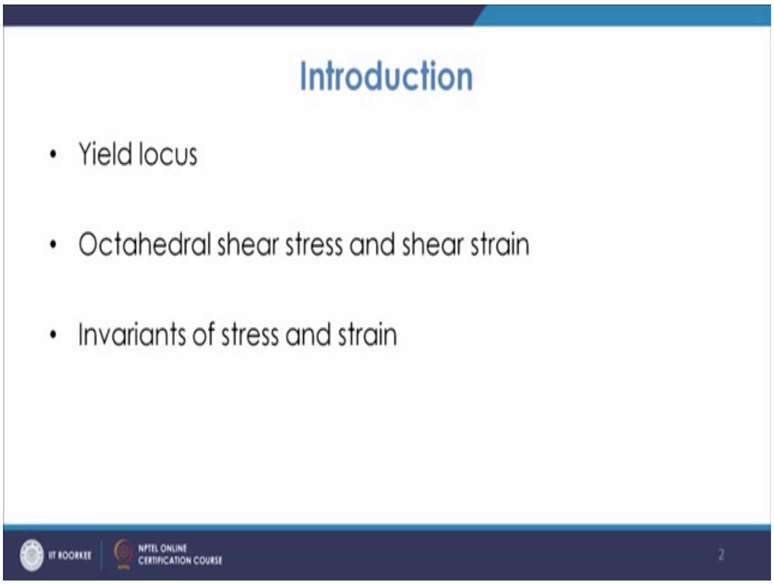
Theory of Production Processes
Dr. Pradeep Kumar Jha
Department of Mechanical Engineering
Indian Institute of Technology, Roorkee

Lecture - 27
Flow rules, Plastic stress strain relationships

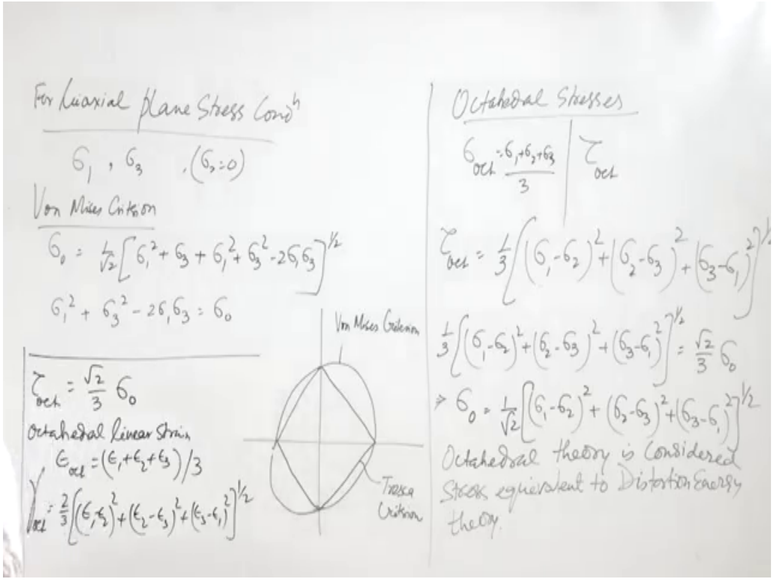
Welcome to the lecture on Flow rules and plastic stress strain relationships. So, now we will discuss about some of the relationships which describe about the ideal plastic flow or the elastoplastic type of a strain relationships. So, before that we will have some introduction about certain terminologies and one of the terminology is; the yield locus. So, what is this yield locus known as?

(Refer Slide Time: 00:53)



Now yield locus will talk about the points; it is the locus of the points. So, the points on this locus will talk about those points which we arrive by these yielding criteria's. So, suppose we are talking about the biaxial plane stress condition.

(Refer Slide Time: 01:34)



So for biaxial plane stress condition, you have sigma 1 and sigma 3, sigma 2 will be taken as 0; in those cases. Now in that case what we get is; so for biaxial plane stress condition, what you get is you have sigma 1 and sigma 3 will be there and sigma 2 will be equal to 0. Now in that case, if you use the Von Mises criteria what you get is; using Von Mises criteria, so you have sigma naught will be 1 by root 2. And then sigma 1 minus sigma 2 square; plus sigma 2 minus sigma 3 square and plus sigma 3 minus sigma 1 square.

