

**Theory of Production Processes**  
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**Lecture-32**

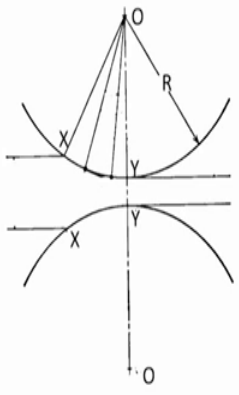
**Analysis of rolling operation: Forces and geometrical relationships**

Welcome to the lecture on analysis of rolling operations. So, in this lecture we are going to discuss about the forces which act in rolling and also the geometrical relationships because as you know this is the schematic of a rolling operation and in this operation as you see strip will be coming of the width here this will be the width initially and then this will be the final width. So, as we see that there are 2 rolls the  $O$  is the centre of the rolls those are the 2 rolls.

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**Introduction**

- A metal sheet entering the roll (of radius  $R$ ) at entrance plane  $XX$  and leaving exit plane  $YY$ .
- Velocity of sheet increases from entrance to exit (if width of sheet is assumed constant, before and after the rolling).
- At neutral point, velocity of roll and velocity of sheet is same.



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So, this line is the line which is meeting the 2 centres then when the, strip of this thickness suppose we take the initial thickness as  $h$  naught. So, the strip of thickness  $h$  naught will be going inside the rolls and then you get if the gap between the rolls is  $h_f$  that is final height. So, certainly the strip which will be going out it will have the final thickness as  $h_f$ . So, the strip width height  $h$  naught is reduced to the height of  $h_f$  so.

That's how the rolling process is carried out and in that you have the 2 rolls and as you see that this basically it is having the entry at the plane  $XX$  and the exit plane is  $YY$ . So,

at this plane the roll is just touching the strip is just touching the rolls and at this plane it is just leaving the rolls. So, that is how this rolling process is schematically presented.

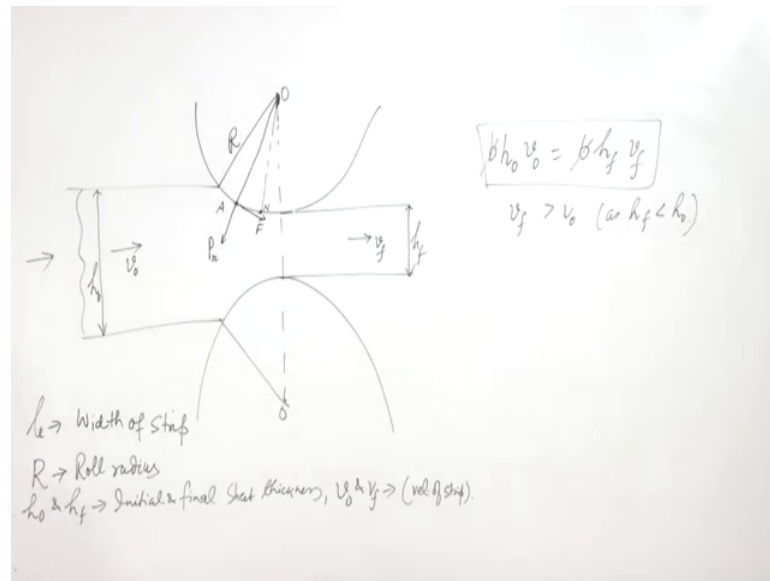
Now, in this case what we see is that when the basically the rolls are moving in this direction here this is the direction of the movement of the roll. So, you know they are moving like this and from the bottom roll is moving like this. So, basically that helps in getting them fed into. So, this way it is moving now what happens that if you look at some position here now at this place you will have 2 kinds of pressure.

Now, what happens that at this point you have a force component, that is going in this direction and another force component is in this direction. So, basically this is the rolling load that the load with which it is basically pushing the material or compressing the material and that is also known as they are all separating force because the same way you have the reaction forces from the strip..

