Theory of Production Processes
Dr. Pradeep Kumar Jha
Department of Mechanical Engineering
Indian Institute of Technology, Roorkee

Lecture - 29
Mechanics of metal working

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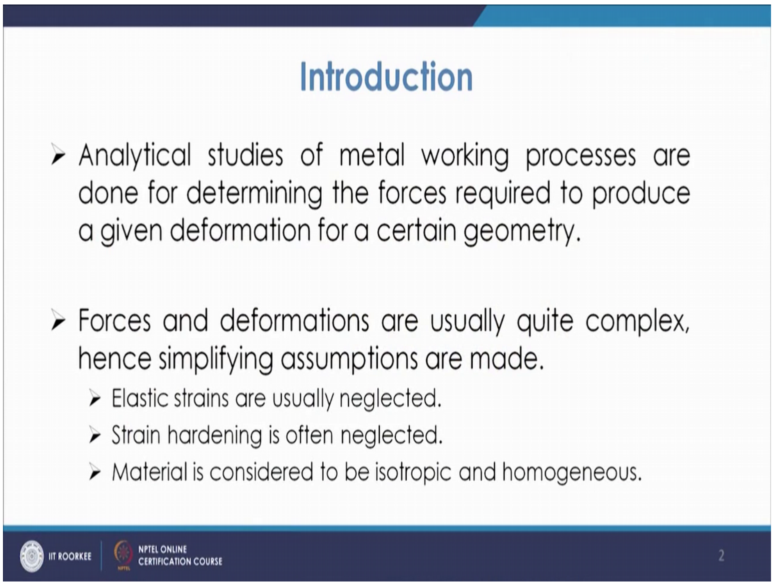

Welcome to the lecture on mechanics of metal working. So, in this lecture, we will study some of the principles based on which they will do the analysis of metal working processes. So, basically analytical studies of metal working processes are required to be done for finding the forces which are required to produce a given deformation for a given geometry. So, as we have discussed that we have different kinds of processes where you have different type in the different way the stresses interact, you have complex state of stresses and the provide the deformations.

So, basically you are required to find the forces which should be able to give a certain deformation in a particular geometry. So, in that basically you will have to have certain assumptions, because if you try to analyze the process in actual, they are quite complex. So, some of the assumptions which normally are made are that the elastic strains are normally neglected. We assume that there is large plastic strain and there is very small elastic strain, so that we normally neglect. So, only you we use the plastic stress strain relationships which are required to be further used, so that is the one.

Then you have the strain hardening is normally neglected, because if the working is done at very high temperature in those cases the strain hardening anyway has no much of the role important role. So, a strain hardening will be neglected. Also, we assume that the material is isotropic as well as homogeneous, so these are the normal assumptions which are basically assumed.

Now, another considerations what we do what we have to keep in mind during the plastic deformation that you have the constancy of volume. So, the constancy of volume condition is already we have seen in that is epsilon 1 plus epsilon 2 plus epsilon 3 should be equal to 0, so that is that talks about the constancy of volume relationship, so that also is one of the condition which will be used while doing the analysis. So, we have to be aware about it also. Then early we have seen that in most of the cases like in forging or rolling or so, most of the times you apply a compressive stress and then you reduce the height of the specimen from certain height suppose h naught to h 1. So, like that you have the change in the height.

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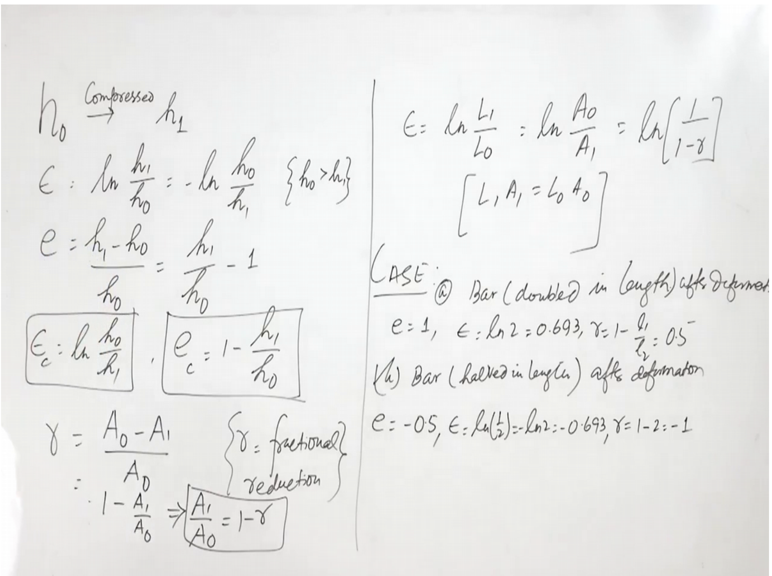