So, sigma 1 minus sigma 2 will be sigma 1 square; similarly you will have sigma 2 minus sigma 3; in that you will have sigma 3 square. And plus you will have sigma 3 minus sigma 1 square, so again you will get sigma 1 square plus sigma 3 square minus 2 sigma 1 sigma 3 and based on that you will have half. So, this is the formula which we use in case of the Von Mises criteria.

Now in this case what we can get, if we take the squares on both the sides; you have 2; so, 2 will be cut here. So, you will have 2 sigma 1 square; plus 2 sigma 3 square, so it will be sigma 1 square plus sigma 3 square and then minus 2 sigma 1, sigma 3 will be sigma naught; that is what you get in this case. Now if you look at this equation; this equation is nothing but; this is a equation of an ellipse and whose major semi axis is; root 2 sigma naught and the minor semi axis will be root 2 by 3 sigma naught. So, if you draw the locus; if you find, you will have like that.

So, you will have an ellipse and this part will be little bigger. So, this will be symmetric in that way; then now the thing is that yield locus tells that the point which will be predicted will be falling on this a line, but the thing is that this is the locus, when you use the Von Mises criteria. So, you will have the different combinations of sigma 1 and sigma 3. And if you try to see the conditions by Von Mises; Von Mises will be predicted by these lines.

So, basically these inner lines they will be predicting the Von Mises; I mean maximum shear stress. So, this line will be by the Von Mises criteria and this prediction is because of the Tresca criteria. So, you will have certain error when you predict using the Tresca criteria at some of the points; they are coinciding whereas, so for the different cases you will have sigma 1 by sigma 3 at certain places. Or you may have the different values of sigma 1 and sigma 3; inter relationship. And for that what it means that; this is a yield locus and yield locus using the Tresca criteria will be like this.

So, you can see that you may have differences in the values; if you take the values in between this or in between this, but at these points they are matching; so that is known about the yield locus. The second terminology which we would like to know is about the octahedral shear stress and shear strain. Now what is octahedral shear stress and shear strain?

So, as we know the normal regular octahedron; now in the case of stresses which act on a 3 D octahedron, so you will have a 3 D octahedron and when the stresses are acting on the phases of a 3 D octahedron, then it has the property that the phases of the plane make equal angle with each of the 3 principal axis. So, that is the property of this octahedron; that the phases of the planes make equal angle with the 3 principal directions.

And then that is a value of 54 degree; 44 minute and basically the cosine of that angle is 1 by root 3; so that is very unique. In that case, they are making equal angle with all these principle directions of the stresses. So, because of that this octahedral shear stress or octahedral shear strain; play an important role and basically if you talk about the octahedral shear stress concept, you will see that it is stress equivalent of the Von Mises criteria; so, how it is that we will see.

So, basically that plane which makes the angle with the principal direction of stress of 54 degree; 44 minute whose cosine is 1 by 3. So, basically that is nothing, but the representing the 1, 1, 1 plane and that is close packed plane, which we say; which is there in the case of plastic deformation, where we encounter; so, this is a close packed plane for a shear occurring.

So, that represents that plane; so that is basically the significance of this octahedral shear stresses. Now shear stress, which is the stress which is acting on the regular octahedral phases. Now each phase on that which were; the stress is acting that can be basically resolved into two components. So, that we are talking about the octahedral stresses.

Now, this octahedral stress will further we having the resolved component and one will be the shear stress component, another will be the normal stress component. So, you have the normal octahedral component; so, if you talk about the octahedral stresses. And in that basically when we talk about the octahedral stresses on each phase of a octahedron.

Then we are basically dividing into normal octahedron phases and then shear octahedron phase; shear forces, octahedral shear stress and octahedral normal stress. As we have earlier also seen, that in the case of normal component or the mean component; what we do is, we take the mean of that value. So, you will have; this is sigma 1 plus sigma 2 plus sigma 3 by 3.

So, that is how the normal octahedral you saw that is basically; this is something same as the hydrostatic component of the stress; so, total stress. Then you have the shear stress component; so, octahedral shear stress and that is basically again defined by octahedral shear stress is 1 by 3. And then sigma 1 minus sigma 2 square; plus sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square and then raised to the power 1 by 2.

So, octahedral shear stress is basically defined as sigma 1 minus sigma 2 square; plus sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square and raised to the power 1 by 2. So, this is how this octahedral shear stress is defined; now as we know this octahedral shear stress, this will be responsible for the plastic deformation. So, it is a analogous to the stress deviator because we know that the stress deviator part only is responsible for any kind of plastic deformation. Now the thing is that it is assumed that when this value; so, critical octahedral shear stress that will be required to find the yielding.

