Theory of Production Processes
Dr. Pradeep Kumar Jha
Department of Mechanical Engineering
Indian Institute of Technology, Roorkee

Lecture – 24
Mean and deviator stresses, Elastics stress and strain relationships

Welcome to the lecture on mean and deviator stresses, and elastic stress and strain relationships. So, in the last lecture, we discussed about the types of strain. In the strain also you have two parts one part was responsible for any volumetric changes; another part was responsible for the shape changes. So, one is basically related to the elastic type of deformation, another is for the plastic type of deformation, so that way we defined that time we defined as mean part and stress strain tensor part another was, further earlier we had discussed about the (Refer Time: 01:06) tensor and all that.

So, now, again in the stress also you have the two parts one is the mean part or hydrostatic part, and another is the deviator part or deviatoric stresses. So, in that you have deviatoric component of strain, here you have the deviatoric component of stresses. Now, the mean stress tensor that will be because of the pure tension or compression, so you will have this will generate that mean component and it will only produce the elastic volume changes, so that is the you know trait of that.

And then the other part the in the total stress part you will have one is the mean stress part, another will be the remaining part that is known as deviatoric part, so that is known as deviator stresses, so that is basically representing the shear stresses. So, the quantity which you see along the diagonal in that basically sigma x plus sigma y plus sigma z by 3 that basically is in that among that part you will have one part as hydrostatic part coming out, and the rest part which will be remaining that will be your deviatoric component.

So, once you have sigma x x sigma x y sigma x z similarly you have. So, sigma x y you can write as tau x y. So, similarly you have sigma y x, sigma y y, sigma y z, sigma z x, sigma z y, sigma z z. So, similarly in from that along the diagonal part you will remove the mean part sigma mean sigma mean will be sigma x plus sigma y plus sigma z by 3. So, that way you will have two matrices, one will be with the mean component, another will be with the other component that will be known as that the deviator component. So, basically the yield stress will be independent of these the mean component of a stress all the fracture strain is dependent upon that basically. So, it is influenced by the hydrostatic stress levels.

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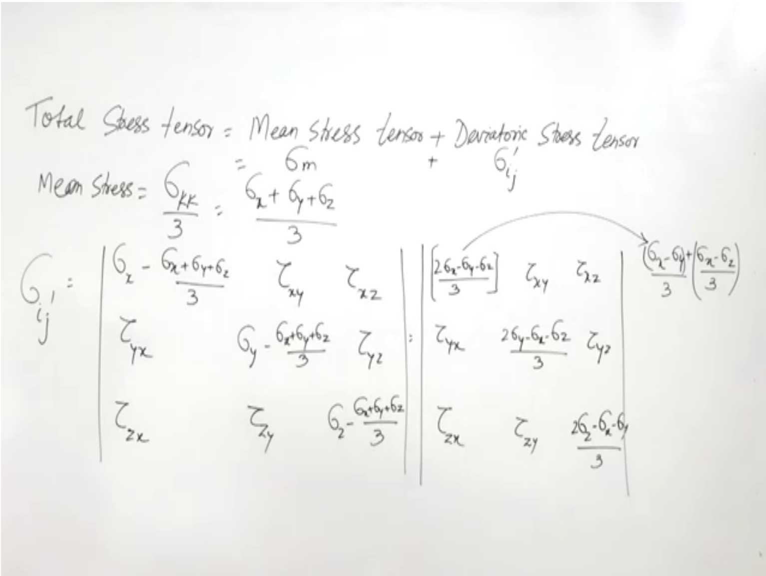

Now, what we mean to say that you have the total stress and total stress you can define as one is the mean stress tensor. So, total stress tensor will be defined as the addition of mean stress tensor plus deviatoric stress tensor. So, you will have total stress tensor and that will be mean stress tensor plus deviatoric stress tensor. Now, as we discussed that the mean part is responsible for the change in volume or here in this case the elastic changes. And then the deviatoric part will be basically representing the shear part; in the case of stresses that leads to the tension or compressive stresses and then there you they represent the shear stresses.

