Theory of Production Processes
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Lecture - 38
Analysis of Drawing Operation

Welcome to the lecture on analysis of drawing operation. So, in the last lecture we discussed that when we go for drawing operation you apply the pulling load from the other side of the dye and you have the compression forces acting inside the dye on the inclined surface and then you have the state of stress acting and that leads to the plastic deformation of the material. So, we need to calculate the value of stresses. So, for that we have already done the analysis of these forming operations for rolling we have we had done for also we did for the forging operations. So, we will do for the rod drawing also. Now what we see is in the case of rod drawing you have basically a rod of suppose this height

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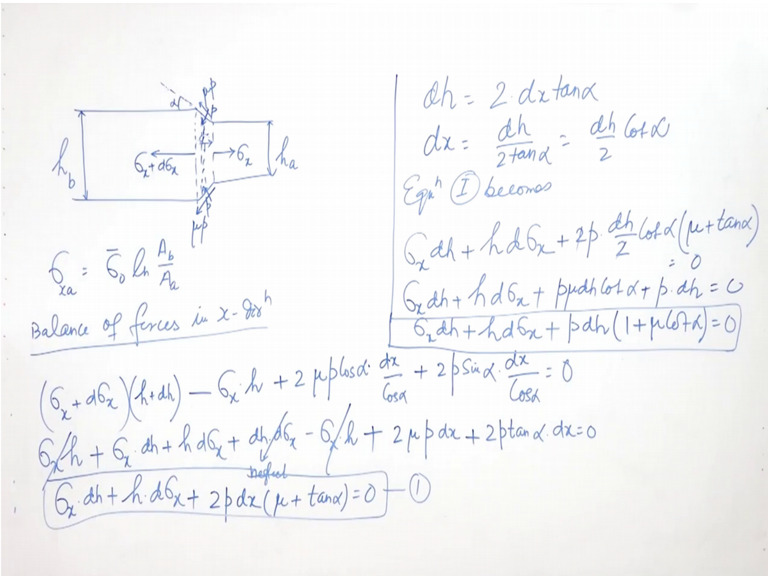

Hb and so this is hb and suppose this is going through the dye and then you are converting to this height of ha so this is the portion of the dye and you have basically the dye here so from here the stresses are acting the force is acting because you are pulling in this direction.

Now in that case if you take this as the so this will be dx if you take, now what we see is if you take this element which is subjected to the stresses, what we see is at this point since this is that in contact with a dye. So, you will have the pressure p acting here. And then what we see is that you have this hb and ha and this will be now, at this point the if you take this as dx.

So, you will have this as sigma x plus d sigma x and that will be your sigma x. So, you will have such kind of derivations we have already done and what can be shown using the uniform deformation energy method, it can be shown that if you want to find the sigma xa so basically sigma xa. You know that can be calculated like you have sigma naught mean value of the yield stress multiplied by ln of ab by aa. So, that is your you do it using this. That is how using that deformation energy method uniform deformation energy method this has been already proved.

So, in that basically work done which is there so based on that you get this expression, but this expression basically is not taking into account and the friction because you have friction acting here. So, the friction will be acting in the opposite direction.

So, frictional force will be mu times p and mu p will be acting in this direction. So, you will have mu p acting here and also mu p acting here. Now this is the half angle is taken. So, if you take this. So, this angle basically is taken as alpha. So, you will have mu p mu p will have both the components one will be acting in the x direction another will be acting in the normal direction.

So, mu p also has the components so basically when we got this expression, which has been proved using the uniform deformation energy approach, in that basically we are neglecting the friction or and also the influence of the transverse stresses because we are only talking about one kind of stress. So, if we take the case of you know friction into account what we see is that you have mu p on both these sides. Now you will have so and this this will be dx so as we know this will be dx. So, you will have x you know at that point and you will have dx here positive exit is this direction or whatever you can say and then so that will be your x plus dx position

So, you will have sigma x plus d sigma x and then this is sigma x. So now, if you do the force balance in the x direction, if you look at and if you do this force balance for the uniform I mean unit width so what will happen? Now sigma x plus d sigma x and this is your h plus dh. So, suppose you take at any point. So, if suppose you are taking at any point in between which is of dx you know dx length. So, it is not that this is dx basically you have that way if you look at you take any element in between and then it is length will be dx.

