about mesh one.

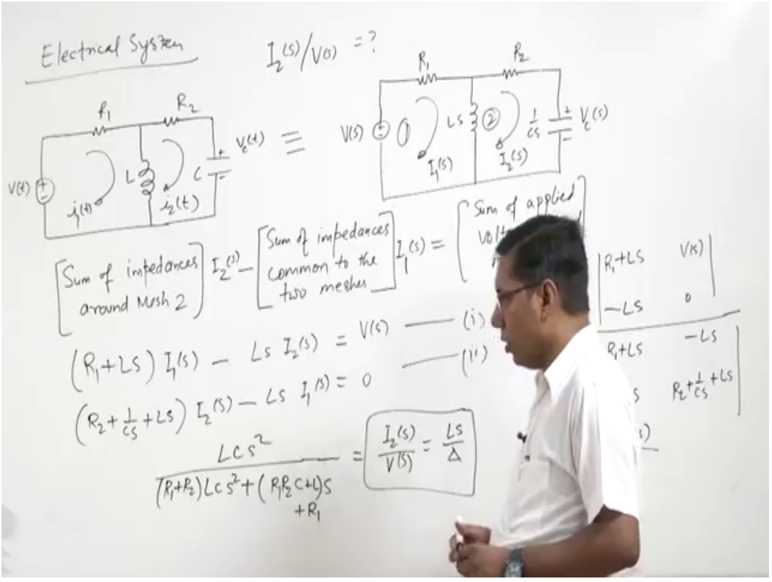
So, we can see that some of impedance around mesh one. So, what is the impedance here? R 1 and R 1 and L s; so, in this mesh one there is impedance R 1 and L s. So, R 1 plus L s into I 1 s minus, sum of impedance common to the 2 meshes. So, here this inductor is common to the 2 mess. Because this is in mesh one as well as also in mesh 2. So, this minus L s into I 2 s equal to sum of applied voltages around mesh one. So, that is we apply here v t or here v s in this mesh one. So, equal to v s this is equation number one.

Now, for mesh 2 we can write sum of impedance around mesh 2 into I 2 s, minus sum of impedances common to the 2 meshes into I one s equal to sum of applied voltage around mesh 2. So, sum of impedance around mesh 2 is R 2 plus 1 by c s plus L s, into I 2 s minus sum of impedance common to the 2 meshes; that is, L s into I one s and the sum of applied voltage surround mesh 2. So, about mesh 2 there is no any applied voltage. So, it is 0, this v c s is the measured voltage. This is we are going to measure, this is not some applied voltage.

So now we have we can find I 2 s. So, again this we can apply cramers rule, because this is a 1 x 1 plus a 2 x 2 equal to b 1 and here let us say c 1 and x 1 plus c 2 x 2 equal to b 2. So, we can apply cramers rule. So, I 2 s equal to; so, we are going to find I 2 s. So, we have R 1 plus L s and here minus L s and this is v s and 0. Upon the determinant R 1 plus L s minus L s and this minus L s R 2 plus 1 by c s plus L s.

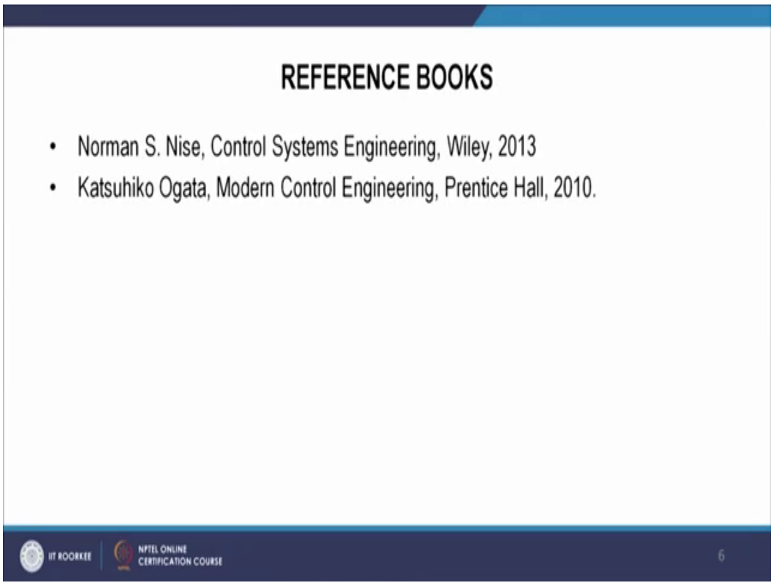
So, this is 0 plus L s into v s. So, this is equal to L s into v s upon delta. Let us say this is delta. So, we can find the transfer function I 2 s by v s n. So, I 2 s by v s is L s by delta, where delta is given with this determinant.

(Refer Slide Time: 27:10)



When we solve this we will find L c s square upon R 1 plus R 2, L c s square plus R 1 R 2 c plus L s plus R 1. So, this is the final transfer function I 2 s by v s. So, here we saw that these 2 examples are taken from the reference books.

(Refer Slide Time: 28:04)



Normal S Nise Control Systems Engineering; so, here we saw that by using the impedance method, we can directly write the equations of motion in complex frequency domain or s domain. And we can find very easily the transfer function of a system. Here we saw the 2 examples one of the mechanical system and electrical system. So, here I stop and I see you in next lecture.

Thanks.