So, mean stress part mean stress part will be defined as it is defined as sigma K k by 3. So, it will be basically sigma x plus sigma y plus sigma z by 3 or sigma 1 plus sigma 2 plus sigma 3 by 3. So, it will be sigma x plus sigma y plus sigma z by 3 or you can also write it as sigma 1 plus sigma 2 plus sigma 3 is normal a stress components they are so that will be basically your mean part. Then so this will be basically sigma mean and that basically is denoted by sigma i j prime. So, this is basically defined as sigma mean and then this is defined as sigma i j prime. Now, the sigma i j prime is the deviator component of a stresses and that you can further find by subtracting this part from the total stress tensor part.

So, if you look at that you will have the stress tensor in the form sigma i j prime and sigma i j prime will be. So, what will happen from this sigma x, it will be subtracted, sigma x plus sigma y plus sigma z by 3. Now, you have other this. So, you will have tau x y, you will have tau x z, similarly you will have tau y x. Again here you will have sigma y minus sigma x plus sigma y plus sigma z by 3 and then it will be tau y z. Then further you can see it will be tau z x, it will be tau z y, and this will be sigma z minus sigma x plus sigma y plus sigma z by 3. So, this is how you define this deviatoric component of stresses. This is known as deviator stresses.

Now, if you look at the deviator stresses all the portions here you can see it is the shear component, this is the shear component, this is a shear component. Now, if you look at this it is nothing but this part is 2 sigma x minus sigma y minus sigma z by 3. And similarly, you will have now here you have tau x y, tau x z, tau y x, this will be 2 sigma y minus sigma x minus sigma z by 3 and this will be tau y z. Similarly, tau z x, tau z y and then you will have 2 sigma z minus sigma x minus sigma y by 3, this is what it looks like.

Now, if you look at this table this matrix, now in this you can see that easily you have these are the shear components. However, if you look at this component also, if you look at this component, this component basically you can see you can write it as sigma x minus sigma y plus sigma x minus sigma z and then by 3. So, basically what you see sigma x minus sigma y by 3. Now, sigma x minus sigma y by 3 or sigma 1 minus sigma 2 by 2 that when we discussed about the maximum value of shear stress which we achieve, so that will be basically tau, it is maximum shear stress. So, this will be tau x y, so that is how you can define.

So, again this part is also the addition of two stress components. So, basically they are again the part of shear kind of stresses. So, altogether what we want to say that is whole part this is the part which is responsible for and it is the deviatoric stress tensor and this is basically all the shear you know part which is responsible for the plastic deformation that basically is taken care of by this component of the stress. So, you have two stress components, mean stress tensor and you have the deviatoric stress tensor. So, we discussed about this mean part and the deviator type of stresses. And as we have seen that that is also a tensor, sigma i j prime that is also a tensor. So, again you will have to have the properties of tensors applicable there, and you have the invariance of that deviatoric stress tensor. So, you can have, there also you get the invariants.

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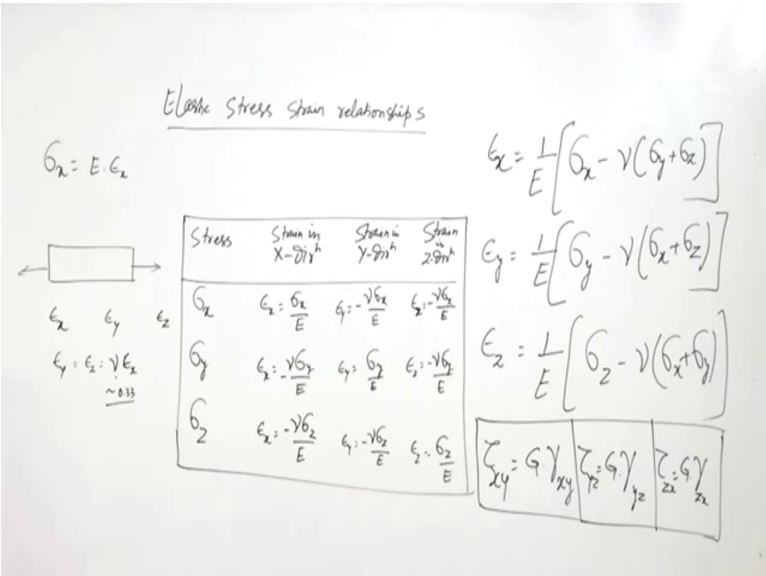

