

**Theory of Production Processes**  
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**Lecture - 35**  
**Problem solving on rolling and forging processes**

Welcome to the lecture on problem solving on rolling and forging processes. So, we discussed about the characteristics of rolling and forging processes and in this lecture we will try to solve few problems based on the rolling process as well as the forging process.

So, starting with the problem on forging process, let us assume a problem where the strip is forged. So, we will write the problem.

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Q1 Initial dimension of Strip: 24 mm x 24 mm x 150 mm  
 forged to final size: 6 mm x 96 mm x 150 mm  
 Coefficient of friction,  $\mu = 0.25$   
 Forging force = ?  
 Average yield stress in tension = 7 N/mm<sup>2</sup>

$K = \frac{6\gamma}{\sqrt{3}} = 4.04 \text{ N/mm}^2$   
 $\alpha_s = \frac{6}{2 \times 0.25} \ln\left(\frac{1}{2 \times 0.25}\right)$   
 $\therefore 8.3 \text{ mm}$

$p_1 = 8.08 e^{0.027x}$   
 $F = 2 \int_0^{8.3} p_1 dx = 159.85 \text{ N/mm}$   
 $F_{\text{total}} = 0.732 \times 10^6 \text{ N}$

$p_2 = 8.08 [0.614 + 0.167x] \text{ N/mm} [8.3 \leq x \leq 48]$   
 $F = 2 \left[ \int_0^{8.3} p_1 dx + \int_{8.3}^{48} p_2 dx \right]$   
 $\therefore 3602.5 \text{ N/mm}$   
 Length of strip = 150 mm  
 Total forging force =  $3602.5 \times 150$   
 $= 0.54 \times 10^6 \text{ N}$

Q2 When  $\mu = 0.08$   
 $\alpha_s = \frac{6}{2 \times 0.08} \ln\left(\frac{1}{2 \times 0.08}\right) \approx 68.72 \text{ mm}$

The problem is that we have a strip of dimension, so initial dimension of the strip initial dimension of strip that is 24 mm by 24 mm by 150 mm and then this is forged to final size. So, forged to final size that is 6 mm by 96 mm and by 150 mm. So, this width remains the length remains the constant, width is changing from 24 to 96 and the height is reduced from 24 to 6. So, this is a typical material for that typical material is the lead material.

Now, for that the coefficient of friction, coefficient of friction value between the work piece and the die that is given as 0.25. Also you have to find the forging force, so forging

force is to be found out, forging for maximum forcing force is required to be found out and the average yield stress in tension average yield stress in tension is given as, it is given as 7 Newton per mm square. So, this is the problem statement which is given for a particular forging problem may come like this. So, you have we have just in the last lecture we have done the analysis of forging process and in that we have seen that we have a strip which is forced and you see that you have 2 regions up to the half of the you know width, so we will do the analysis up to 48 mm. So, you have width is that was basically 96 mm. So, you will have half length of 48.

Now, in the 48 you will have certain regions. So, ultimately you have this as 96. So, this is whole is 96 and then in that the half is 48. Now, in that you will have from here to here. So, this will be  $x_s$ . So, you will have here 0. So, it will be  $x_s$  and then you will have this is as 48 half of the 96. Now, this excess needs to be removed found out the process is that you find the value of  $x_s$ ,  $x_s$  we know and we know the expression for  $x_s$ . So, up to this you will have the sliding friction zone and then this zone is the sticking friction zone. So, first job is to find the value of  $x_s$ , if the  $x_s$  value is less than 48 it means that up to that zone the expression for  $p_1$  will work and then you will have the expression for  $p_2$  that is for the sticking zone you have got the expression for  $p_2$ . So, that will work and then ultimately total maximum forging force that will be  $p_1 dx$  plus  $p_2 dx$  and  $dx$  will be changing from 0 to  $x_s$  for the first case and  $x_s$  to 48 will be the second case.

So, finding the  $x_s$  in that case. Now,  $x_s$ , first of all once you have been given the yield stress in tension 7 Newton per meter millimeter square. So, you have to find the value of  $K$  and  $K$  will be  $\sigma_y$  by root 3. So, that will be your 4.04 Newton per mm square. So, using the yielding principle you can find this  $\sigma_y$  will, so  $K$  will be  $\sigma_y$  by root 3 and that will be your, basically this is the shear yield strength value and that will be coming as 4.04 Newton per mm square.