So, that tries to separate the rolls. So, that is why this is also known as rolls separating forces. So, you have this force another force is in this direction tangent to the roll surface and that surface is that you know force is the component of the frictional force. So, that is basically that is the frictional force  $f$  here because there is a movement of the 2 surface now there when they are meeting. So, at that time you have the friction force generated and that goes along the tangent direction.

So, once the rolls touch at this point then this force is generated you have  $f$  here and  $p$  here in this radial direction. So, you have one radial direction one is tangential direction and that radial direction talks about that force which is basically squeezing the material or compressing the material. Now what happens that when it is touching this point in that case basically you have a force and the frictional force basically? So, the frictional force will be acting and that because the frictional force here. So, if you look at this rolling load force which is acting in in this direction it has a component in the negative X direction negative direction this way. So, if you try to understand like this.

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So, if you have suppose you have one roll and you have. So, this will be your and you have another roll like this now what happens suppose you have one strip coming like this. Now what happens that if this is the radius now when so you have this strip of larger thickness initially and then you have the strip of smaller thickness going out now in this case what we have seen that this basically the this point suppose this is a point O. So, again you will have this as O.

So, this angle is known as angle of bite as we have discussed earlier in the lecture that this is known as angle of bite, now the thing is that when it is touching this point or at any point suppose as you come to any point now at this point basically as you saw suppose you see the this point now this point if you take as suppose A. So, in this case what happens, you have a force in the radial direction and that is P radial. So, that will be the rolling load basically that basically is the force which basically is pressing the material.

Now, if you look at this it is it is vertical d it has a vertical component in the downward direction and it well it has a horizontal component in the negative direction. So, this way, so, basically roll if tries to go into it now this roll. So, if suppose you have you are talking about this point at this point when it touches you will have a P acting in P r acting in this direction whose one part is in the vertical downward direction which is the compressive force another component is in the in this direction.

Now this direction will try to inhibit the roll go into the metal the strip go into the in between the roll gap. So, what happens at this point there is another force or at this point if you look at in generalized way you have another force which is acting  $F$ . So, even if you look at this point also. So, you have another force which is acting in the tangential direction and this direction is basically the move opposite to the movement. So, that is why this move direction is the direction of the frictional force and because of this frictional force the roll is able to go into it.

So, what we see is, at this point you have the so, initially the because of the friction itself if the friction is less it will not be able to go. So, you have must have the coefficient of friction at the surface or at the in between the roll and the strip if there is no friction it cannot go inside there will be. So, that it will be slipping here itself it will be rotating. So, coefficient of friction or friction is required for this strip to go in between the rolls.

Now, what we see is that if you look at if so, here  $b$  is the width of the strip if suppose we take  $b$  as width of the strip you take this height initial height as  $h_{naught}$  and this height as you know you can take as  $h_{final}$ . Similarly it will be moving with velocity  $v_{naught}$  here and it will be moving with velocity  $v_f$  here, this has a radius of  $R$  roll has radius  $R$ . So, capital  $R$  is roll radius if you look at the  $h_{naught}$  and  $h_f$  initial and final sheet thickness sheet thickness or height you can say. So, this is initial or final means before rolling and after rolling so, that is your  $h_{naught}$  n  $h_f$ .

Similarly, you have  $v_{naught}$  and  $v_f$ . So, that is the velocity of the strip before and after the rolling. So, what we see is that in this place what we see that when the strip is touching here because of the frictional forces it is able to go into it. Now if the  $b$  is constant suppose the width of the you know strip is remaining constant. So, basically you are and since the constancy of volume will be applied in case of plastic deformation, in that case because of that the element which is vertical which is  $v$  in the vertical shape itself in that case the this condition will be applied  $b h_{naught} v_{naught}$  as  $b$  of  $h_f v_f$ . So, this way this condition needs to be satisfied.

Now,  $b$  basically  $b$  we have assumed as constant in that case now in that case that using of constant using constancy volume constancy of volume condition that will be the case. So,  $b$  will cancel what you see is  $h_{naught} v_{naught}$  equal to  $h_f v_f$ . So, what we see that since  $h_{naught}$  is more than  $h_f$  because in rolling your purpose is to reduce the thickness.


