Theory of Production Processes
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Lecture - 26
Yield criteria for ductile materials

Welcome to the lecture on yield criteria for ductile materials. So, in the last lecture and even before that we discussed about different kinds of stresses and strains, then we discussed about mainly we are focused towards the deformation in the elastic region so, in that elastic region you have, we have seen that you have different kinds of constitutive relationships between the different values like different type of constants of the material, then you have the poissons ratio all that you have different types of relationships.

Now, these relationships are required because once you compute the strain then once you know the strain and since in the elastic region the stress is proportional to strain, so you can find the value of stresses. Now, when we come to the plastic deformation. So, as we discussed that in the case of plastic deformation instead of the engineering stress and engineering strain we are confining our studies more practically towards true stress and true strain. So, that basically defines the domain for which you know we will talk today.

So, what are the yield criteria for the ductile materials now, when we try to stretch the material when the material is subjected to stresses then what should be the yield criteria. So, for that there has been certain formulas, certain observations and given by the different types of researcher; different researchers and that we will discuss, before that what we studied was that the yielding that is macroscopic plastic flow, so that is known as yielding.

So, once you have the plastic flow starts macroscopically then we say that it has started the yielding. So, at that time the yield stress value is reached. So, we had seen in the curve that you have a value, yield stress value. So, that is reached then onwards we said that I will yielding has started and that is the plastic deformation also has started and if you recall the true stress true strain curve what we have seen that after that you have the increase in the value of true stress constantly because of the strain hardening taking place.

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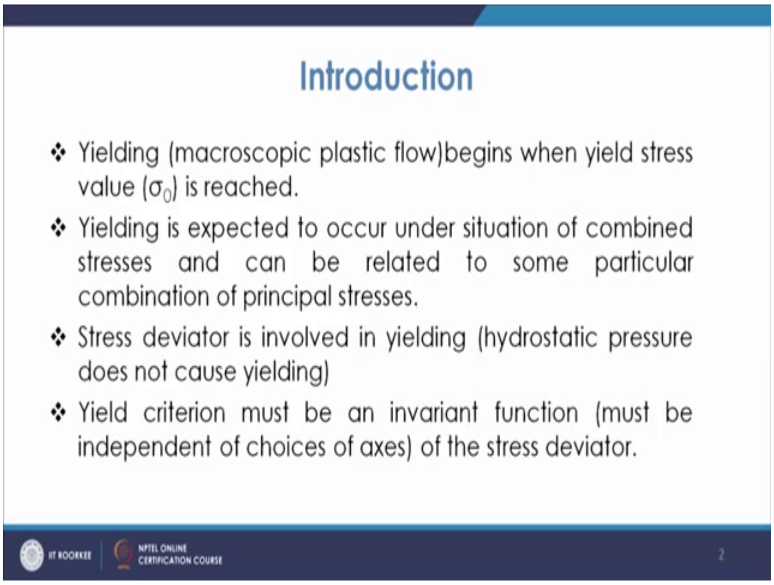

So, anyway that we will discuss; now, yielding is expected to occur under situation of combined stresses. So, you have the material is subjected to different combination of stresses and then, it can be related to some particular combination of principal stresses. So, basically that the meaning is that it must be and the combination of certain type of principal stresses so, basically it should be a one invariant quantity, which will decide that because that will be a generalized case. So, that is how you can say that with some combination of principal stresses you will have you expect that it will be related and it will tell about yielding situations.

Now, what we further got the information in the earlier lectures that you have as a 2 part and the second part that is the deviator part, the stress deviator this part is basically responsible. So, one is yours hydrostatic part, another is the deviator part. So, the hydrostatic part is basically for not for the yielding to take place basically for yielding to take place the deviator part is important. So, the stress deviator will be involved in yielding not the hydrostatic pressure.

So, we had seen that you have one we will be sigma m that is mean that hydrostatic component and then rest is your the deviator part. So, the deviator part every component is the stress shear stress component and even the element along the body diagonal also where we have seen that it will be sigma 1 minus sigma 2 by 2 or sigma 2 minus sigma 3 by 2 something like that, they are also part of the shear stress. So, that way what we see that for yielding to take place these stress deviator element is basically involved. So, that yield takes place.