So, once this value reaches the critical values that will be critical octahedral shear stress. And in that case you can write that this value has to reach to root 2 by 3 sigma naught. So, if you say 1 by 3 and sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square. And that value has to be do equal to root 2 by 3 sigma naught. So, how this came? Basically that it needs to be known; now again if you look at this situation and if you take the uniaxial tension test case; in that case sigma 1 will be equal to sigma naught and sigma 2 or sigma 3 will be 0.

So, if you put that into this; so, this will be again sigma naught square plus sigma naught square here and that it is under root; so, root 2. So, that is why root 2 by 3 sigma naught; it has come here, so this quantity you get from here. So, if you further solve this what you get is; so, root 2 by 3. So, sigma naught you are getting as 1 by root 2 of sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square and raised to the power half. This is what you get and if you recall the same thing you get in the case of Von Mises criteria.

So, many a times this equation; so, actually this is the same equation which we derived from the distortion energy theory or Von Mises theory. Basically for some Von Mises theory is also known as the distortion energy theory which has been given that name by another scientist; based on this distortion energy concept or strain energy concept.

So, that basically is giving the same result by these two methods. So, basically this octahedral theory is considered as the stress equivalent of the distortion energy theory. So, that is why; we say that octahedral theory is considered stress equivalent to distortion energy theory.

So, using this octahedral shear stress concept; you see that you are coming to the prediction of yield stress; same as that being predicted by the Von Mises criteria; now octahedral shear stress corresponding to the yielding in uniaxial stress; that also can be found out. And that is found out as root 2 by 3 sigma naught, so that is basically coming as again 0.471 sigma naught.

Now if you talk about the octahedral shear strains; then octahedral shear strains are basically again you have the mean strain. So, linear strain that will be epsilon 1 plus epsilon 2 plus epsilon 3 by 3 and then octahedral shear strain is further defined as 2 by 3 of sigma epsilon 1 minus epsilon 2 square plus epsilon 2 minus epsilon 3 square plus epsilon 3 minus epsilon 1 square and raised to the power half.

So, this way we define the octahedral shear stress and shear strain. So, what we found is that in the case of octahedral shear stress, you get as root 2 by 3 sigma naught. And then octahedral strain, when we talk about the linear strain octahedral linear strain that will be defined as epsilon 1 plus epsilon 2 plus epsilon 3 by 3; so, that is known as octahedral linear strain.

Similarly octahedral shear strain; so, that will be octahedral and this you can define as epsilon 1 minus epsilon 2 the square plus epsilon 2 minus epsilon 3 plus epsilon 3 minus epsilon 1; raised to the power half. So, this way we define the octahedral linear strain and octahedral shear strain and they are basically; if you look at these terminologies, you see that these are the invariant functions.

You have the use of all these sigma 1, sigma 2 and sigma 3. So, they are basically used when we try to define these invariant functions; when we try to make their use in the case of plastic stress strain relationships. Another thing which we would like to know is the invariants of stress and strain; so, that is what here it is; invariants. Now many a times; so, it is basically better to simplify the representation of the complex state of stress.

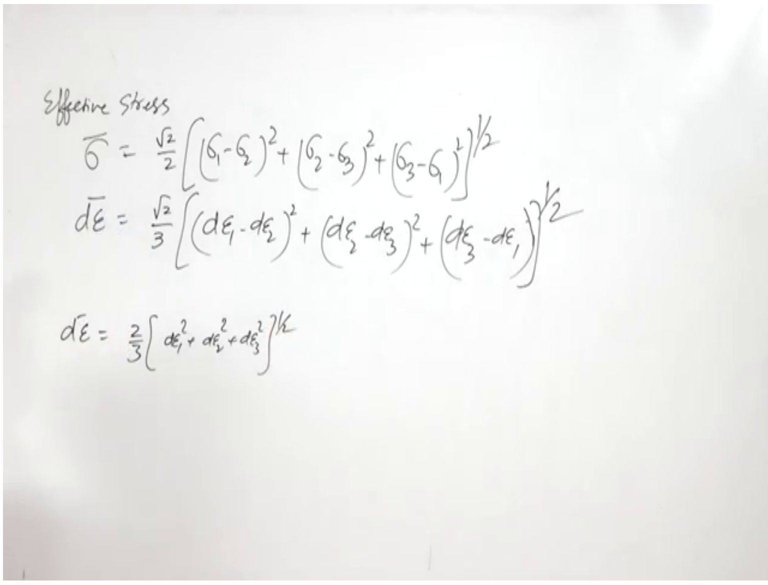
So, we basically if we use the invariant functions of stress and strain then it becomes simpler. And if the plastic stress strain curve is plotted in terms of these invariants of stress and strain, then the same curve approximately would be obtained regardless of the state of stress. So, that is what the advantage of this invariants functions have; that if you use them to plot this, then it will be similar and it will be respective of the state of stress which is used.