So, at that position you will have sigma x sigma x plus d sigma x that way. Now if you take the force balance of forces in x direction. Now if you take the balance of forces in x direction what we see is. So, that is for per unit to width then in that case what will happen sigma x plus d sigma x and that is on this height so this height basically will be you know h plus dh, then minus of this is opposition acting in opposite direction so sigma x into h then further you will have the mu p acting now if this mu p is acting and it is component is cos alpha.

So, mu p cos alpha and from here also mu p cos alpha. So, you will have mu 2 of mu p cos alpha, now this is acting on this surface now this surface. So, this is basically dx if you look at and this being alpha so that will be dx by cos alpha. So, this will be into dx by cos alpha, further you will have p so p it is acting like this so p cos alpha will be the compressive force. P is sin alpha will be acting in the in this x direction. So, you will have 2 times p sin alpha because p sin alpha will be here and this side and p sin alpha will be this side. So, you will have 2 p sin alpha further it is acting on the same surface that is dx by cos alpha.

So, again it will be dx by cos alpha and that should be equal to 0. Now if you solve this equation what we see is that you will have sigma x h plus sigma x dh plus h d sigma x plus dh into d sigma x so this we neglect minus sigma x into h. So, that will further be you know cancelled plus this will be 2 mu p and dx. So, it will be 2 mu p and dx and then here it will be 2 p 10 alpha into dx. So, that will be equal to 0. So, what we see is again sigma xh and this is minus sigma xh will be cancelled. So, what we see is sigma x dx plus hd sigma x plus. So, what you see is sigma x into dh plus h of d sigma x plus what you see is here you have 2 into p into dx is common. So, 2 p is common you will have and here you have dx also common.

So, you will have mu plus 10 alpha that is equal to 0. So, you what you get is you will have the sigma x dh plus h into d sigma x plus you know 2 p 10 alpha dx plus 2 mu p dx. So, that is equal to 0. Now if you look at the geometry phi by the geometry if you look at you can write dh. So, dh basically is the difference of this now dh will be basically dh will be 2 times dx 10 alpha. So, what you see is you will have dh will be 2 times dx 10 alpha, dh basically is the change in the height. So, on both the sides you have. So, dx 10 alpha will be the height in this S direction and if you take both side together dh will be 2 times dx of 10 alpha. So, you can keep it that value here now in that case you can write this dx as dh by 2 10 alpha. So, you can write dx as dh by 2 10 alpha that is dh by 2 cot alpha.

So, further taking now if you take this equation so equation 1 becomes so you have to put dx as dh by 2 cot alpha. So, it will be sigma x dh plus h of d sigma x plus 2 p into dh by 2 cot alpha into so dx is dh by 2 cot alpha into mu plus 10 alpha and that will be equal to 0. So, what will happen sigma x dh plus hd sigma x. So, sigma x dh plus h of d sigma x plus, if you go further say this 2 and 2 will be cutting down you will have p and dh cot alpha and into mu. So, p into mu and dh cot alpha plus p into dh that is all because cot alpha into 10 alpha will be 1.

So, if you take p into dh as outside. So, it will be p into dh into 1 plus. So, 1 plus mu cot alpha. So, that is equal to 0 that will lead to sigma x dh plus h of d sigma x plus p into dh you will take into that and you will have 1 plus mu cot alpha it will be equal to 0.

So, this is what the equation you are getting and this equation in this equation further we are going to use the yielding condition and the yielding condition will tell so this is the condition which you get. Now the yielding condition will tell that sigma x plus p it should be equal to sigma naught prime. So, based on that you will have the equation which is further to be integrated and we will have the expression for sigma x a or so. So that we will see. So, as we discussed that we will provide the yielding condition and the yielding condition will provide.

