Theory of Production Processes
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Lecture – 25
Theory of plasticity: True stress and true strain, Flow curve

Welcome to the lecture on theory of plasticity. So, in this lecture we are going to discussed about true stress, true strain and the curve between true stress and true strain that is flow curve. So, we discussed about the deformations, we discussed about different as stresses we have stress component being divided into one responsible for the volumetric change, another responsible for the shape change.

So, the shape change basically means there is deformation I mean basically either by plastic or by elastic.

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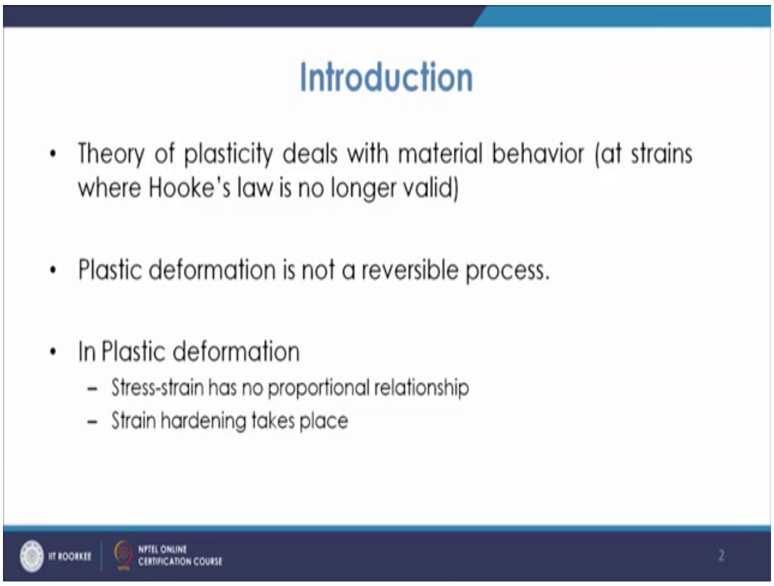

So, now, when we deal with this plastic deformation means or the concept of a plasticity, it means we deal with the strains which are higher and once the strain becomes higher, then in that reason the Hooke's law is no longer valid because Hooke's law will be valid between up to certain strain and then the Hooke's law will no longer be valid, they will not be the proportionality, any proportional relationship between the stress and strain.

So, we need to know the behavior of material when it undergoes the plastic deformation. There are certain characteristics of the plastic deformation and one of the characteristic is that it is a irreversible process, it is not a reversible process in the case of elastic deformation as we have come across the process is reversible. So, you go up to a certain strain and once you unload then the material comes back to it is original position. So, that is what the range of the elastic deformation is or that is what how you define the elastic deformation.

Whereas, in case of plastic deformation the reversibility is not there. So, once you are going into that plastic range then you are not able to come back to the original shape of the material. So, basically it has gone some sub there is come permanent, shape change, so that is what; so, that is why you do not have the validity of these Hooke's law or so.

Again the elastic deformation basically depends on the initial and final state of stress and strain. So, that is what the characteristic of elastic deformation is, whereas in the plastic strain case it will also depend upon the loading path, so how the loading has been done; so, that also decides the plastic strain or so. So, this is the major difference between the elastic and plastic deformation, again the in plastic deformation, so you cannot correlate the stress and strain easily.

So, that relationship is not there just like in the case of elastic deformation you can correlate the stress and strain with the property the modulus of elasticity. So, young's modulus of elasticity. So, it means if at any point you know the strain, then you can find the stress because you know the property of the material that is young's modulus of elasticity that is the that is slope of the stress strain curve. So, that way you can have the value of stress.

Whereas, in the case of plastic strain that is the challenge and you cannot correlate that easily. Further as we see that in the case of plastic deformation, so that is what we discussed that it has no proportional relationship, then the strain hardening is also taking place in the case of plastic deformation. So, that is basically difficult to be accommodated. So, what is the basic underlying principle that is we know that this is strain hardening because of that material gives hardened and the stress required to further deform because will be more and more. So, that is why you have a price in that curve, but then how to incorporate that in the form of law or so, or how to have a relationship for that, that basically is a challenge in the case of plastic deformation.

So, basically you have the use of these plastic deformation when we go for having certain designs or in the case of human buildings or so, we have these considerations that we how much plastic strain we can accommodate. So, based on that we design the structures, we think that how to further design or what should be the strength up to what the deformation can be accepted like that.

So, apart from that when we deal with the plastic deformation what is there that there is a the true stress and true strain curve. So, if you talk about this true stress true strain curve if you see what is the true stress true strain curve? Now, the thing is that in normal case what we see that the in case of engineering stress or a strain we do the calculations based on the original dimension of the specimen.

So, whenever we draw the engineering stress strain curve and the stress strain curve comes like this. So, you have and there you are at this point is the fracture point, so this is how the typical engineering stress strain curve looks like and so, this is engineering stress and this is engineering strain.