So, based on that yield criteria also it has also to be understood that the yield criterion must be an invariant function. So, basically the invariant functions are the one, which must be independent of the choices of axis. So, that will be applied for any case and that is why the invariant function of stress deviator. So, one is that the yielding has to take place by the stress deviator; you have 2 types of stresses mean stress and deviator stress. Mean stress does not cause does not involve in yielding. So, the deviator stress is involved in yielding, then it should be one invariant function.

Now, for the stress deviator part you have 3 invariant functions first invariant function was that sigma 1 plus sigma 2 plus sigma 3 is a constant quantity. So, sigma x plus sigma y plus sigma z something like that. So, that will be a constant quantity that was the first stress deviator, we can recall the lecture which we had earlier attended. Then, the second stress deviator that stress deviator must be the one which should be instrumental in telling us that what should be the condition for yielding to take place. So, based on that basically we are going to see the different cases they are in a see the condition under which the yielding will take place.

So, there has been certain methods, certain rules which has been proposed and one of the rule proposed is by Von Mises. So, if you come to the yield criteria the Von Mises has proposed yield criteria and this criteria tells that yielding occurs when second invariant of stress deviator reaches a critical value.

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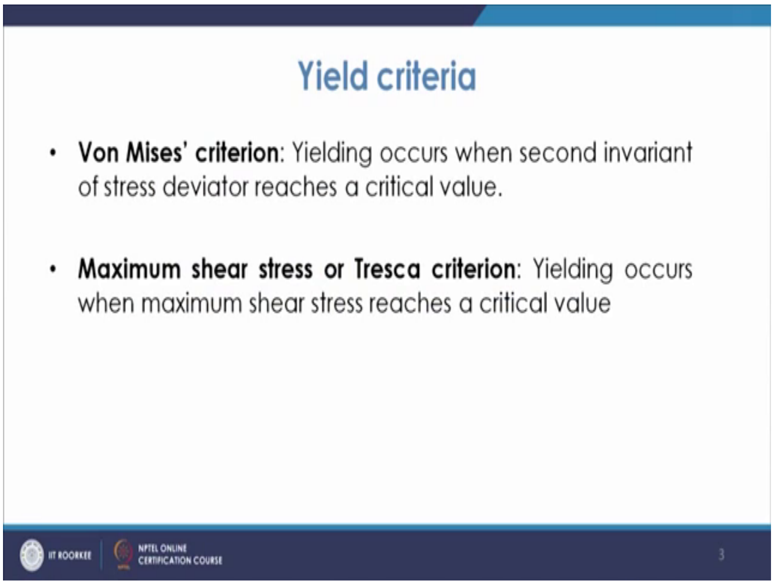

So, basically this second stress or second invariant of stress deviator. So, this was basically encountered when we talked about the stress deviator portion and in that if you try to find the second invariant of stress deviator in that way. So, that has to be reached or to some critical value then only the yielding will occur.

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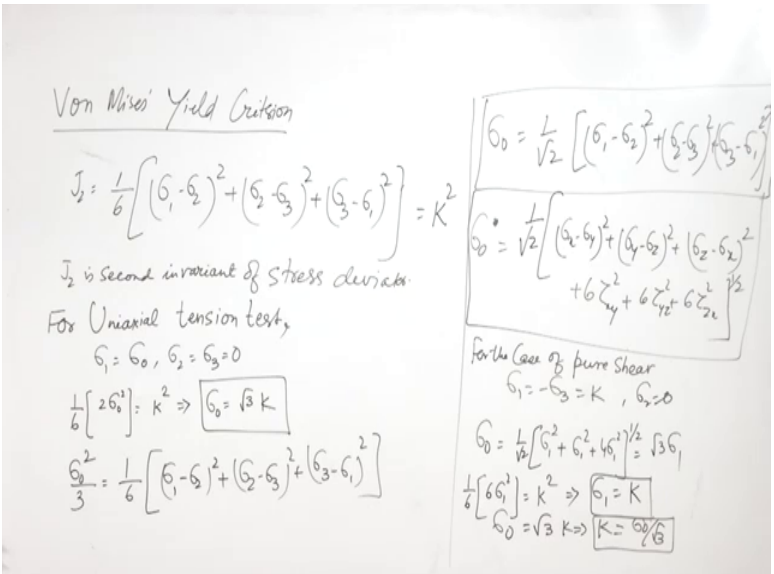

So, if you look at this here Von Mises criteria; so, what it tells that the second invariant of stress tensor must reach a critical value so that yielding will take place. The second invariant of stress tensor is basically so that, if you recall the second invariant first invariant was the sum of the normal stresses or principal stresses you can say sigma 1 and second stress was 1 by 6 and sigma 1 minus sigma 2 square; plus sigma 2 minus sigma 3 square; plus sigma 3 minus sigma 1 square raise to the power half.