So, if you talk about the strain which is induced so normally if you say that your height is compressed from h naught to h 1. So, if height h naught is compressed to h 1, now this when you have h naught height and you are compressing is to h 1, so height is reduced. In that case, if you try to find the compressive stresses strains, so if you take at the true strain value true strain in compressive. Now, true strain basically if you look at that will be ln of h 1 by h naught. So, now ln of h 1 by h naught will be a negative quantity because this value is less than 1. So, we can write it as minus of ln h naught upon h 1. So, then you know this is how the strain is defined in these cases.

So, what we see that in normally in the normal practice we call this as we take it because if you take in that way, it becomes minus of ln. So, we will see that in the later part in the case of such cases we take it as a positive case, positive values, because in the case of compressive strains you know or compressive cases or in the case of plastic deformation this height always be decreasing. So, always either you have to take the negative value, so we conventionally we take as the positive.

So, let us see how we do it. Now, if you take the engineering strain in these such case, so it will be h 1 minus h naught by h naught. So, if you take at the, so we simply assume as this is the true strain basically when we know do not tell that. Once we put this conventional compress compressive then we take it as positive. Now, if you take the engineering strain, engineering strain will be basically the change in height divided by original height, so that will be h 1 minus h naught by h naught. So, it will be h 1 by h naught minus 1, so that is what we get.

Again h 1 by h naught will be less than 1. So, again we have we see that this value is also negative, this is also negative, because this h naught by h 1 now because here in this case h naught is more than h 1. So, this term will be positive and then whole term will be negative. What we see that in such cases the strains are seen that it is negative. So, in the metal forming processes, its normal practice that you reverse this convention, so we are reversing this convention, we take it as ln of h naught by h 1 like that. So, we take it as the positive. So, if we talk this value as compressive which we had written here then we simply take it as ln of h naught by h 1. So, this is the normal practice in the case of this metal forming analysis where because every time it will be coming as negative, so we take it as the positive.

Similarly, if you take the engineering strain also compressing engineering strain and then also we will define as 1 minus h 1 by h naught. So, then these negative quantities are to be taken as a positive quantity that is what the convention is in the case of the plastic deformation. Now, in case of metalworking, the deformation is also expressed many a times as the reduction in cross section area. So, what is the reduction in cross section area. So, suppose cross section area is a not initially and it is going to A 1. So, sometimes we define this reduction in cross section fractional reduction is defined as A naught minus A 1 upon a 1. So, this is r is fractional reduction in the cross sectional area. So, basically here r is fractional reduction.

So, if you further see now this will be equal to A naught by A 1 minus 1. So, this will be that. So, no, this is basically we have this original area, so we are defining by A naught. So, this will be A naught 1 minus A 1 by A naught. Actually, this is the original area and this is the final area. So, difference is A naught minus A 1 and then you have 1 minus A 1 by A naught. So, now, this 1 minus A 1 by A naught, now we can have we have to see that in the case of metal forming, you have the constancy of volume. So, as the volume will be same, so A 1 l 1 will be equal to A naught L naught. So, A 1 by A naught will be equal to L naught by l 1 something like that. So, it will be, so r will be, now you can see A 1 by A naught if you look at here you can write A 1 by A naught as 1 minus r, this is one of the definition. This value or this relationship will be coming frequently when we discuss about these values.