So, since  $h_{naught}$  is reduced to then  $h_f$ . So, it seems  $v_f$  will be more than  $v_{naught}$  as  $h_f$  will be less than  $h_{naught}$ .

It means the now, what is this velocity. So, this is velocity of the strip basically now the thing is that if. So, it means the roll at this point when it is touching and trying to move that time roll has certain velocity now what happens this means that at certain point this will have. So, there is no slipping you will get a point where basically you have the velocity of the strip in between  $v_f$  and  $v_{naught}$  and that will be equal to the velocity of the roll and this point that point will be basically somewhere here and this is known as a neutral point.

So, those that point where the strip velocity and the roll velocity are equal that is known as the, you know neutral point and basically it is a plane. So, you can we call it as a point, but it is a plane where basically the velocity of roll and the velocity of sheet both are taken as constant.

Now, what will be the roll pressure?

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**Roll pressure**

- Rolling load is the force with which rolls press against the metal. (it is also called roll separating force).
- Specific roll pressure is roll pressure divided by contact area.
- Roll contact area is product of width of rolls and projected length of arc of contact.

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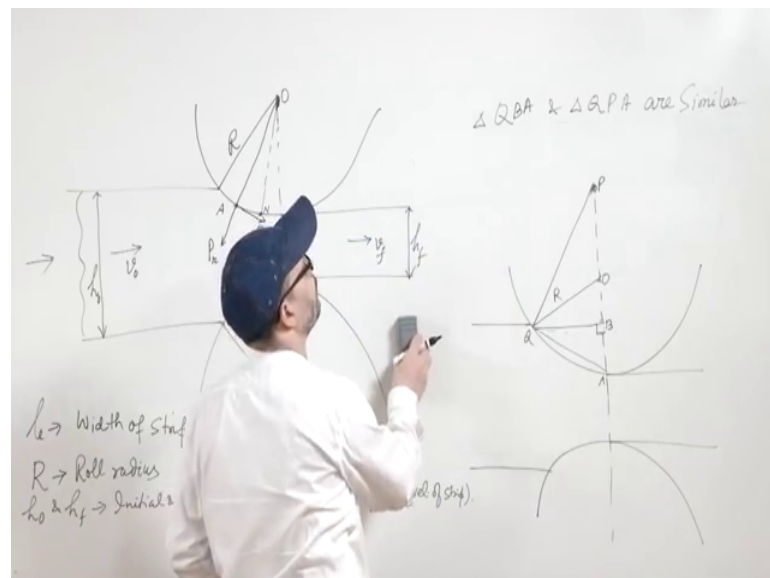
So, if you try to see the value of the roll pressure before that we have to see that how what you have to find now, before that you must know that what will be the roll pressure basically now there are terms like specific roll pressure. So, specific roll pressure means roll pressure divided by contact area. So, the contact area this is the projected contract

contact this projected length of the arc L P. So, this length we call it as L P. So, thus this length is basically the length L P.

Now what happens that in this case what we see that, you have the width of this strip is the one and this is the, this is the area this is the cord area arc area. So, this will be the arc this arc basically multiplied by the width that will be the area of contact and this area of contact which is when it is divided by the roll pressure I mean roll load that will give you the roll pressure. So, specific role pressure will be the role pressure divided by contact area.

Now, how to find this contact area? So, for that again we have to see that, how can we find the role contact area? So, for that if suppose what we see is suppose you want to find the roll contact area in that case you have this and this is your.