So, that is how it is very much important; now what we see that if we use many different kinds of cases and there has been certain findings by some of the researchers like Nadia. And they have shown that these octahedral shear stress and octahedral shear strain, we have seen that; the octahedral shear stress that is 1 by 3 and then 1 by further you have sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square whole raised to the power half.

That component and again the octahedral shear strain component that is 2 by 3 into again sigma 1 minus see; that is epsilon 1 minus epsilon 2 square plus epsilon 2 minus epsilon 3 square plus epsilon 3 minus epsilon 1 squared raised to the power half. Now these 3 quantities has been proved by Nadia; he was a researcher that they are basically the invariant functions in the analysis. Now further, you have another the most frequently used invariant functions which are basically used and one of them is the effective stress and effective strain.

So, effective stress and effective strain are also the most commonly used invariant functions in the case of the plastic deformation. Now what is the effective stress and effective strain? Now effective stress will be; again you will have the stress symbol and it will be a bar. So, you will have sigma bar that is effective stress will be root 2 by 2 and again sigma 1 minus sigma 2.

(Refer Slide Time: 20:53)



So effective stress, this we normally represent it as and this is basically you call it as; so, sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square and then we take its under root; so, that is basically the effective stress.

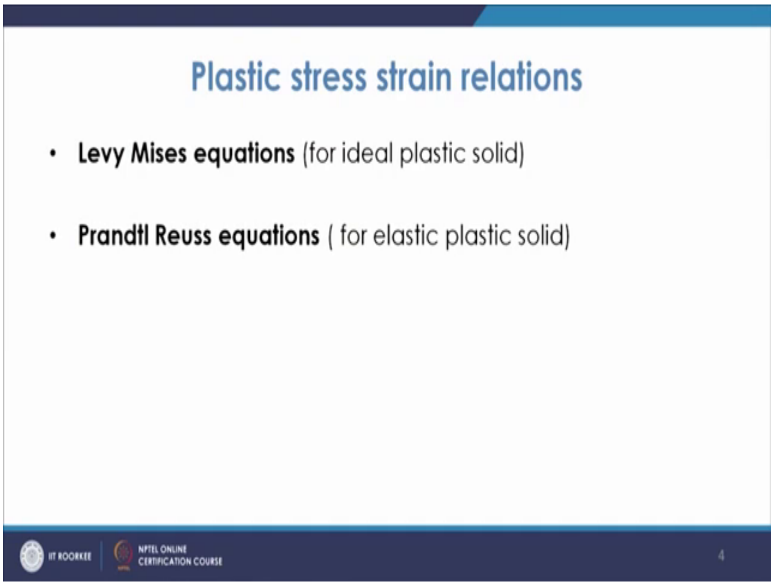
Similarly you take the effective strain and if you take the effective strain, you define it as this. So, it will be root 2 by 3 and then we define as d epsilon 1, d epsilon 2 and then plus d epsilon 2 minus d epsilon 3 plus d epsilon 3 minus d epsilon 1 and then whole raised to the power half. So, this is how the effective stress and effective strain; these are very commonly used; the invariant functions which are used in the plastic deformation studies, when we talk about the equations for plastic stress strain; then we will see their use.

So, these are normally defined like that; now many a times we also define d epsilon. So, if you take that out; so, if you further root 2 by 3, if d epsilon bar will be taken as; so, from here if you take d epsilon 1 square. So, if you expand it then this will be two times d epsilon 1; so, this root 2 comes out. So, it will be 2 by 3 of d epsilon 1 square plus d epsilon 2 square plus d epsilon 3 square raised to the power half.