Now, the thing is that if you look at the true strain value, now true strain value will be ln of basically l 1 by L naught. So, if you take true strain will be ln of L 1 by L naught. Now, if you use this constancy of volume A 1 into L 1 should be A naught into L naught. So, because of that A 1 by A naught or L 1 by L naught will be equal to A naught by A 1. So, if you can further write it as ln A naught by A 1, why so because L 1 A 1 will be equal to L naught A naught, this is because of the constancy of volume. So, you can write this ln of L 1 by L naught as ln of A naught by A 1 and that can be written as so A naught by A 1 will be 1 by 1 minus r.

So, you can further write it as ln of 1 by 1 minus r. So, this is another relationship, epsilon you can write it as ln of 1 by 1 minus r, when we are discussing about or when we typically discuss about this term like fractional reduction in the area. At that time, we come across these situations, where we can express this true strain in the case of either change in length to original length, no, final length by original length and then it is ln. Or, if you know the fractional reduction in the area in that case you know the r value then also you can find the true strain value.

So, suppose we have a case where A bar is basically doubled in length. So, if the case is like the bar is doubled in length; and second case is that bar is halved in length. Now, the bar is doubled in length in that case, the engineering strain will be 1, because the change in length will be. So, if the so case a, is for bar doubled in length. So, for bar double in length, if you find the engineering strain, it will be 1. If you find the true strain, it will be ln 2. So, ln 2 is something close to 0.693. So, this view you get as 0.693. And then if you try to find the r, now if you try to find the r that is you know fractional reduction in the area, so it will be 1 minus A 2 by A 1 and that will be 1 minus L 1 by L 2.

So, if you find 1 minus L 1 by L 2, so in that case L 1 by L 2 is 0.5. So, it is doubled. So, the first length was L and it has gone to 2 L. So, it will be 1 minus L 1 by L 2. So, it will be 0.5. Similarly, if you have A bar which is halved in length, this all these cases after deformation. So, this is after deformation. So, this is also after deformation. Now, for this also we can find the engineering strain, and if you try to find the engineering strain again here, the change in length will be minus 0.5 and original length is 1. So, e will be minus of 0.5.

Similarly, if you take the true strain, it will be ln of 1 by 2. So, minus of ln 2, so that will be again minus of 0.693. So, this will be ln of 1 by 2, so that will be minus of ln 2 that is minus of 0.693. And further again we can use 1 minus L 1 by L 2, and that again further r will be that, so r 1 minus L 1 by L 2 is L 1 by 2. So, that is 1 minus 2, so that becomes minus 1. So, 1 minus 2 that is, so this is how you can compute these values of these strains or the fractional reduction.

Next, we are going to discuss about the approaches how we solve and how we further discuss about the mechanics of metalworking. So, next when we talk about the different you know theories, so at that time, we must be able to make the accurate you know prediction of stresses that is what when we talk about the analysis of metalworking processes or mechanics of metalworking processes we need to know that how to predict the stresses at every point or the strains and velocities at every point. So, basically for that you will have to consider certain set of equations and you will have the equations that represents that how to so you will have first of all equations that needs to be found by examining the type of forces which are occurring in certain directions. Suppose, you do the analysis of forces which acting certain direction like in x direction or in y direction or in z direction, so that way you are making the forces equal and that way you get certain equations.

Then the second is the plastic stress-strain equation. Now, the stress and strains are related to each other that is plastic stress and strain using certain formulas, you have certain rules that is flow rules or plastic stress strain you know curves, not curves basically they are the plastic stress-strain you know expressions so that also is applied. So, you have if you talk about only the plastic rigid plastic, ideal plastic curve in that case you use the Levy-Mises equation that we have already discussed that is applied in the case of such deformations where only the plastic strain is there in the elastic strain is anyway we are assuming that they are negligible.

And the third is the yield criteria. So, what will be the yield criteria at which that can be applied. So, basically you have the theory. So, it will be you have three sets of equations consisting in this theory.