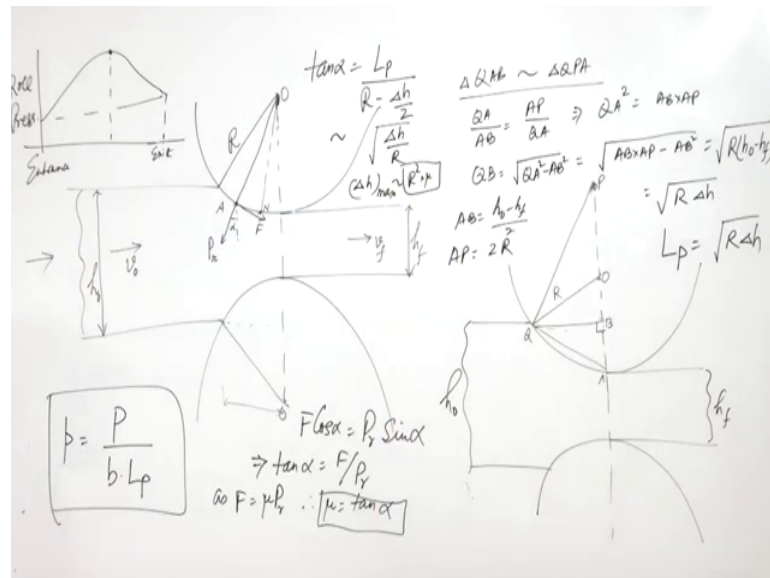
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So, suppose this is the strip and this is going from here. So, suppose this is the final sheet now in this case you have this as the final this as the radius. So, now this is R now what we do is now in this case, you just make it this way and you have to make a horizontal line. So, what we see is, if you further extend it and this is the 2 R. So, in that case we have to have a drawing where if suppose this is. So, just let us rub it now if you try to see that this is the final point which is on the periphery of this role in that case if you try to match it now what we see is that in this, you have to do the nomenclature.

So, suppose this is the P this is Q this is this is R anyway this is suppose A, this is B and this is O, now what we see is that, this is a horizontal line. So, this triangle and the this triangle so like triangle QBA and triangle QPA both are basically similar. So, this is a horizontal line. So, basically this is 90 degree. So, here similarly this is the 90 degree and so, this triangle QBA and triangle QPA are similar.

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So, in this way what you can find by using the similarity condition. So, if you look at this triangle QAB and QPA are similar. So, QAB is similar to triangle QPA. So, what will happen, QA upon AB so, QA is opposite to this angle. So, this angle is 90 again it will be AP divided by and AB is the, this angle, so this angle opposite is same as this. So, that will be QA. So, that is by similarity of the 2 triangles QBA and QPB, QAB and QPA. So, what we see is, QA square is AB into AP so, this is what you get from this.

Now, so, what we see is you can find QB now if you find try to find QB, QB will be equal to QA square minus AB square. So, it will be under root QA square and minus AB square. So, QA square is you can write as AB into AP and then minus AB square. Now if you look at this figure AB and AP. So, AB is nothing, but if you see this is the height  $h$  naught and this is the height  $h$  f. So, AB is nothing, but  $h$  naught minus  $h$  f by 2 and so, this is AB and AP is AP is the 2 times the radius  $2 R$ .

So, neglecting this term basically if you neglect this term because this will be again further small if  $h$  naught minus  $h$  f is very small in that case this term may be neglected

and you can take it as  $2R$  multiplied by  $h_{naught} - h_f$  by 2. So, it will be  $R$  naught  $h_{naught} - R$  multiplied by  $h_{naught} - h_f$ . So, sometimes we also write it as under root  $R \Delta h$ . So, this is how this contact area. So, in that case basically if you take this area this is same you can take if that is very small in that case this can be taken as equal to this or, so in that case this will be your roll contact area.

So, this roll contact area multiplied by the specific roll pressure give you roll load or roll load divided by this roll contact area that gives you basically the you know. So, this this will be the length basically this is the length and then you are multiplying it with  $b$ . So, this is roll contact length and then contact roll contact area will be  $b$  times  $L P$ . So, you write this as length of this projected area of the arc that as  $R \Delta h$ .

Now, if you want to find the roll load or you find want to find a specific roll pressure in that case  $b$  times  $L P$ . So, where that way you have to multiply the pressure and you get the roll load. So, that is how the role load is calculated. So, what we see is that we have seen that you can calculate this roll load or roll specific pressure depending upon the area of contact and area of contact will be basically  $b$  times this roll contact area. So, this is the contact length multiplied by the  $b$  that will be give you giving you that final pressure. So,  $P$  will be roll load divided by  $b$  into  $L P$ . So, this is how you find the role pressure.