So, this was the second invariant of, so J2 is second invariant of stress deviator, yeah. So, it has to reach certain critical value that is K square. So, that once this condition is reached, then the yielding will take place. Now, how to find this constant K? So, suppose you are taking a uniaxial tension case, in which the sigma 1 is basically equal to sigma naught that is yield strength. So, if you take that, so for uniaxial tension test; so, if suppose you are doing one uniaxial tension test it means you are applying the stress in one direction. So, one of the principal stress is there, so it will fail under this stress when this value will reach to the yield strength.

So, in that case sigma 1 will be equal to sigma naught that is your real strength of the material and then sigma 2 and sigma 3 will be 0. So, if you put this into it what you get is sigma 1 will be sigma naught. So, sigma naught square plus again this term will be 0 and this will be again sigma naught square. So, 2 sigma naught square and it is under root and then by 6 will be K square. So, what you see is, so if you look at 1 by 6 of 2 sigma naught square will be K square.

So, which will be sigma naught and is 2 half. So, sigma naught will be equal to square will be, so no here you solve it. So, it will be 1 by 6 into root 2 into sigma naught will be K. So, what you see is, so basically there was a mistake here. So, once we have taken K square this will not come here . So, you will have not this coming. So, once you have this as K square then what you get is, you get sigma naught is root 3 into K. So, this is your stress invariant as being the square term. So, we are equating it to K square.

So, you get sigma naught as root 3 K, so this is what you get from the Von Mises criterion. Now, once you put that sigma naught is as root 3 K, so your K will be sigma naught by root 3. So, once you put that into that equation you look how this equation becomes. So, K will be basically sigma naught by root 3, so your sigma naught square by 3 will be 1 by 6 into sigma 1 minus sigma 2 square; plus sigma 2 minus sigma 3 square; plus sigma 3 minus sigma square, this is what you get the equation for once you put this sigma naught as root 3 K.

So, K will be sigma naught by root 3 and from here you get the generalized equation sigma naught as 1 by root 2. So, because this 3 will go here 3 by 6, so 1 by 2, so it will be 1 by root 2 and this quantity raised to the power half . So, this is what you get by the Von Mises yield criteria, that is what you get once you provide this constant there. So, this is sigma naught yield strength will be 1 by root 2, sigma 1 minus sigma 2 square; plus sigma 2 minus sigma 3 square and plus sigma 3 minus sigma 1 a square whole raised to the power 1 by 2. So, 1 by root 2 is coming to this and, so this value once this value comes more than this sigma naught the in that case the yielding will take place.

So, this is basically involving that the 3 principal stresses in those cases if you try to use the normal and shear stress components in that case it will be sigma x square plus sigma sigma x minus sigma y square plus sigma y minus sigma z square plus sigma z minus sigma x square and plus you have 6 times tau x y plus tau y z taus plus tau z x square.

So, you can further write it in terms of normal axial stresses or shear stresses, so it will be sigma x minus sigma y square; plus sigma y minus sigma z square; plus sigma z minus sigma x square; plus 6 tau x y square; 6 tau y z square; plus 6 tau z x square, so like that and then raised to the power half. So, this is also further this equation can be written as this, so this is the Von Mises yield criterion.

Now, if you take the pure shear case. So, in the case of pure shear sigma 1 will be equal to minus sigma 3 and then that will be equal to K in the case of plane stress condition. If you take sigma 2 as 0 and if it is a case of pure shear it means sigma 1 will be minus sigma 3 that is equal to K. So, in that case if you look at if you say now for the case of pure shear, so sigma 1 will be minus sigma 3 will be K.