Now, we will try to find the value of  $x_s$ . So,  $x_s$  will be, now if you find the value of  $x_s$  it will be basically  $6 \ln \left( \frac{1}{1 - 0.25} \right)$  and then you will have  $\ln$  of  $1$  by  $2 \mu$ ,  $1$  by  $0.2$  into  $0.25$ . So, we had found that expression for  $x_s$   $h$  by  $2 \mu$   $h$  by  $2 \mu$  into  $\ln$  of  $1$  by  $2 \mu$ . So, this  $h$  becomes 6 mm and then  $1$  by  $2$  into  $\mu$  and  $\ln$  of this and that expression if you find this will be coming as 8.3 mm it means from 0 to 8 point. So, this is becomes 8.3 mm. Now, from 0 to 8.3 mm you will have the expression for  $p_1$ . So,  $p_1 dx$  and that

will be basically integrated from 0 to 8.3 and then you will have  $p^2 dx$  and that will be integrated from 8.3 up to 48. So, for that we will find the forging load.

So, you will have the value of  $p^1$ . So,  $p^1$  will be, the expression for  $p^1$  will be you know 8.08 and  $e$  raised to the power  $0.083x$ . So, this will be Newton per mm square and  $x$  is varying from 0 to 8.3. Now, how this comes because we know the expression for this  $p^1$ ,  $p^1$  is nothing but you will have you have got the expression for  $p^1$  as  $2K e$  raised to the power  $2\nu x$  by  $h$ . So,  $p^1$  is  $2K e$  raised to the power  $2\nu x$  by  $h$ . So, 2 into  $K$ ,  $K$  is 4.04, so 2 into 4.04 that is 8.08 and then  $2\nu x$  by  $h$ , 2 into  $\nu$  that is 0.25, 0.5 into  $x$  by  $h$ ,  $h$  is basically 6. So, that is how it becomes  $0.083x$ .

Next you have the expression for  $p^2$  and for expression for  $p^2$  we have already calculated the value for  $p^2$  and  $p^2$  is nothing but  $2K$  multiplied by  $1$  by  $2\nu$  into  $1$  minus  $\ln$  of  $1$  by  $2\nu$  plus  $x$  by  $h$ . So, that is how you have got to the expression for  $p^2$  and if you find the expression for that. So,  $p^2$  will become as again 2. So,  $2K$ ,  $K$  will be 4.04, so that will be 8.08 and then you will have 0.614 this comes because the term that is  $1$  by  $2\nu$ . So, that way it will come  $x$  by  $h$  this 0.614. So, that will be coming as one 0.167  $x$ . So, we can have that. So, that will be 0.614 and plus 0.167  $x$ , so that you can just verify from the expression. So, it will be  $1$  by  $2\nu$  into  $1$  minus  $\ln$  of  $1$  by  $2\nu$  comes out to be 0.614 and then the term  $x$  by  $h$  comes out to be  $x$  by 6, so that is 0.167  $x$ .

Now, in the first case you will have. So, this is also unit of Newton per mm square. Now, in this case  $x$  is varying from  $x_0$  to 8.3 and here it is varying from 8.3 to 48 because this is the  $2l$ , this is  $l$ . So, from access  $2L$  it is going on. So, you can find finally, the value of  $F$ ,  $F$  will be 2 times then you have  $p^1 dx$ . So,  $p^1 dx$  integral will be from 0 to 8.3 and then you have  $p^2 dx$  it will be integral will be from 8.3 to 48. So, this can be calculated. So, it will be 8.08 and  $e$  raised to the power  $0.083x$ . So, if you and then  $dx$ , it will be same thing divided by 0.083 and then the value will be changing from 0 to 8.3.

Similarly, in this case you will have this linear function. So, in that you can directly get this into 48 minus this plus this into  $x$  square by 2. So, this square by 2 minus this is square by 2. So, this way you can find the values and this value comes out to be 3602.5 Newton per mm. So, this way you get the value of the force per unit length in this case. And if you find the total forging force then this value is to be multiplied with the length. So, the length of a strip is given as 150 mm. So, you have total forging force maximum

forging force, total forging force will be equal to  $3602.5 \times 150$  so that will be  $0.5 \times 4$  into 10 raised to the power 6 Newton. So, this is how you get the value of the total forging force.

Now, this is the case where basically you are getting the 2 regions, one is the sliding region another is the sticking region. Suppose you are given a situation where the coefficient of friction is given to be different, you may be told that the coefficient of friction is different than that. So, if the coefficient of friction is changed in that case what will be the situation? Now, when the coefficient of friction is taken as 0.08, now in the second part of the problem is for that case when question to a when  $\mu$  becomes 0.08. In the earlier case is 0.25. Now, in this case the  $\mu$  is 0.08. So, what will be that situation?