Now, further if you look at the roll pressure value, roll pressure value will be basically varying and if you look at the roll pressure value it will vary like this. So, basically this is a role pressure and this is your entrance and this is exit. So, in this case this is maximum is basically achieved near the neutral point. So, that is how that is the significance further of the neutral point.

Further we discussed that you have the 2 kind of forces one force is trying to will have the effect like it will try to try not to allow this strip go into it another force is basically the force that that component of the frictional force which basically you know or the force which is at the here the tangential force that is. So, this force it is component in the  $x$  direction it will try to put it inside the rolls. So, basically what happens you have if you look at the horizontal component here you will have this is the  $P r$ . So, you have this is as  $l \alpha$ .

Now if you look at this it is horizontal component is  $P r \cos \alpha$  and similarly  $F$  has the horizontal component as  $F \sin \alpha$  so,  $P r \cos \alpha$  so, no  $P r \cos \alpha$  will be here.



So, it will be  $P r \sin \alpha$ . So, its horizontal component will be  $P r \sin \alpha$  here and  $F$  will have  $F \cos \alpha$  in this side and both have to balance in fact,  $F \cos \alpha$  has to be more, so that the metal goes into it. So, in the limiting condition this  $P r \sin \alpha$  has to be equal to  $F \cos \alpha$ . So, you have  $F \cos \alpha$  has to be  $P r \sin \alpha$ .

Now, so,  $\tan \alpha$  will be  $F$  by  $P r$  that is what you see from here if you look at this point you this point is suppose you have  $F$  in this direction and you have  $P r$  in this direction. So, you will have  $F$  by  $P r$  as  $\tan \alpha$ , now the  $F$  will be basically  $\mu P r$ . So,  $F$  actually is as  $F$  is  $\mu P r$  because you will have the coefficient of friction at the metal interface. So, you will have  $\mu$  as so,  $\mu$  will be  $F$  by  $P r$ . So, that is why you get  $\mu$  as  $\tan \alpha$ .

So, basically now what happens that one piece cannot be drawn into the roll if the tangent of the contact angle will exceed this coefficient of friction. So, that gives you a limiting case that when the roll will not be going inside I mean the strip will not be going inside the rolls, what that component must exceed so, that this strip will go in between the rolls. So, that is why that is the limiting condition where when the roll cannot enter the you know in between the strip cannot enter in between the rolls, you can also find the, you know  $\Delta h$  maximum and basically if you find that the expression for  $\tan \alpha$ .

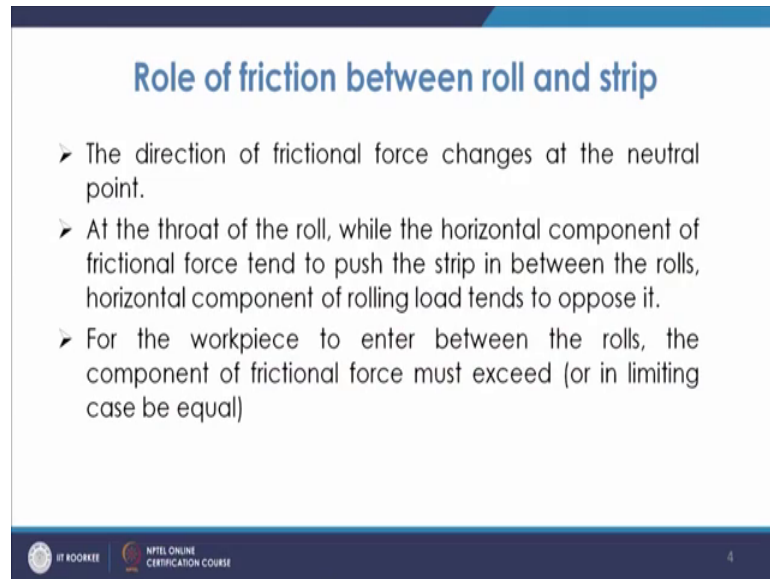
So,  $\tan \alpha$  will be basically the  $L P$  and divided by this  $R$  minus  $\Delta h$  by 2. So, that is what if you look at this  $\tan \alpha$  you see that  $\tan \alpha$  will be you will have  $L P$  divided by  $R$  minus  $\Delta h$  by 2 because you will have this projected length. So,  $\tan \alpha$  if you look at you will have this length that is your  $L P$  and then this this height. So, it will be basically this is your whole is our and this height is  $\Delta h$  by 2 so,  $h$  naught minus  $h f$  divided by 2.

