Theory of Production Processes
Dr. Pradeep Kumar Jha
Department of Mechanical Engineering
Indian Institute of Technology, Roorkee

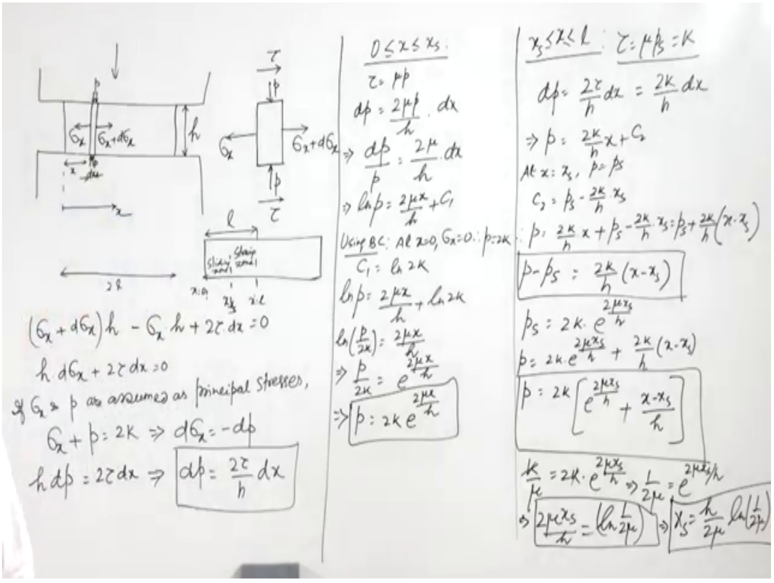
Lecture - 34
Analysis of forging process

Welcome to the lecture on analysis of forging process. So, we discussed about the introduction of forging processes and in that we discussed about the open die forging, close die forging, then different type of forging processes, different equipments which are used in forging that is hammers or press. So, what we have understood by forging process is that you have the die, lower die is there which is normally fixed and you have the top die which is normally going in the downward direction it comes in contact with the work piece and then it tries to compress it.

So, what happens that in this case normally, there will be frictional forces also acting and then once it presses the object in between the dies then, stresses are developed. So, the purpose is to compute these stresses or forces and you will be given the dimensions of the job you will be given the pressure by which you are applying and based on that you need to find what are basically the stresses, how to calculate the load. So, that can be you know found out.

So, what is being done in the case of so, we need to find the forging forces. So, if you look at the schematic of the forging process what we see is that you have basically a top die and you have one bottom die which is normally fixed.

(Refer Slide Time: 02:12).



So, this will be the bottom die and this will be top die which will be moving and there is one element which is there and this element needs to be forced. So, what happens that when you apply the pressure in that case and the stresses will be developed and the material basically flattens this way and here we assume that the thickness is uniform throughout the material and for that we have to assume one element suppose so, this.

So, suppose what we do is, what we see that, you have you have overhanging to flattens this will be top flatten and this will be bottom flatten and that is fixed and then this is the work piece which is basically. So, you are applying the pressure from this side and certainly the pressure will be also applied from this side. So, what happens because of that, you will have the application of pressure on this element.

Now, what we do is we assume that this is x direction. So, suppose we take this as x direction. So, you take this distance as x and we take a strip of dx this this is the dx. Now what happens that once you apply the load now under that load on the surface you have shearing stresses acting now, in this case what we see is you have p acting here. So, you are acting p here and basically the shear stress is acting and you know from for that because of the movement there will be frictional forces acting. Now the direction of the frictional force will be depending upon the position where it is. So, it will be normally towards the middle plane of the element or the whole work piece.

So, in this case if you look at the direction of the frictional force will be in this direction because the middle portion is here. So, this part will be tend to move so, half of the portion in this side they will try to move in this fashion and if the element is on this side then that element will try. So, in on that there are frictional forces because movement will be in this side.

So, frictional forces will move in this direction here similarly in the element on this side will try to move in this fashion because of the force applied. So, the frictional force will apply in that opposite direction. So, in this direction at this place the friction force will be acting in that direction. So, if you try to find the free body diagram in this case so, what we see is you have application of pressure p here, from both the sides and then you have frictional forces acting here that is frictional force..