So, you can further write if you simplify then this root 2 will come out and then you will get d epsilon 1 square plus d epsilon 2 square plus d epsilon 3 square; that also is the simpler explanation to that. Further you can have; epsilon bar will be 2 by 3 of epsilon square plus epsilon 2 square plus epsilon 3 square raised to the power half; that way you can further make it simpler. So, these are the typical invariant functions.

Now, we are going to discuss about the plastic stress strain relationship. So, we know that in the case of plastic deformation; the strain will be depending on the entire history of loading. So, I mean we must know that; it is necessary for us to know the increments of plastic strain; so, through the loading path. So, how the increment is going on? In that case, we have to then we can have the total strain by integration or summation.

(Refer Slide Time: 24:09)



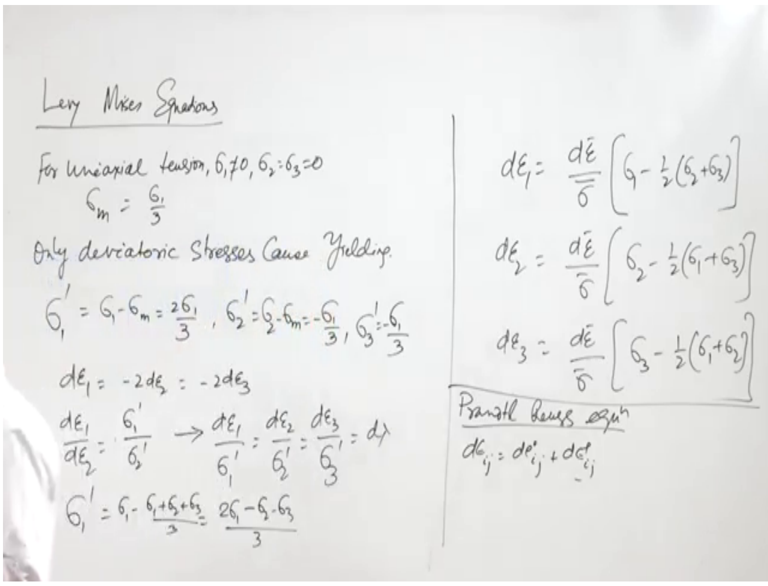
So, when we talk about the plastic stress strain relationships; basically you have two kinds of plastic stress strain relationship; one is the Levy Mises equation; which normally talk about for ideal plastic solid; so, it talks about the large plastic strains.

So, it calculates that the value of stresses in the case of large plastic strains. And then, the another kind of relationship is basically suggested by the Prandtl Reuss; who has given also taken the elastic basically strain component. So, basically total strain is taken as basically the elastic strain plus plastic strain.

So, plastic strain component again will be taken by the use of Levy Mises equation which is for the plastic solid; ideally plastic solid. And then elastic component we will be taking from the back, where we have calculated the value of elastic strain. So, for an elastoplastic type of solid; you have the use of Prandtl Reuss equation.

So, we will see how; what are the relationships and equations for these two equations? So, as we discussed that in the case of this Levy Mises equations; it gives for the ideal plastic solid. And here the elastic strain is considered to be negligible and then these flow rules are known as the Levy Mises equations.

(Refer Slide Time: 25:49)



So, we will discuss about first the Levy Mises equations; so, these equations talk about those cases when there is negligible strain in the elastic region and there is ideally this is a plastic deformation. So, suppose we are taking the example of the uniaxial tension. So, in that case you have basically sigma 1 is not equal to 0 and sigma 2 and sigma 3 is equal to 0.

So, in that case for uniaxial tension; sigma 1 is not equal to 0 and sigma 2, sigma 3 is 0. So, sigma 1 means stress will be; we know that mean stress is sigma 1 plus sigma 2 plus sigma 3 by 3; so, it will be sigma 1 by 3. Now you have you can see that you have deviatoric stresses and you know that the components along the diagonal; what we see is we subtract the mean stress from the total stress component.

So, if you talk about the deviatoric stresses; so, only deviatoric stresses cause yielding. So, that we know because yielding is caused only by the deviatoric components of stresses; not the mean component or hydrostatic component of stresses. Now the deviatoric component of stresses are basically presented by this prime.

So, sigma 1 prime and if you talk about the sigma 1 prime will be what? It will be basically sigma 1 minus sigma m; so, it will be 2 sigma 1 by 3. Similarly you will have sigma 2 prime and that will be basically; this is sigma 2 minus sigma m and sigma 2 is 0. So, it will be minus sigma 1 by 3 and similarly sigma 3 prime will be minus again sigma 1 by 3; so, that is the deviatoric stresses in these cases.