Now, it means we see that the frictional value coefficient of friction value is quite less as compared to this. Now, what it leads to? You have to calculate the value of  $x_s$ , now  $x_s$  if you calculate in that case. So, if you calculate the  $x_s$ . Now,  $x_s$  we have already calculated. So, in this case  $x_s$  will be  $6 \times 2$  into this value will change  $6 \times 2$  into 0.08 and then under root 1 by and now,  $\ln$  of 1 by 2 into 0.08. So, this is the expression for  $x_s$  that is  $6 \times$  we have got the expression for  $x_s$ .

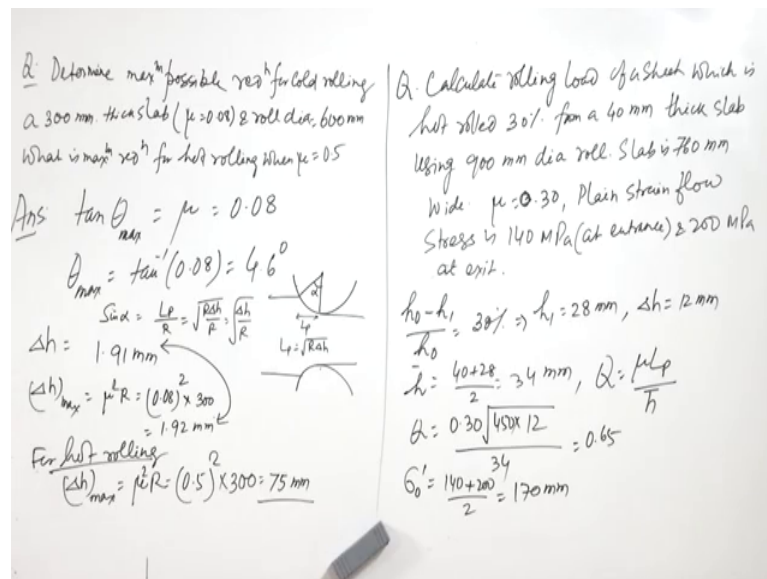
And if you calculate this value this comes out to be 68.72 mm. So, what we see that this sliding region comes out to be 68 mm which is more than its half length also it means the whole zone becomes a zone of sliding friction. So, in that case you will have only this expression working  $p_1$  will be again you will have the values you will have the  $2K$ ,  $K$  is  $e$  raised to the power  $2 \mu x$ . So, that  $\mu$  will change here  $x$  by  $h$  other things are remaining constant and that will be integrated from 0 to 48. So, that will simply tell you 2 times. So, you will have 2 times you know that value. So, you will have 8.08 and then  $e$  raised to the power  $0.027x$  in that case it comes out to me and then you can find further 2 times that value you can calculate the load. So, in that case it comes out to be 1588.5 Newton per mm and the load becomes  $0.23 \times 8$  into 10 to power raised to the power 6 Newton you can verify it by solving this problem. So, this way you can practice this problem.

So, let us see that how you will get it. So, if you further move from here then what you get is  $p_1$ . So, you are in this zone only. So,  $p_1$  will be  $8.08 \times 2K$  is the same then  $e$  raised to the power 0.083. So,  $e$  raised to the power basically  $2 \mu x$  by  $h$ . So, in this case  $\mu$  is

changing  $\mu$  is changing from 0.25 to 0.08. So, it will be 3 times less by 3 itself. So, it will be  $0.027 \times$  because this  $\mu$  earlier it was 0.25, now it is 0.08. So, this will be you know reduced by 3 factor, so this will be this and then this will be basically integrated from 0 to 48. So, what we do is  $F$  will be 2 times integral 0 to 48  $e$  raise to the power. So, at 8.08 into exponential raised to the power  $0.027 \times$  and  $dx$ . So, if you compute this value this comes out to be 1588.5 Newton per mm and if you calculate the force. So, forging force will be. So, this is force per unit length. So, total force will be again this multiplied by 150. So, if you multiply this with 150 it will be 0.238 into 10 raised to the power 6 Newton.

Now, look at these 2 values in the first case when there was sticking friction range zone in that case you got this  $0.54$  into 10 raised power 6 Newton whereas, in the case of no this was both zone sliding as well as sticking zone, but when we have only sliding zone and coefficient of friction is less then we get this force as  $0.238$  into 10 raised to power 6 Newton. So, this way you can solve such problems and similar problems.