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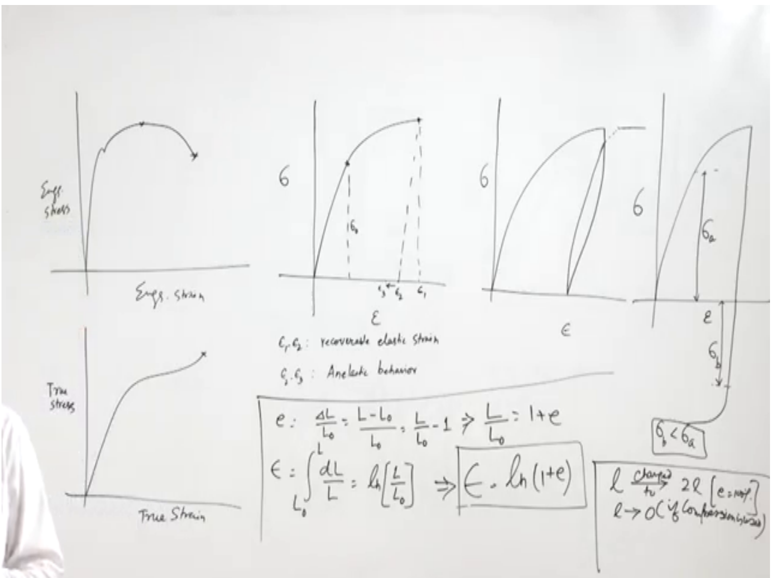

Now, in this case what we see is that the stress is increasing and then after that you will be reaching the maximum point and then further it is decreasing. Now, this curve basically is based on the original area of this specimen. So, if you have the original area, some cross section is there, so always we find the stress based on the load divided by that area. So, that is why after this point it will go on decreasing because after that the, but in actual after this point there is considerable decrease in the cross section although whenever yielding starts from that point itself there will be change in the dimensions of the specimen gauge length will be changing, cross section will be changing.

Now, that is not incorporated when we calculate or when we do the analysis based on the engineering stress strain curve. So, basically what is done is that you have to do that consideration, you have to consider the actual cross section. So, when we do the stress strain curve based on actual cross sectional area then that curve is known as the true stress true strain curve.

So, because as the deformation progresses, the cross sectional area goes on changing. So, actually the in the case of actual this so, it goes like this, it does not decrease basically the true stress true strain curve because this is the reason where this strain hardening takes place and also the cross sectional area goes on increasing, decreasing. So, although the load is required is decreasing, but cross sectional area is also decreasing. So, that is why, but so a stress is basically not decreasing in that sense.

So, this is basically true stress true strain curve . So, this is how, so you have some proportional point and then further it will go on and then at this will be the flexure point where it will fracture. So, that is base the basis of the true stress and true strength. Now, the thing is that when we discuss about the nature of this true stress true strain curve then in that case there are few points which are to be seen. The one thing is that you have to see that up to this point suppose the Hooke's law is valid and after that the Hooke's law is not valid. So, suppose you take this curve, so suppose here up to this point you have, so suppose this is the yield strength, so this will be sigma 0.

Now, the thing is that once we go at any point, at any point if you unload in that case this is your total strain here. Now, what happens once you unload then in that case the strain will be decreased to this point and this epsilon 1 minus epsilon 2 this is because of the elastic property of the material, so it is known as recoverable elastic strain. So, basically the total strain was at this point it was epsilon 1 and then once you release that you come to the 0 stress condition, so you are coming, then at that time you are seeing that this is coming to this point and this strain which is recovered. So, because of the elastic property. So, that is known as recoverable elastic strain. Then, further it is not stopping here, it has further the tendency to move. So, it may come somewhere to epsilon 3.

So, strain will further be reduced and this is basically because of that any elastic behavior. So, basically this is because known as the n elastic behavior. So, this is what is found in the case of these true stress, true strain curves. Now, the thing is that when we deal with such stress strain curve there are few more observations. The observation is another observation is like this which is known as the hysteresis effect; before that we must know that what is hysteresis?

So, a loop is formed basically once you have this and this curve. So, once you have supposed you have this stress strain curve. Now, if suppose you have tried to unload, so in that case it will coming as a bend here and then you if you further load in that case it will be going like this. The, so, it will be further traversing in that fashion and with little bit more strain what was there earlier, you will have again coming to this point. So, it will be following this.

So, this kind of studies is seen, when you further reload the material in the case of these plastic deformation. So, basically this also is neglected, when we will whenever we do the analysis of plastic deformation. So, this is the hysteresis concept, further there is another effect that is known as bauschinger effect.