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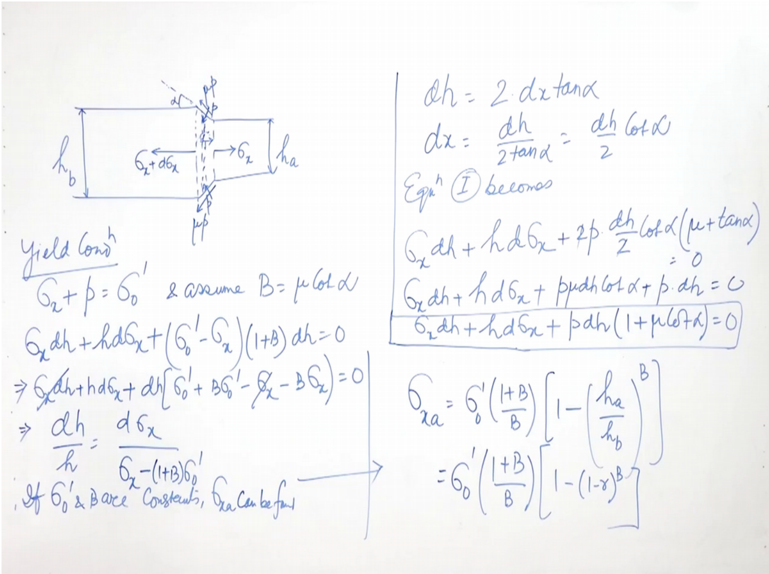

Yield condition will be like sigma x plus p it should be equal to sigma naught prime. So, this we will put it. So, that in place of p we will have the expression sigma naught prime minus sigma x. So, based on that you will have the dx is there and then you have already dh term.

So, then you can further integrate it, and the mu cot alpha term that we will replace it with b and assume b as mu cot alpha. So now, if you look at this equation what it becomes. So, you will have sigma x dh plus h of d sigma x plus p is sigma naught prime minus sigma x into 1 plus b into dh this is equal to 0. So, if you further you can expand it so let us go further now what will it become sigma x dh plus hd sigma x plus dh into sigma naught prime plus b sigma naught prime minus sigma x minus b of sigma x equal to 0. So, sigma x dh and minus of sigma x dh that will be cancelled from here. So, what we see is h d sigma x plus dh into this will be equal to 0. So, that is what we are getting from here.

So, further you can write so now, dh and this is h. So, if you look at dh by h so you can have dh upon h that will be equal to d sigma x upon then you will have here sigma naught prime plus b sigma naught prime minus sigma x. So, it will be negative so it will be sigma x and then you will have minus. So, that will be coming here and then 1 plus b of sigma naught prime. So, that is what you get once you do that the terminologies if you try to rearrange. So, this will be h will be going to this side and this being plus it will be minus.

So, this minus of this will be sigma x minus sigma naught prime multiplied by 1 plus b. So, this way you will have this term now this can be integrated. So, you will have this term can be integrated from ha to hb you we will have d sigma x. So, that that can be further you know integrated. Now in that if you try to so here what we see is that the b and sigma naught prime if these are constants then you can have the you know calculation of draw stress. So, if sigma naught prime and b are constants, sigma x a that is draw stress which is this is the sigma x a the draw stress can be you know computed. So, can be found now how it can be found.

So, you will have the expression from here, now if you try to see that what will be the expression. So, basically sigma x a will be equal to sigma naught prime into 1 plus b by b and then you will have 1 minus ha by hb raised to the power b. So, that will be sigma x a that will be sigma naught prime into 1 plus b by b. So, sigma naught prime into 1 plus b by b and then you will have 1 minus ha by hb and raised to the power b. So, it will be this is what you get because in both the cases you will have the logarithmic turn it will be basically cancelled. So, you will have the expression for sigma xa directly and these logarithm terms will go away and you will have that term. So, this since that b comes as the you know multiplication that goes as the exponent in that side.

So, you will have sigma xa as 1 plus sigma naught prime multiplied by 1 plus b by b into 1 minus ha by hb and raised to the power b. Now if you look at that fine terminology ha by hb raised to the power b now that can ha by hb can be written as 1 minus r. So, that can be further written as sigma naught prime 1 plus b by b and 1 minus this is 1 minus r raised to the power b. So, basically the ha by hb and we know that you will have the reduction is basically our term is related to the reduction in area. So, because of the constancy of volume this this term becomes 1 minus r. So, you can write it as 1 minus r raised to the power b. So, this is how the expression can be found and if you know this mean value of the now this value if you know the value of sigma naught prime if you know 1 plus b by b you know because b is mu cot alpha.