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Now, we will deal with another problem based on rolling. So, the problem specifies like this that you have to find the determine maximum possible reduction for cold rolling 300 mm thick slab  $\mu$  is given as 0.08 and roll dia is given as 600 mm. So, what is maximum reduction for hot rolling when  $\mu$  is 0.5? Now, this problem is the one where you have to find the maximum possible reduction for cold rolling we are rolling a 300 mm thick slab

where  $\mu$  is 0.08 and roll diameter is 600 mm. So, for that we just recall the schematic of the rolling process where we have 2 rolls of certain diameter and then the strip is has to go into in between the rolls you have  $\mu$  given.

Now, in that case what we have if you recall our solutions what you see is that you have the limit of  $\theta_{\max}$  which you can achieve and that will be  $\theta_{\max}$  is actually  $\mu$ . So, you have  $\theta_{\max}$  as  $\mu$ . So, this is what the maximum reduction can be achieved and  $\mu$  is given as 0.08. So, we will have, if you the  $\alpha$  is the maximum reduction I mean that is angle of bite which is possible in that case. So, that is given as 0.08. So, if  $\alpha$  is the value that is maximum, so  $\theta_{\max}$  that is  $\alpha$  that will be  $\theta_{\max}$  inverse 0.08. So, that comes out to be approximately 4.6 degrees. So, this is the case of cold rolling.

Now, the thing is further what we want to achieve from here. If you try to see the  $\Delta h$  value, what will be the  $\Delta h$  value? Now,  $\Delta h$  if you try to see the figure here, what we see is you have  $R$  here and suppose this is how it goes. So, in that case you have if it is  $\alpha$   $\sin \alpha$  will be  $L_p$  upon  $R$ . So, this is basically  $L_p$  which is  $R \Delta h$ ,  $L_p$  is under root  $R \Delta h$ . So, what we see is you can find the  $\Delta h$ ,  $\Delta h$  will be. So,  $\sin \alpha$  will be  $L_p$  by  $R$ . So,  $L_p$  is anyway  $R \Delta h$  by  $R$ , so  $\Delta h$  by  $R$  under root. So, if you find  $\Delta h$  you get from there,  $\Delta h$  will be you can find it will be  $\sin \alpha$  and  $\Delta R$ , root  $R$ . So, you can directly get this  $\Delta h$  you because you know this value of  $\alpha$ . So,  $\Delta h$  can be found out as  $\sin^2 \alpha$  and then this value of  $R$  multiplied by that.

So, that comes out if you do the calculation it will be coming out as 1.91 mm. This even can be found out using the other correlation also and the other correlation is that  $\Delta h_{\max}$  if you look at it is  $\mu^2 R$ . So, if you find the  $\mu^2 R$  value so that will be  $\mu^2$  is your 0.08 square and  $R$  is your 300, so it will be 300. So, you come out again to be 1.92 mm. So, this how you see that both are same. So, this is how you can calculate this value of maximum possible reduction  $\Delta x_{\max}$  is nothing but this will be, this is the degree of reduction which you can achieve  $\Delta h_{\max}$ .

Now, the thing is that if it is the cold hot rolling case then in that case again you will have  $\mu^2 R$  is the maximum reduction. So, in case of hot rolling, for hot rolling maximum part is possible reduction will be again  $\Delta h_{\max}$  that will be  $\mu^2 R$ ,

so you will have 0 point. So, now, your  $\mu$  is changed, so this is square into  $R$ . So, your this value comes out to be,  $0.25$  into  $300$  and that comes out to be  $75$  mm. So, this is how you can see that in the case of cold rolling you can achieve maximum  $1.92$  mm in the case of hot rolling you get on the value of  $75$  mm.

The next question is regarding the analysis of forging load. So, here you have to calculate the not forging rolling load. So, you have to calculate a rolling load for a sheet. Now, before the sheet which is hot rod 30 percent from  $440$  mm thick slab using  $900$  mm diameter roll. So, slab is  $760$  mm wide and  $\mu$  is  $0.30$  and plain strain flow stress is given as  $140$  mega Pascal at entrance at entrance and  $200$  mega Pascal at exit.

Now, this is a problem of finding the rolling load that we have done the analysis of rolling process and in that these are the values which are given. So, we will approach this problem what we see is if you see the reduction is of 30 percent. So,  $h_{naught}$  is known so you have to find the  $h_1$  value for finding the average height of this, so  $h_{bar}$  for that you need to find the  $h_1$ . So, basically what you give are given is  $h_{naught}$  minus  $h_1$  by  $h_{naught}$  not you are given as 30 percent. So, from this expression it is given 30 percent. So,  $h_1$  comes out to be  $28$  mm. So, once  $h_1$  comes out to be  $28$  mm, your  $\Delta h$  comes out to be  $12$  mm.