Now, bauschinger effect tells that if you have this as the stress strain curve, then and if you have suppose further came back to, so you have unloaded and then, you are further going. So, suppose in this case you have this is the sigma a that is in tension. So, you have done the analysis based in tension. So, your access is stress in one particular direction you have unloaded and then further you are reloading.

So, in the case of reloading you will get suppose here, this is as sigma b, this is the yield strength in compression. So, suppose you have done the pulling in tension and then further after coming back to it is normal state again you have done the pulling in the compression side. So, what is seen is that this sigma b is seems to be less than sigma a. So, this effect is known as bauschinger effect. So, basically that effect is also neglected, you assume that whether you go for a tension or compression, the sigma a would be equal to sigma b. So, these are the points you should know, that this is how it is observed in the case of true stress true strain diagram. Now, the thing is that what is true stress? What is true strain? So, true stress if you define, now let us see true strain if you look at the true strength basically whenever we talk about the you know, strain it is changing length by original length.

So, what we do is where we talk about change from one to other, then one value to other values are other well 2 next other values or so. Now, in the case of engineering strain it is e is basically delta L by L naught, this is what you get. So, that is basically if you have final value as L and final length and initial length was L naught. So, this is how the engineering stress is defined. Now, the thing is that when we talk about the engineering true strain, in that case true strain will be basically change in length by original length, but length is basically every time changing.

So, this time you have gone from L naught to L 1, next time L 1 to L 2. So, this time it will be by L naught, but next time it will be from L 1 to L 2. So, in the denominator it will be L 1. So, in the next instant it will be e 2 L 2. So, basically that basically is changing as a variable. So, in the case of this, what we do is we do the integral. So, this L will come because this being a variable it is going on changing and it is from L naught to L.

So, that is why it becomes as ln of L by L naught, now we defined e as L by my by L minus L naught by L naught that is L by L naught minus 1. So, L by L naught is coming as 1 plus e. So, what we see from here, we see from here that this epsilon that is true strain is coming out as ln of 1 plus e.

So, this is the relationship between the true strain. So, that is how the true strain can be defined you can find. So, once you know the engineering strain, you can find the true strain using this formula. How it is advantageous? What is the basically, why this is required basically this true strain concept is required in case of plastic deformation that can be understood by the example, suppose a bar of certain length L is increased to 2 L.

So, suppose for a bar of length changed to 2 L. So, what is the engineering strain? Engineering strain will be change in length by original length. So, change in length is L and original length is L. So, engineering strain will be 100 percent, it is 1. Now, the thing is that if, so this is in tension. So, it means if we try to achieve that in compression, it should be similar. So, if you try to achieve this value in compression it means you will have to compress it to 0 length. So, that does not have any meaning, if you try to further compress; so if you want to have this strain itself, but in the compression side, then by that L should be decreased by L. So, once L is decreased by L it will be 0, so basically it will come to 0, if compression is used. So, which does not have any meaning.

So, basically if you use this as, in this case if you use the concept of true strain what you will see is true strain for this case will be ln of L by L naught, so ln 2. So, true strain will be ln 2 and if you want to achieve the same strain. So, in that case you have to go for minus of that value. So, basically if you compress it to L 2 L by 2 then the same true strain will come with negative value. So, that carries certain meaning that carries meaning. So, that is why this true stress and true strain has more meaning in the case of plastic deformation.

Further, the thing is that this L by L naught is there and in the case of plastic deformation, if you assume that the material is isotropic there is an homogeneous or so, there is no defect in anything. So, in that case there will be constancy of volume; so, volume will be constant and once the volume is constant means L by L naught can be replaced with a naught by a, because if the area of cross section is a at a particular instant and a naught is at initially. So, area of cross section into length should be a conserved quantity that is volume. So, basically you can also take it equal to ln of a by, so a naught by a.

So, that way also you can define the this true strain concept, we can also see that if you take certain incremental steps strain, suppose 1 bar is increased to certain length and you go for 2 or 3 repeats and if you see the final strain, the engineering; final engineering strain will be different you cannot see that it will be addition of all that because suppose a length of 50 millimeter is there, you further change to 55. So, strain is something like 0.1, then further 55 to 60, 60.5. So, again a strain will be point 1, so something like that. So, 1 percent point like that then if you go further, so that way, but if you add them the addition of all these stress will not be equal to total strain, whereas in the case of true strain you will see that the altogether the strain will be same as the sum of all these incremental strains.

So, that is how in the case of these strains plastic strains these true stress true strain has basically a practically more significant meaning. Now, let us know something about other terminologies. So, basically this true stress true strain curve is known as the flow curve.