So, that will be depending upon the position that we will discuss later and then what happens that since it is at x. So, there will be stress acting that will be sigma x here and then stress acting at the distance x plus dx you can take it as at this point as sigma x plus d sigma x. So, what we see is you have x. So, that is sigma x and then here you have sigma x plus d sigma x, now that is what the basically free body diagram is and you have the shear stresses acting here. So, that will be tau this side and tau this side also. So, you have tau on this surface is acting towards the positive x direction because the element is taken in the left half from the centre plane.

So, now in this case you have to assume certain things like you have to assume that the forging force will attain the maximum value at the end of the forging process, similarly you have to assume that the coefficient of friction between the metal material and the die is assumed to be constant. So, as the time progresses then you have to assume that the thickness is very small. So, that is why the variation inst of stress in the in that perpendicular direction will be negligible so, that is another assumption in that case.

So, also you have to find that the length of the strip is quite more and then it is width. So, in this case basically for the dimension given the whole dimension here this is taken as total as 2 l. So, you have to do up to l and from both the sides once you get you can find the total forging load. Now what you see is, that if you are basically and you are taking this is the width basically and you are taking the length as unity. So, in this case your whole width is 2 l and length is taken as unity in that case. So, that remains. So, we will find the force per unit length. In fact, now and this thickness or height you can have that as h.

Now, you have to do the force analysis. So, if you try to do the force analysis what you see is, now here this is h so, h and multiplied by unity. So, if you look at that if you do the force analysis in the x direction, what you see is sigma x plus d sigma x into h. So, if you do per unit length so, area becomes h into 1 and then minus sigma x into h and then it will be plus 2 tau into dx. So, this is the d x length. So, you have tau plus tau, that is 2 tau into dx, and that should be equal to 0. So, if you do the force analysis equilibrium force equilibrium analysis in the x direction you get this equation. So, if you do that this term will cancel you get h into d sigma x plus 2 tau dx will be equal to 0. So, this is what you get from this force balance equation in the x direction. So, you get the expression that is h into d sigma x plus 2 tau dx ok.

Now, further what we see is that you have you can consider this sigma x and p we are if you consider them as sigma x and p as the principal stresses. So, in that case you can apply the conditions of yielding and you can have the relationship between these 2 forces sigma x and p. So, if you consider them as principal stresses that way you can have the co relationship. So, what we get so, once you assume the p and sigma x. So, as the principal stresses in that case you can apply the condition of yielding and you can find that if. So, you can write if sigma x and p are assumed as principal stresses..

So, (Refer Time: 11:24) conditions of yielding you can write sigma x and minus of minus p. So, plus p will be 2 K. So, p is compressive in nature. So, that is why you have taken minus p so, minus of minus p that will be 2 K. So, from this expression you can have the finding here. So, sigma x plus p it will be 2 K. So, what we get is you can get d sigma x equal to minus dp so, because K is constant. So, d sigma x will be basically minus dp. So, further you apply this d sigma x here in place of that you can have minus dp. So, what you get h dp will be equal to 2 tau dx.

Now, that is. So, what you get is, dp will be 2 tau by h dx, this is the expression for dp now what we have to do is you have to assume that where we are basically studying where we are doing the analysis. So, what we discussed is that we are doing the analysis between here up to the midpoint. So, up to the midpoint you have 2 regions there can be 2 regions considered, one will be now the thing is that for deformation to occur there has to be relative motion. So, there has to be sliding and the sliding will occur up to certain distance and then after certain distance that sliding will be stopped so, you will have the sticking zones.

So, you have 2 zones you can specify 2 zones or one zone is for the sliding zone, another zone is the sticking zone, now in that case what you can see that from here to certain distance. So, if you take the 2 zones 0 to x s. So, you will have this as the 2 l. So, if you take this length as l. So, in that we assume that there is in this zone from x equal to 0 to x equal to x s there is sliding occurs and then in this is the zone from x equal to x s to x equal to l. So, this will be your sliding zone and this will be your sticking zone.

Now we have to do the analysis in this zone because when the x is small the sliding and so, for the small value of x sliding has to be incorporated and so, that the required expansion takes place in the material and once we go beyond the certain value of x in that case. So, once we go above this value then basically in that case that zone is the sticking zone.

So, basically there will be no sliding because of increasing frictional stresses. So, at that point what happens that, that value reaches the shear stress value. So, that time the maximum shear stress that the frictional stress which is achieved that becomes equal to the shear yield stress. So, that this is the, these are the 2 zones.