So, if you put this condition because this is a case of plane stress condition where sigma 2 is 0. Now, you can further put the values in the expression and what you see is that you will have sigma 1 square plus again sigma 3 will be minus sigma 1, so again sigma 1 square plus 2 sigma 1 square, so in the bracket you will have 4 sigma 1 square. So, you will have 4 sigma 1 square and to the power 1 by 2, so 4 plus 2 6. So, this will be, so if you look at if you put in this equation it will be and you will have sigma 1 square plus again sigma 1 square plus 4 sigma 1 square and raised to the power 1 by 2.

So, what you get is you get 6 sigma 1 square. So, you get root 3 sigma 1, you get this root 3 sigma 1. Now, further if you put this value into this equation what you see is 1 by

6 into, again sigma 1 square plus sigma 1 square plus 4 sigma 1 square. So, 6 sigma 1 square that will be K square. So, what you see is, so in this case 1 by 6 of 6 sigma 1 square is K square, so sigma 1 is K. So, what you see is from here you can write K will be basically, K will be sigma 1 and here this is sigma naught is root 3 sigma 1's sigma naught will be root 3 K.

So, sigma naught will be root 3 K or K equal to sigma naught by root 3. Now, in this case what you see is K will be sigma naught by root 3 and that is what it comes here and what it is tells that basically yield stress in torsion. So, in the earlier cases you got something different sigma 1 was sigma naught and in this case K is and that was equal to K and this case K is sigma naught by root 3. So, it will be I mean lesser, so it means a yield stress in torsion will be lesser than yield stress in tension. So, this criterion tells that yield stress in torsion will be lesser than the yield stress in tension in that case.

Now, we will deal with another case another criteria that is known as maximum shear stress or tresca criterion. So, before that we can also see that if you have a problem, now you can solve what you see in this kind of criterion that it involves the stresses of in the all the 3 directions. So, you are and once you know the values of the sigma x sigma y or sigma z and these shear stresses, then you can find the yield stress values. Now, next is the case of maximum shear stress criterion. Now, the next criteria of yielding is the maximum shear stress or tresca criterion. So, this criterion tells that yielding occurs when maximum shear stress reaches a critical value.

So, we have seen that if you have 3 principal stresses sigma 1, sigma 2 and sigma 3 and sigma 1 is the largest and sigma 3 is the smallest in that case the maximum shear stress developed will be sigma 1 minus sigma 3 by 2. So, the thing is that this because the yielding is based on the shear stresses, so this shear stress is based on this concept that when the maximum shear stress developed will cross a critical values or will reach a critical value then the yielding will occur. So, that way the maximum shear stress is defined as actually, so you have tresca criteria.

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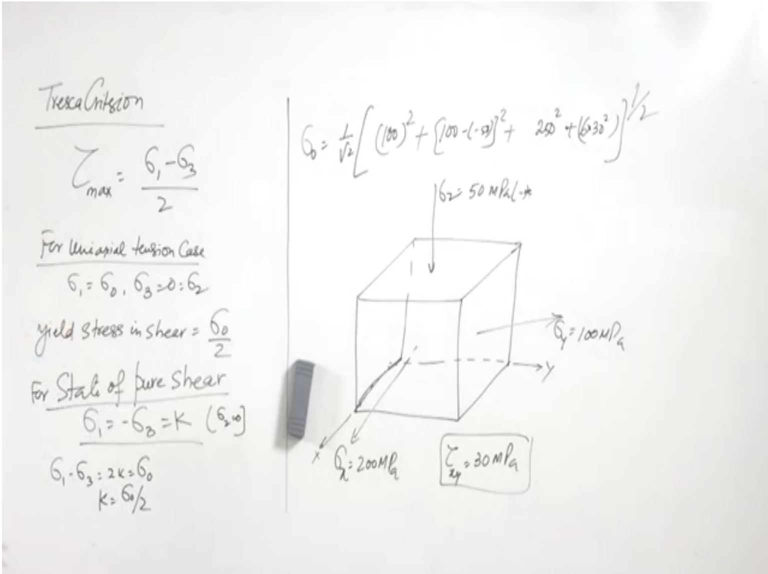

So, tau max will be sigma 1 minus sigma 3 by 2, when your sigma 1 is the algebraically the largest principal stress and sigma 3 is algebraically the smallest principal stress, in those case the maximum shear stress which can be defined is sigma 1 minus sigma 3 by 2. Now, if you take the example of suppose the uniaxial tensile condition uniaxial condition, now in that case sigma 1 will be sigma naught and sigma 2 and sigma 3 will be equal to 0. So, that is what we have assumed that in the case of uniaxial tension you have sigma 1 equal to sigma naught and so, and sigma 3 will be 0 so for uniaxial tension case.