Now, we will come to the elastic stress-strain relationships. So, now what is said there are certain constitutive relationships, which are there between the constants. Now, what are these constraints which are we come across when we deal with this stress tensor and we deal with the strain tensor then once we have to you know find the relationship between them you will have to put the material properties you know in question. And in between them and then how then you get certain relationships between them and these are known as constitutive relationships.

Now, we know that you have the Hooke's law, and Hooke's law talks about the linearity between the stress and strain. So, Hooke's law tells that for as I mean certainly for isotropic materials, so for isotropic materials, we defined the Hooke's law as sigma x into E of epsilon x that is how the Hooke's law is defined. So, this is how you define the Hooke's law means you are in the case of the tensile stress-strain diagram, the slope is known as this E that is your elastic modulus so in tension or compression.

Now the thing is so this is a constant which we know this is the property of a material. Now, the thing is that when we apply the tension or compressor in certain directions. So, suppose, we apply the tensile force or tensile stress is applied in certain direction, then basically there is deformation in a particular direction, so there will be strain in that particular longitudinal direction, but then a along with that you will have a strain in other two directions also.

So, if you take the three-dimensional picture into mind then you will have if there is tension in the bar, so if you have a bar, and if you are applying, so if there is tension in the x direction, and because of that you have the longitudinal strain in the so length is increased or a strain is there in the positive in the x-direction then there will be a reduction of the dimension in the other two directions that is y and z. And it was seen that the ratio of these strains in the two directions to the ratio in the longitudinal direction that is normally a constant. So, that was seen that say suppose you are doing the tensile giving the tensile loading in certain direction x direction, then in that case if the epsilon x is the strain in one direction, and you have epsilon y and epsilon z as the strain in other two directions.

Now, the thing is that epsilon y and epsilon by epsilon x will be a quantity known as nu. Similarly, epsilon z by epsilon x will be a quantity known as nu. And this constant is known as Poisson constant. So, certainly if this is tension, and these two are compression, so they will be having negative values. So, if you take minus of epsilon y by epsilon x, it will be nu and its value is normally 0.33 normally for the solids. So, epsilon y or epsilon z, so this epsilon y or epsilon z, it will be basically nu times E x and that is basically this value is normally typically 0.33 for metals, so that is found. So, that is another constant which is normally a constant for the material when we discussed. So, when we tried to have the relationship constitutive relationships between the constants then this nu also comes into picture more frequently.

Now, the thing is that if you take the stress that is if you make a table, so you should if you have stress and strain in x direction, and then strain in y direction and then strain in z direction. Now, you can finish this table. So, once you have the stress as you know sigma x, so this will produce a strain that is epsilon x and this epsilon x will be that will be sigma x by E. So, this is what we get. Now, because of that if you look at the sigma y if you take this component, this will induce also a strain in this direction. So, for that it will be nu times sigma x by E, so that is what we get. And since this is negative, so if I put it here as negative.

Similarly, you have sigma z, and then for this, it will be minus nu of sigma x by E, no, this is sigma y and this will be sigma z. So, similarly you can fill all this, you will have due to y direction you will have epsilon y as minus of nu and sigma x by E, and this will be you know again epsilon y, so that will be sigma y by E. And this will be again epsilon y, this component will be minus nu of sigma z by E. So, value you have epsilon x epsilon z. So, this will be minus nu sigma z by E, then you have epsilon z as minus nu sigma. So, this is x basically and this is y by E and this will be sigma z by E that is how these strains in other directions other components of strain can be found out.