So, if you know the mu that is the coefficient of friction between the dye and the metal and you know the degree of reduction in that case you can find this value of the draw stress that is sigma xa and that is how you calculate this draw stress value. So, if you have any problem any numerical problem to be solved where the value is given and if the for the flow stress calculation the formula is given in that case, the sigma naught value can be found out the mean value can be found out and from there other things being known you can find the value of the draw stress, now this is the case in the case of drawing operation, now when we talk about the tube drawing process in the case of tube drawing process basically as you know that you have a mandrel.

So, and so you have 2 surfaces where the frictional force is acting one is in between the mandrel and the tubes internal surface and another is in between the external surface of the tube and in between the dye and the dye.

So, you have 2 surfaces. So, in the case of tube drawing you know in that case if you take the plane strain condition.

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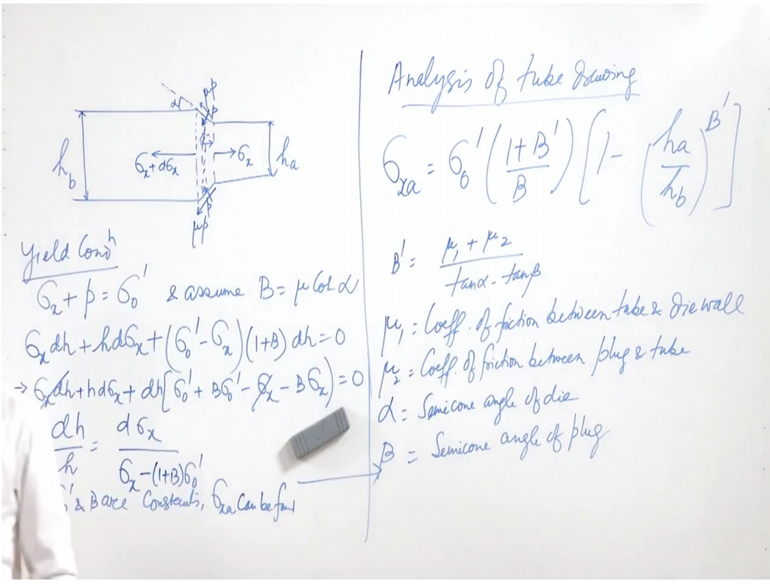

So, analysis of tube drawing. So, the frictional forces are acting or friction is acting at 2 places so in that case the same expression is basically modified and the analysis based on the plane strain condition the analysis is sigma xa will be basically, in that case you have sigma naught prime multiplied by 1 plus b by b and then 1 minus ha by hb raised to the power b now b is mu cot alpha. In this case you will have same thing, but b will be replaced with b prime. So, in that case sigma x a will be sigma naught prime into 1 plus b prime by b and then 1 minus ha by hb raised to the power b prime. So, this is how in the case of tube drawing this expression will be simplified. Now what is b prime? Now b prime is mu 1 plus mu 2 divided by 10 alpha minus 10 beta.

So, b prime will be mu 1 plus mu 2 divided by 10 alpha minus 10 beta. Now what are the mu 1 and what are the mu 2s alpha and beta. So, mu 1 is coefficient of friction between tube and dye wall. So, mu 1 is coefficient of friction between tube and dye wall. Similarly, the mu 2 will be in between the mandrel and the you tube or the plug and the tube whatever you can use so if you use the plug. So, it will be in between the plug and the tube. So, mu 2 will be coefficient of friction between plug and tube. Now you will have the semi dye angles. So, alpha is the semi cone angle of the dye and beta is the semi cone angle of the plug. So, semi cone angle of dye and beta is semi cone angle of plug.

So, this is how the calculation of the draw stress in the case of tube making is computed and you can do the analysis and you can solve the problems because once you have been given the geometry, you have been given the you know outer diameter inner diameter I mean degree of reduction which is required to be computed alpha and beta then finally, the sigma x a can be computers you can solve and the problems based on such you know formulas and get yourself having you know more confidence by solving certain problems from the standard books of you know dieter or other books and get confidence about the problem solving abilities.

Thank you very much.