Now, this is that situation. Now, further you have to find the average height. So, average height will be  $40$  plus  $28$  by  $2$ . So, it will be  $34$  mm. Now, in the expression for forging load calculation you have the value coming  $Q$ , so  $Q$  will be  $\mu L_p$  by  $h_{bar}$ . So,  $Q$  is coming as  $\mu L_p$  by  $h_{bar}$ . Now, you know  $\mu$  value,  $L_p$  you know  $L_p$  will be  $R \Delta h$  under root. So,  $L_p$  you know that  $R$  is given  $\Delta h$  you have calculated. So, you can find and  $h_{bar}$  is, mean value of the, mean value of height  $34$  mm. So, from there you can find the value of  $Q$  and that comes out. So, you can find  $Q$  as, this is your  $\mu 0.30$  is the  $\mu$  then  $L_p$  is  $R \Delta h$ ,  $R$  is  $760$  mm by roll, no; rolls diameter is dia roll is  $900$  mm. So,  $R$  is  $450$ ,  $450$  into then  $\Delta h$   $\Delta h$  is  $12$   $L_p$  is  $R \Delta h$  by  $h_{bar}$   $h_{bar}$  is  $34$ . So,  $Q$  comes out to be  $0.65$ .

Now, you have to have calculate this, this rolling load. Now, rolling load for that you have to find  $\sigma_{naught prime}$ . Now,  $\sigma_{naught prime}$  is basically you have to calculate that is the average value. So,  $\sigma_{naught prime}$  will be at the entrance it is  $140$  and at the exit it is  $200$ ,  $140$  plus  $200$  by  $2$ . So, that is coming out to be  $170$  mm. So, in

the expression we have sigma not average that average term comes out to be here otherwise you have 1 by root 3, 2 by root 3 term coming out. So, you can take this term as sigma not prime as 170. Now, you have to put this in the expression. So, p will be. So, after this calculation of sigma naught prime you can just put in the expression for the load and that comes out to be; the expression is you have you might have seen.

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Q. Determine max<sup>n</sup> possible red<sup>n</sup> for cold rolling a 300 mm thick slab ( $\mu = 0.08$ ) & roll dia. 600 mm  
What is max<sup>n</sup> red<sup>n</sup> for hot rolling when  $\mu = 0.5$

Ans  $\tan \theta = \mu = 0.08$   
 $\theta = \tan^{-1}(0.08) = 4.6^\circ$

Q. Calculate rolling load of a sheet which is hot rolled 30% from a 40 mm thick slab using 900 mm dia roll. Slab is 760 mm wide.  $\mu = 0.30$ , Plain strain flow. Stress is 140 MPa (at entrance) & 200 MPa at exit.

$h_0 - h_1 = 30\% \Rightarrow h_1 = 28 \text{ mm}, \Delta h = 12 \text{ mm}$   
 $\frac{h_0}{h_1} = \frac{40}{12} = 3.33$   
 $\alpha = \frac{\mu \sqrt{L_p}}{h} = \frac{0.30 \sqrt{4500 \times 12}}{12} = 0.45$   
 $G_0 = \frac{140 + 200}{2} = 170 \text{ N/mm}^2$

$P = \frac{2 \sqrt{G_0}}{\sqrt{3}} \left[ \frac{1}{\alpha} (e^\alpha - 1) \right] b \sqrt{R \Delta h}$   
 $= \frac{2 \sqrt{170}}{\sqrt{3}} \left[ \frac{1}{0.45} (e^{0.45} - 1) \right] \cdot 0.76 \sqrt{0.45 \times 0.012}$   
 $= 13.4 \text{ MN}$

That expression comes out to be 2 by root 3 sigma naught prime bar and then you have 1 by Q, e raised to the power Q minus 1 and then you have b under root R delta h. This was the expression. So, this term will be replaced by this average value here because you have got the average value of 170. So, you will have 170 and then 1 by Q, Q value is taken as 0.65, then e raised to the power 0.65 minus 1 and then b value is 0.76, 760 mm wide and then multiplied by R, R is 450 mm to 0.45 into delta a 0.012. So, this way if you calculate this value this comes out to be 13.4 mega Newton.

So, this way you calculate the rolling load in such kind of problems you can practice with more and more kind of problems and you can get even the angle of bite maximum reduction rolling load. So, depending upon the coefficient of friction values the values may be calculated.

Thank you very much.