So, we are going to consider these 2 zones and accordingly you will have the pressure variation calculated according to this expression in these 2 zones. Now coming to the first zone the sliding zone now what will happen in the sliding zone. So, in the sliding zone basically this tau can be taken as mu times p. So, coming to the zone where x is from 0 to x s. So, 0 to x s once we are doing in this zone then this is the zone where the, there is sliding.

Now in this zone you can take the tau as mu dp. So, if you take that it will be dp equal to mu p basically tau will be mu into p. So, it will be tau equal to mu p. So, it will be dp equal to 2 mu p divided by h into dx that is what we get from here tau will be mu p. So, that is what we get now we can further write dp by p equal to 2 mu by h into dx. So, you can write from here if you integrate it will be lnp will be 2 mu x by h plus constant. So, there is a constant of integration that is C 1.

Now, this constant of integration can be found by using the boundary condition values. So, this C 1 can be found out. So, using boundary condition at x equal to 0 basically sigma x is 0. So, this from this expression sigma x plus p equal to 2 K at x equal to 0 at this point since sigma x is 0. So, p will be 2 K. So, you will have p as 2 K. So, you can have. So, your C 1 becomes a ln 2 K.

So, what we get is lnp equal to 2 mu x by h plus ln 2 K. So, this ln 2 K will come this side so, ln of p by 2 K will be equal to 2 mu x by h this we can further write p by 2 K will be equal to exponential 2 mu x by h. So, p will be equal to 2 mu 2 K multiplied by e raised to the power 2 mu x by h. So, this is the expression for the pressure expression for this zone that is this zone is known as the sliding zone. So, this will be varying from 0 to x s.

Now we will move to the next zone, that is your sticking zone now in the case of sticking zone what happens. So, in the case of sticking zone if you move to sticking zone now your x will be varying from x s to l. So, in this case you can have the. So, here what will happen you can have tau as mu p s. So, in that case the shear strength value reaches the value of K. So, you have tau as mu of p s. So, suppose p becomes p s in that case. So, that will be reaching to the value of K.

Now, in that case you will have to further use this formula. So, here it will be dp will be now you can find in terms of. So, dp will be 2 tau by h and dx. So, it will be tau is k. So, it will be 2 K by h dx. So, you will get dp as 2 tau by h dx it is. So, this will be equal to 2 into K by h dx. So, if you can further integrate it will be p equal to 2 K by h x plus C 2.

Now, you have to find the constant value of C 2 now for that what we see is for finding the value of you know C 2 now at x equal to basic the x s, p becomes p s. So, we have assumed that x equal to. So, at x equal to x s p becomes p s. So, in that case you can find the C 2. So, C 2 will be. So, this will be p s minus 2 K by h into x s. So, you can further write your expression becomes like this p will be equal to 2 K by h into x plus p s minus 2 K by h into x s that is p s plus 2 K by h into x minus x s. So, your expression becomes equal to p equal to so, you can have the s decide and your expression will become p minus p s will be equal to 2 K by h x minus x s. So, this is what you get from there.

Now, for finding the p s you got this p as the expression here. So, you can find the p s value from this expression because this expression is valid up to the x equal to x s. So, you can have p s value now p s will be equal to p s you can calculate by putting the x equal to x s here in this expression. So, p s will be 2 K into e raised to the power 2 mu access by h. So, you will get this expression p is equal to 2 K e raised to the power 2 mu x s by h. So, you can further write p equal to p s plus this.

So, you will have 2 K e raised to the power 2 mu x s by h plus 2 K by h x minus x s. So, you can have the 2 K common and then you will have e raised to the power 2 mu x s by h plus x minus x s by h this is the expression you get for the reason here in that second one you get p equal to 2 K and e raised to the power 2 mu x s by x plus x minus x s by h.