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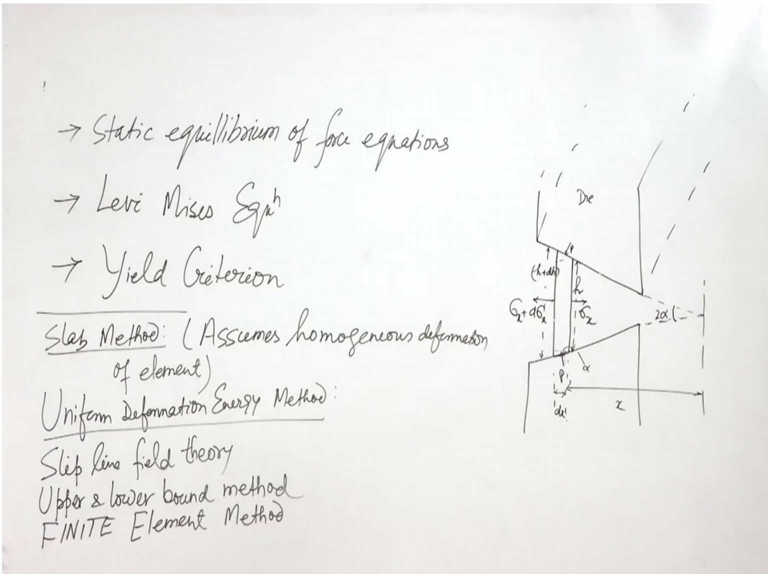

And you have the first is that you have static equilibrium of force. So, whenever we are going to analyze about any metalworking processes, we are going to focus on these aspects. So, first of all we will have the deformation zone making. So, in that, you will make the geometry we will see that where the die is touching the workpiece, what we the forces are applied in which direction the forces are getting applied, so then the stresses are generated at this point. So, this way you will have the. So, then you will apply this static equilibrium of force equations. So, you will apply these forces because if the system is in equilibrium, it means the forces which are acting in suppose plus x l minus x direction, they must be equal in magnitude certainly opposite in direction. So, similarly plus y n minus y n and plus z n minus z like that, so you will have to have that. So, this way you have the static equilibrium of force equation.

Second is that Levi-Mises equation. So, the Levi-Mises equations that talks about the plastic stress strain relationships. And you know they will let you know because this is the case of whole plastic deformation, no elastic deformation. So, you give like that we relate the stress to strain. So, this way you will have again three equations coming up. So, three equation you will get from the first case, three equation will you will get from the second case, and then you will have the yield criterion.

So, the yield criterion you will have. And using this basically you have the nine independent equations, and you have nine independent unknowns also that way you have six basically stress components, and three velocity component that is strain rate component and then you can solve these equations. So, it is not that is not possible, you can solve it using the analytical techniques, but then it becomes you know you have to apply sufficient number of boundary conditions, so that you can solve these equations and try to find the stresses, strains and all that and relationship between them how they vary. So, based on that you can do the analysis, so that is possible with the application of mathematics or so.

Now, most of the you know problems are basically confined a 2D or 3D symmetric type of problems because the equations which are coming they are becoming quite you know complex and hues. So, mathematical complexity is there. So, normally for that either you convert into 2D case or into the 3D symmetric cases. Now, you have different methods of solving these equations and predicting the you know details for finding and the methods are like you have the slab method.

So, one is slab method. What is slab method of analysis? These are the analysis methods based on which we analyzed. So, slab method basically it assumes the homogeneous deformation. So, this assumes homogeneous deformation of element in the deformation zone basically. So, it assumes that in the deformation zone that deformation of the element is basically the homogenous one means there is you know one element which is in rectangular in nature. So, it after the deformation also it will be in rectangular in nature like that it will have whatever cross section, so that way you have the homogeneous deformation. So, this method assumes that and in most of the cases we are going to have the slab method analysis, because this is the you know easy way this is the least complex way of analysis. So, we go for the such methods.

So, again you have the second method. So, second method is the uniform deformation energy method. So, the second method which is known as uniform deformation energy method. Now, this is basically this calculates the average forming stress from the work of plastic deformation. So, from there again you further you know simplify the expressions, and you try to get the you know relationship between stress and strain. So, on that concept, this uniform deformation energy method is used.

Third is the slip line field theory, where basically you know you go for or it permits you for point to point I mean calculation of stress for the plane strain conditions. So, in that slip line field theory, we do that analysis.