Now, if you look at  $L P$  is  $R \Delta h$  under root and then divided by. So, this can approximately be taken as basically  $L P$  is so, that will be under root  $R \Delta h$ . So, that way  $\Delta h$  by  $R$  will come. So, that will be approximately oblique of  $\Delta h$  by  $R$ . So, if you look at that  $\tan \alpha$  has to be  $\mu$ . So, if you take that value  $\mu^2$  will be  $\Delta h$  by  $R$  and  $\Delta h$  maximum can be that is why you get  $\mu^2$  by  $\mu^2$  into  $R$ . So, since this  $\mu$  this is  $\mu$ . So,  $\Delta h$  maximum this will be basically you know  $\Delta h$  maximum will be  $R$  square into  $\mu$ . So, this you can say how you can find that

maximum of the reduction depending upon the radius  $R$  and the  $\mu$ . So, that can be basically found out so, these are the points.

So, now what we see is rolling load that we have already discussed roll contact area is already discussed friction.

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**Role of friction between roll and strip**

- The direction of frictional force changes at the neutral point.
- At the throat of the roll, while the horizontal component of frictional force tend to push the strip in between the rolls, horizontal component of rolling load tends to oppose it.
- For the workpiece to enter between the rolls, the component of frictional force must exceed (or in limiting case be equal)

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So, direction of friction we have already seen that the direction friction force will be changing at the neutral point throat of the roll when the horizontal component of frictional force tend to push the strip in between the rolls the horizontal component of rolling load tends to oppose it. So, that is what we have already discussed that this rolling loads it is horizontal component is trying to oppose this and for the work piece to enter between the rolls the component of frictional force must exceed or in limiting case be equal.

Now, before that we have further to see that what are the other parameters basically how we calculate the rolling load in case of the rolling that we will see. So, what are the main parameters in the rolling? Basically roll dia is there then you know deformation resistance of the material. So, that is basically influenced by the metallurgy you know and temperature also strain rates. So, these are the parameters in the case of rolling friction between roll and the strip surface is also one important parameter and as we have seen that rolling load will be basically the roll pressure and multiplied by the area of

contact. So, if we neglect the friction in that case. So, your pressure becomes the yield stress of the material.

So, in that case the you know you can find the roll load and roll load will be  $p \cdot b \cdot L$  multiplied. So, that is pressure.

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Neglecting friction:

$$P = \bar{p} \cdot b \cdot L_p$$

$$= \bar{\sigma}_0' \cdot b \cdot \sqrt{R \Delta h}$$

$\bar{\sigma}_0' \rightarrow$  plane strain yield stress value

$$\frac{\bar{p}}{\bar{\sigma}_0'} = \frac{L}{a} (e^{\alpha} - 1)$$

$$\alpha = \frac{\mu L_p}{h} \quad P = \bar{p} b L_p = \frac{2}{\sqrt{3}} \bar{\sigma}_0' \left[ \frac{L}{a} (e^{\alpha} - 1) \right] b \sqrt{R \Delta h}$$

So, in that case as you neglecting the friction so, if you neglect the friction in that case the pressure becomes the yield stress. So, in that case the roll load will be pressure multiplied by  $b$  times  $L \cdot P$  and again as you know you have  $b$  as the  $L \cdot P$  as under root  $R \Delta h$  and  $p$  in that case the you know plane strain yield stress value in that case you can take it as  $\sigma_0$  prime and  $R \Delta h$ .