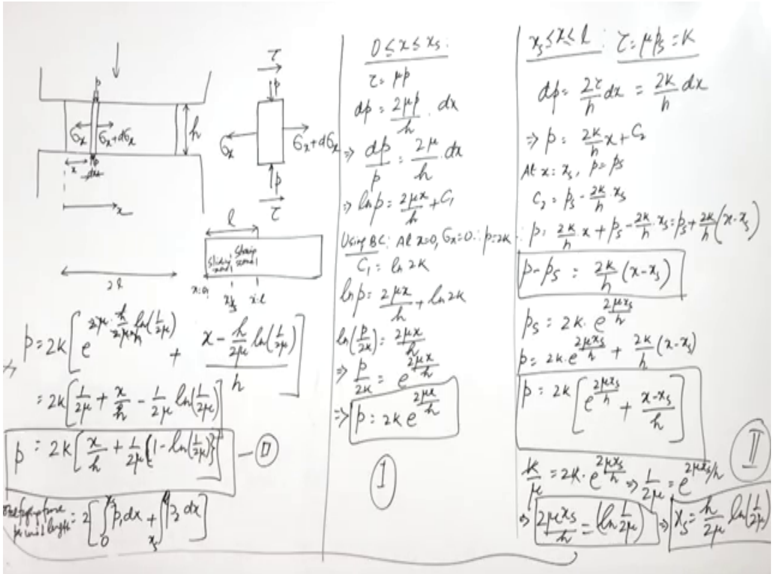
So, from there now at x equal to x s basically we have got the p as p s. So, tau is basically mu p s at x equal to x s we have found the tau as mu p s that is equal to K. So, what you see is the p s will be basically 2 K times and then e raised to the power to mu access by h so, that is what we got. Now again the this mu p s equal to K from there you can find the other values. So, you will have now if you look at this from here now in that mu p s is K. So, basically you can have in the place of p s you can have K by mu you can write. So, if you write here in this case now p s as you can be written as K by mu as equal to 2 K into e raised to the power 2 mu x s by h .

So, what you see is again further it will be going away what you get is 1 by 2 mu. So, you get 1 by 2 mu equal to e raised to the power 2 mu x s by h it means 2 mu x s by h will be equal to ln of 1 by 2 mu. So, 2 mu x s by h will be equal to ln of 1 by 2 mu this is what you also get from here you get this from this you can have the p s.

So, from this expression you are getting K by mu p s will be K by mu. So, p s will be K by mu and that will be equal to this. So, from here you get this and you get 2 mu x s by h it will be equal to ln 1 by 2 mu from here you get x s as x s will be taken as h by 2 mu and ln of 1 by 2 mu. So, x s is basically taken as h by 2 mu into ln of 2 mu.

Now, this x s is to be substituted. So, we reached first here. So, we got first here.

(Refer Slide Time: 27:41)



Then we got this as here, now from here, we will further move and we will find the expression now we have got the expression p equal to 2 K into 2 e raised to the power 2 mu x s by h plus x minus x s by h, now in that we will substitute the expression for x s. So, p we can find as 2 K into e raised to the power 2 mu x s by h.

So, e raised to the power 2 mu into x s, x s is h by 2 mu ln 1 by 2 mu. So, h by 2 mu and ln it is 1 by 2 mu. So, it is 2 mu x s upon h. So, h will come here in the bottom. So, this way you will have these terminologies cutting up and that we will do it later on plus x minus x s by h. So, x minus again x s value we will put in h by 2 mu ln 1 by 2 mu h by 2 mu and ln 1 by 2 mu. So, x minus x s divided by h. So, this is what you get.

So, this can be further simplified 2 K will be here and if you look at this 2 will cancel mu will cancel h will cancel. So, e raised to the power ln 1 by 2 mu that becomes 1 by 2 mu plus then you have x by h then, you have h by 2 mu by h. So, it will be 1 by 2 mu minus 1 by 2 mu ln 1 by 2 mu. So, all together you get 2 K into x by h plus 1 by 2 mu into 1 minus ln 1 by 2 mu. So, this is what the expression is. So, this becomes p as equal to 2 K into this. Now, you got the so, this is the number 2.

So, you have got pressure this term expression as. So, this will be 2 K into x by h plus 1 by 2 mu and plus x by h plus now you can find the force per unit length, if the total force per unit length is to be calculated. So, total forging force per unit length per unit length if you look at it will be basically pdx. So, p is there and then dx you have to calculate from.

So, this, it will be 2 times p 1 dx. So, p 1 is nothing, but here. So, in this dx and this for this the x is varying from 0 to x s up to this zone. So, it will be varying from 0 to x s similarly p 2 dx integral and for this the integral will vary from x s to l. So, this will be x s to l. So, this will tell you the total forging load. So, this way you calculate the total forging force per unit length considering this sliding and sticking friction for such cases.

So, you can do the analysis once more and get the confidence how to calculate these values. So, basically you will be given these values like you will have the, these dimensions known and once you know the point where you have to calculate you have all these values known. So, for a given dimension of products you can find these values.

Thank you very much.