Then after that, you will have the upper and lower bound methods. So, basically they use the reasonable stress and you know velocity fields to calculate the bounds within which these actual forming loads must be lying. So, basically you have one upper bound and you have one lower bound. So, you expect that the actual the loads, these values will be actual forming load must lie between these predicted this upper bound and the lower bound values. So, upper and lower bound method.

And the another method which is further used is the finite element method. Now, in the finite element method which is also known as the matrix method. Now, that allows basically large elements of deformation for rigid plastic materials, and you have quite considerable reduction in the you know computation time, so that method is also very much preferred and that is known as finite element method. Now, how there is a, so most of the cases we will whenever we try to deal with certain type of process, we will try to see that how these methods like mostly we will deal with this slab method, and all this as you go down they are getting more and more complex.

So, let us have some idea about this slab method how it works. Now, what I told you that in the case of slab method it is assumed that the deformation in the deformation zone, there is homogeneous deformation of the element. So, if you have a square element, it will be again after deformation, it will be again square or any cross sectional element, so square or rectangular cross sectional element will go into a rectangular cross sectional element like that the analysis goes in this way that you have suppose this is a case of strip drawing problem.

So, suppose you have an element here, and you have one this is meeting at this angle. So, what happens if you will have certain angle here 2 alpha it is taken as to it is having alpha at this. Now, in this case, and you will have this as alpha and this alpha. So, what happens in the case of that strip drawing or so, if you have the this is as a die and this is the strip which is drawn. So, in that case, what we see is it applies the pressure p here and then if this is at distance suppose x, this is at distance x, and you tell this as the distance dx. So, dx is the thickness of the strip.

Now, the thing is that it is this surface will be experiencing a stress of sigma x. So, with distance dx extra, the stress here will be sigma x plus d sigma x. Now, as we discussed that in this case if you look at the first we discussed that we will have few conditions based on which you will have to analyze this situation. So, if you take the example of these such situations, now you can if you see that how you have to analyze the forces in x direction. So, in x direction, you will have sigma x plus d sigma x, and then the force will be applied. So, this suppose this is h, and this is h plus dh this height is basically h plus dh from here to full. So, similarly this is the h.

Now, what you see is that if you feel that it has certain width of w, so this has the width w. Now, this stress which is acting now on this stress is acting on this phase, and then you have a width so here. So, you will have sigma x plus d x into it is acting on h plus dx. So, this way it will have one component this side. Similarly, sigma x into h this side and that will be the force balance equation for the x direction.

Similarly, if you look at the force balance direction in the y direction you will have p and this is alpha. So, this goes, so this is alpha again, this angle is also alpha. So, basically this will be p cos alpha and this direction you will have p sin alpha. So, p sin alpha will be there, and then it will be acting on dx by cos alpha that will be so this line will be since this is d x this will be d x by cos alpha, so that way you will have the all this has this direction. So, this acting is so this sigma x plus dx into h plus d x plus 2 p into then you will have dx by cos alpha into 2 p sin alpha. So, that way you will have the expression and that will be equated to 0, because this is all the forces in the x direction. So, this way you will apply the forces in the x direction.

Similarly, you will have force balance in the y direction. So, force balance in the y direction that will give you, so suppose sigma y is the stress in the y direction. So, that basically gives you sigma y equal to p. So, once you do that because you will have y then again on this surface and then p on this surface, so from both the sides p are basically coming into this direction in the opposite direction. So, basically that results into sigma y equal to p. And then you will use the you know these yielding equations, and from that you will have the expressions. And finally, what you get is that you will have the expression for these stresses in terms of the height. And also in the term of the yield strength of the material or so, so that way that expression is achieved and you try to find the stresses at different heights or source or what is the value of these stresses.

So, this one derivation you can we can see in the further classes that how these derivations are carried out to find the values of the stresses at I mean in such situations and you can solve the problems. Now, in that case, there will be some boundary conditions coming up, and these boundary conditions also are required while solving such equations. So, because you will have the values of this gap, and this gap different, you will have these values. So, what will be the stresses here and here that you know. So, these are the boundary conditions, and they are used further for solving such equations that can be seen in the subsequent lecture classes. So, hope you try to see such problems and try to have the concept about the derivation of such type of problems.

Thank you very much.