So, this is plane strength yield stress value. So, that becomes basically in such situations you have to assume you have to take this expression into account because in this case we have as this is a case of plane strain because we have fixed the  $b$ . So, in one direction the strain is negligible it is 0.

So, we are since we have fixed that in that case you know this  $\sigma_0$  prime that will be the plane stress value and you can find this roll load as  $p$  equal to small  $p \cdot b \cdot L \cdot P$  and that will be  $\sigma_0$  prime and  $b$  into that will be  $L \cdot P$  already we have discovered the relationship for this.

Now, even when we have friction in that case we will discuss in future I mean in the coming lectures about the forging. So, in that case when we have friction and then we when we apply the load the compressive load in that friction heat is generated and we can find the load in that case. So, in that case using the mean concept mean deformation pressure so, you can find the value of the mean deformation pressure and mean deformation pressure by  $\sigma_{naught\ prime}$  this is the case when there is friction so, that basically we want to calculate.

Now in this case you can find it will be equal to  $1 + \mu \frac{L}{h}$  raised to the power  $Q - 1$ . So,  $Q$  basically depends upon the contact where the at which contact the dia is in contact with the strip and basically that will be  $\mu \frac{L}{h}$ . So,  $Q$  is  $\mu \frac{L}{h}$ . So, that is mean height and  $L$  as you know that this is the contact length and this will be  $\mu$ . So,  $h$  will be mean thickness between the entry and the exit at  $h$  will be the, that mean thickness between the entry and the exit.

So, what we see is now in this case if you find the  $p$ . So, what we get is  $p$  will be. So, what we have got this expression  $P$  will be basically  $p$  bar and then multiplied by  $b L$ . Now  $p$  bar value we will get from here. So, that will be  $\sigma_{naught\ prime}$  and then you will have in in that case you get the  $2 \sqrt{3}$  and then you get the mean value of this stress. So, this will be  $2 \sqrt{3} \sigma_{naught}$  so, that will be mean value, then further you have these values  $1 + \mu \frac{L}{h}$  raised to the power  $Q - 1$ . So,  $1 + \mu \frac{L}{h}$  raised to the power  $Q - 1$  and then further you have the value this  $b$  into under root  $R \Delta h$ .

So, this is how you find the value of the load in this case this  $2 \sqrt{3}$  factor is coming because rolling is in the plane stress situation and flow stress should be the flow stress in the plane strain. So, that is why if you recall our previous you know calculations previous findings we get this expression and you can find the values because once we know this  $\sigma_{naught\ bar}$  then we can find. So, this is average stress value. So, once you know that then final finally, you can find the, this rolling load. So, this is how the value of this running load can be found out.

Now, if you look at this, what we see is, that here you see certain values that  $p$  is basically dependent upon this  $Q$  and also this  $R \Delta h$ . So, basically what we see is, it is dependent upon the radius or radius of the roll. Now in in general what has been seen is

that roll dia is important in that case you know both roll load and the length of arc both are important for the calculation of the roll load and smaller dia rolls or are basically giving you that way more effective and they are basically stiffened you know and then. So, that is how you apply the backup rolls to do the rolling operations. So, you know there is certainly a minimum gauge seat which can be rolled. So, there is a limit upon those cases and that is how the rolling operation has to be carried out.

So, among the important parameters which influence this rolling load is that basically the rolling load as well as the length of arc both are basically decreased by you know because of the you know roll diameter. So, basically smaller dia rolls are that is why preferred, but they are to be basically they have to be given the rigidity and that is why you put the supporting rolls in those cases.

So, this is about the rolling load calculations you can also study the effect of hot working. So, in case of hot working you need to have you have to study about the effect of strain rate. So, in that case the strain rate that term comes into picture and in case of cold rolling we have seen that you have what is the maximum height of reduction possible depending upon the  $\mu$  and  $R$ . So, this way you can find the calculation of rolling load and other reductions possible or so.

Thank you very much.