So, sigma 1 is sigma naught and sigma 3 is o. So, that is what in the case of uniaxial tension will look like. So, in that case you will have the maximum shear stress that will be equal to tau 0, I mean sigma 0 by 2. So, yield stress in shear is equal to sigma naught by 2 that is what we get is, from here sigma naught minus 0 by 2. So, that is what we get from here sigma 3 equal to also sigma 2, so from here we directly get it. Now, if we take the state of pure shear. So, in the case of pure shear again sigma 1 will be minus sigma 3 equal to K. So, for the state of pure shear sigma 1 is minus sigma 3 that is equal to K.

Now, if you look at this what will happen in this case. So, in that case sigma 1 minus sigma 3 will be equal to 2 K and that will be equal to sigma naught. So, 2 K by 2, so that will be equal to sigma naught, so you get K equal to sigma naught by 2. So, that is why in this case once you have the state of pure shear you have sigma 2 as 0 in this case and sigma 2 as 0 in the case of plane stress condition. So, once you have that case then sigma 1 minus sigma 3 it will be 2 K. So, that will be 2 K that is equal to sigma naught.

So, that is how you get the values, now K will be basically sigma naught by 2. So, that is what you get in the case of this shear stress criterion. So, this is how you get the different conditions in the case of this yield criteria, in case of tresca and you can write that sigma 1 minus sigma 3 equal to sigma 1 prime minus sigma 3 prime that will be equal to 2 K because being the invariant function, this will be equally valid for any situations any angles or so.

Now, in this case what we see the thing is that it looks the simple of the cases of the 2 yield criteria. However, it has certain disadvantage, you do not know about the second, so you did not involve them, the you did not involve the use of the second principal stress sigma 2. So, you only use the 2 the maximum and the minimum value of these principal stresses. So, that way the Von Mises is having more practical usability and that is preferred approach to be used. Now, even in the Von Mises you use the all the 3 values of the principal stresses.

So, suppose you are given any problem where, suppose you are given that in the cube you are given the different values of sigma x sigma y and sigma z, suppose sigma x is given as 200 mega pascal. So, if suppose you have a cube, so if suppose it comes here. So, if suppose you are given a problem like you have sigma z, so this z axis you have sigma z given here, you have the x axis, you have the y axis. So, you have the values of sigma x sigma y and sigma z is given sigma x is given as 200 mega pascal sigma y is so from here you are given sigma x as 200 mega pascal.

Similarly, sigma y has been given as suppose 100 mega pascal it seems. So, sigma y is given as 100 mega pascal and then, you have also been given tau x y, suppose tau x y is given as 30 mega pascal. So, suppose these are the values which are given to you and you are told to find the value of at which whether it will yield or not sigma 0 value is given, sigma naught value. So, sigma naught value will according to this, you have to find what it comes. So, it you can find it by 1 by root 2, so you will find sigma naught as 1 by root 2 and then you will use sigma x minus sigma y, so this is minus 100, so 100 square plus sigma y minus sigma z.

So, sigma z value is given, sigma z value is given as 50 mega pascal. So, this is 50 mega pascal and this is given as negative, so that is why it will be minus 50. So, again it will be 150 square, 100 minus; minus 50 square then plus again sigma x minus z or sigma z minus sigma x, so 200 plus 50. So, it will be 250 square and then half no. So, it will be sigma x square plus sigma y sigma x minus sigma y square, then you will have sigma y minus sigma z square plus sigma x minus sigma z square plus 6 times tau x y square. So, this is into 30 square and then you will have this good.

So, this way you will have the calculation of the sigma naught value or yield stress value. The stress which is reached using this criterion and then you can say that whether this value is more than the yield strength of the material or not. So, under this condition of stress if the, if we use the Von Mises criteria according to that what is the value this is being reached and then based on that we have to decide that whether it will fail given being the value of the yield strength of the material.

So, it uses all the 3 values of the stresses in such cases. So, you have mainly to practice based on the 4, different type of problems you encounter in such cases.

Thank you very much.