Now using the constancy of volume approaches in the case of plastic; because in the case of plastic deformation you have the constancy of volume that concept is utilized. So, basically according to that d epsilon 1 will be equal to minus 2 of d epsilon 2; equal to minus 2 of d epsilon 3.

So, this is using the constancy of volume relationship. So, what we see is that d epsilon 1 by d epsilon 2. So, if you take d epsilon by 1 by d epsilon 2; it will be basically minus 2 and it will be nothing, but minus of; so, again it will be sigma 1 by sigma 2 prime. So, that you see its ratio if you look at; you will see that you will get and then you have to put it minus.

Because sigma 1 is 2 sigma 1 by 3 and this is; so, you will have minus 1 by 2; is it right? This is minus 2; so, sigma 1 is 2 sigma 1 by 3 and then this is minus sigma; so 2 you will get; so this is ok. So, d sigma 1 by d sigma 2; it will be minus 2; so, you will have minus sigma 1 prime by sigma 2 prime. Now, you can further see that you can generalize it; this sigma 1 by sigma 1 will be equal to d sigma 1 by sigma 1 equal to d sigma 2 by; so, sigma 1 prime will be equal to d sigma 2 by sigma 2 prime. So, you can further write d sigma 1 by sigma 1 prime will be d sigma 2 by sigma 2 prime. it will be d sigma 3 by sigma 3 prim; so, that you can take it as d lambda.

So, this is what you get in the cases of these plastic deformation; now what we see is that at any instant of deformation, basically what we see is at any instant of deformation; the ratio of these increment of the plastic deformation to current deviatoric stresses; they are a constant. So, what we see that the ratio of this strain increment; to this current deviatoric stresses, they are remaining a constant; so, that is what we get from here.

Now again; sigma 1 prime is what? What is sigma 1 prime? So sigma 1 prime; we know it will be 2 sigma 1 minus; sigma 2 minus sigma 3 by 3. So, it will be basically sigma 1 minus sigma 1 plus sigma 2 plus sigma 3 by 3. So, basically it is 2 sigma 1; minus sigma 2 minus sigma 3 that is what we know about the deviatoric stresses. So, if you further use these invariant strain terms, which we had discussed earlier in variant strain terms or effective strain terms; then you can find further.

So, you have invariant stress term for the effective stress; invariant strain term for the effective strain. So, if you use that you will get the correlation or the expression; so, that will be d epsilon 1 will be equal to effective d epsilon 1; divided by effective stress and then you will have the sigma 1 minus half into sigma 1 sigma 2 plus sigma 3.

So, that way you can get the relationship; so, if you talk d; it will be; so, that way and then, so it will be again sigma 1 minus half into sigma 2 plus sigma 3. Similarly, you have d epsilon 2; will be again the effective strain and stress component; sigma 2 minus half into sigma 1 plus sigma 3. And then d epsilon 3 will be again sigma 3 minus half into sigma 1 plus sigma 2.

So, this is basically the equation for this Levy Mises plastic strain; which talks only about the condition of plastic strain. The second equation is about the Prandtl Reuss equation; which talk about the elastoplastic solid. So, here we are treating whenever we have to deal with the problems in which we have the strain also in the elastic region.

So, in that case total strain will be; the strain in elastic region plus strain in plastic region. So, if you take the Prandtl Reuss equation; so, it talks about both the reasons elastic reason, as well as the plastic reason. So, d epsilon i j; total will be d e i j plus d epsilon i j plastic; so, normally; so, you have elastic as well as plastic.

So, this is basically elastic strain and that is the plastic strain. Now the elastic strain, we have seen the strain tensor and there we have studied about this d e i j and d e i j can be taken as equal to 1 plus nu by e; d sigma prime i j. And then plus 1 minus 2 nu by e into d again d sigma k k by 3 and kronecker delta i j; so, that expression will be there for this d e i j.

And similarly you have plastic part; this plastic part we will get from the Levy Mises equation. So, this way you can have the equation for the total plastic strain; so, the main difference is that; you have the additional component here and that is for the elastic part; so that way you can find the equations and you can find the total strain and then you can predict the stresses from there. So, this is how you can practice the; more and more you practice, you will have more and more control over the derivation of these equations.

Thank you very much.