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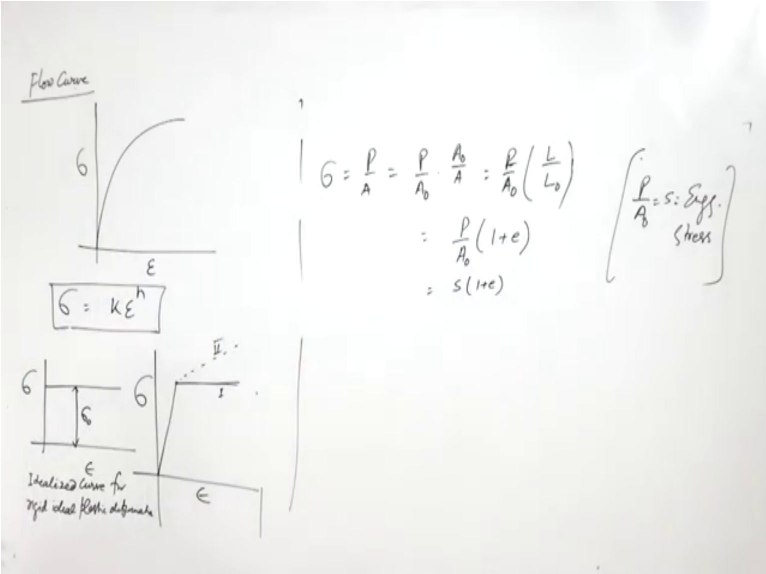

So, this flow curve is nothing but the true stress true strain curve. So, this true stress true strain curve is known as flow curve and top typically you will get such kind of curve which is the true stress true strain curve, it is known as flow curve because you require flow stress, the flow stress determination will be there for having particular plastic strain or so, so that type of relationship you may get from here. Now, in that case flows this it is found that in this case there is empirical co relationship which is developed that is sigma equal to k epsilon m.

So, in this sigma is the flow stress and this is the plastic strain and then further you have k. So, this is basically nothing but the flow stress value when this plastic strain is having 1. So, that is how this k is defined. So, and then n, n is known as the strain hardening exponent. So, strain hardening exponent is nothing but if you take the log log plot of this curve. So, log sigma and log epsilon.

So, the slope of that curve will let you know about the value of n. So, that is known as strain hardening exponent. The thing is that this is also can be this can typically drawn for the different type of engineering materials. So, if you try, so it will basically be valid from the onset of plastic deformation. So, this equation is basically valid from the onset of plastic deformation till the material fractures and the this value, this type of curve we will be there you have different kinds of curves you may have the epsilon sigma epsilon curve may be of different type.

So, if suppose you have sigma in epsilon you may have this curve. So, this is basically this is an example of rigid and ideal plastic curve. So, it is rigid and when this value is epsilon. So, basically such curves are basically idealized flow curve for rigid ideal plastic material. So, idealized curve for rigid ideal plastic deformation.

So, there is no deformation till this point, but then after once the stress value is reached at this value then there will be ideal plastic deformation; similarly, you have another kind of this stress strain curve which is the idealized curve for material with some elastic reason. So, may have you may have such curve like you have this and then this. So, this is basically the flow curve for the plastic deformation, but with elastic reason also. So, this is the elastic reason and then you have idealized plastic deformation. So, at this point it started behaving in a plastic manner in an ideal way.

You may have another type of the deformation and there you may have this is as a linear variation here and then you have the strain hardening reason. So, that is piecewise linear region. So, this one will be elastic region followed by the strain hardening region that is also linear. So, that there may be another reason, so this is 1 and this is 2. So, there may be, this may be another case which is basically the strain hardening region it is known as piecewise linear region and that is a strain hardening reason.

So, these are the idealized kind of a stress strain diagram true stress true strain diagram which you may encounter during the studies. So, this is how you find the I mean from him at this point if you know this plastic strain by using the value of n you can find the stress values and we will also see that how this true stress values and true strain values are going to be used for developing the relationship in the case of plastic deformation.

Now, what do you we see that once we have the engineering stress definition you have the true stress definition. So, basically engineering stress will be sigma will be P by A that is your original area or A naught you can say that is original area and then you have the A at the instantaneous processing area. So, that way you can have the you know P by A that is a true stress, so that is instantaneous area.

So, what you see is if the if you take the true stress you may define. So, suppose you have got basically the original cross sectional area in that case, now sigma will be P by A. So, basically you can write it as P by A naught and then A naught by A. So, if, so P by a naught will be further that will be basically the engineering strain and then you can further use this P by a naught as the s and so c by P now. Now, you will have sigma as P by A naught and A naught by A will be same as L by L naught that is what we are discussed. So, you will have that.

So, it will be P by A naught and then you will have L by L naught v o that will be you know the because of the constancy of volume relationship and once you have that we earlier discussed that you can have P by A naught into 1 plus e and P by A naught will be taken as S into 1 plus e. So, P by A naught is as engineering stress. So, if you are given such curves you have the data like for engineering stress or the original dimension or so, you can further use these formulations and you can find the value of the true stresses or so; so this is how you can solve the different kinds of problems.

Thank you very much.