Now, if you apply the principle of superposition, then you can find the expression for epsilon x. So, epsilon x will be basically sigma x by E minus nu by E into sigma y plus sigma z. So, if you use the principle of superposition, you get epsilon x as 1 by E if you take as common, you will have sigma x minus sigma y plus sigma z into nu. So, it will be sigma x minus nu into sigma y plus sigma z. Similarly, you will have epsilon y as 1 by E sigma y minus nu sigma x plus sigma z, and epsilon z will be 1 by E sigma z minus nu sigma x plus sigma y. So, this is how you get the expressions for these strain components.

Now, there is another you know constant and that is the shear modulus or modulus of rigidity. Now, for that when we do the shear test, so in the case of shear test, you have tau x y is the stress and gamma x y is the shear strain. So, they are defined this so they are having you know stress by strain is G. So, basically they are from you get tau x y as G into gamma x y. Similarly, you have another relationship tau y z will be G into gamma y z, tau z x will be G into gamma y. So, this is another so you have another constant that is G that is your shear modulus. So, you may have this also is a constant, and also you will have the constitutive relationship involving this constant.

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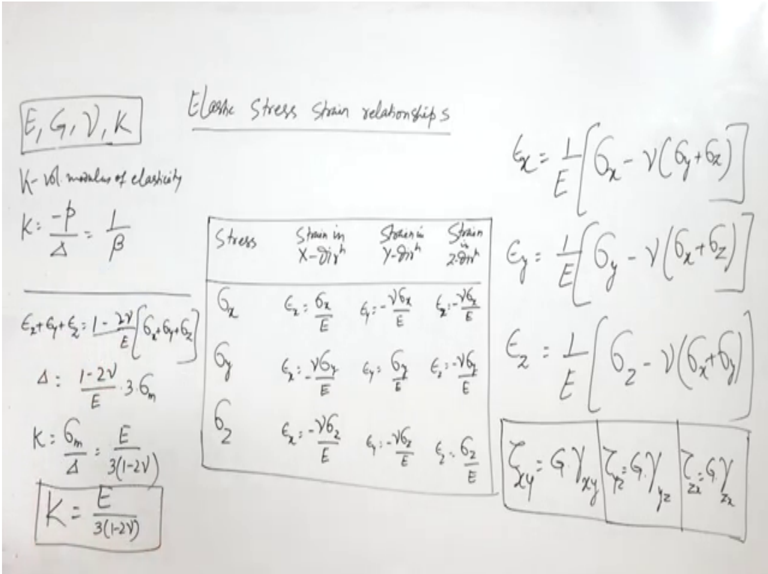

Now, so it means what we see is we have some constant we have the constant defined as E, G and nu. So, we have defined so far the constants for the solid E, G and nu. Now, another constant is related to the volumetric strain and that is basically volumetric modulus of elasticity, so that is known as in the constant K. So, as we know we know that in that case volumetric modulus of elasticity that is v, so this is K, so that is volumetric modulus of elasticity. Now, in this case, you have again you have the stress by strain, and stress you know that that is your mean stress, so that is minus p if the p is the pressure which is equal at all the places, so that it will be minus p and divided by volumetric strain that is delta.

So, once you define that, K will be defined as volumetric, so that will be your mean stress, so that will be p and divided by a strain. So, volumetric strain is delta and that is basically defined as 1 by beta. So, this beta is known as compressibility. So, this is another constant that is your coming into finding the constitutive relationships. So, this E, G, nu and then you also have got K. Now, from these four constants, you will have the equations getting developed. Now, how to develop these equations?

So, what we see is that you get sigma x plus sigma y plus sigma z that we have know whether it is epsilon x plus epsilon y plus epsilon z that you get as 1 minus 2 nu by E into sigma 1 plus sigma x plus sigma y plus sigma z. So, you can have this you can see that if you find the epsilon x plus epsilon y plus epsilon z, now, if you look from there then you will get it 1 minus 2 nu by E, so that will be the term. And then you will have the sigma x plus sigma y by sigma z, and now this is basically the 3 times the mean stress. So, you can write this, and this is the volumetric strain. So, this will be equal to 1 minus 2 nu by E and sigma x plus sigma y plus sigma z this is 3 times the mean stress.

So, basically this expression you have got by adding these equations. If you add these two-three equations you get from here like this. Now, from here, you get this is your volumetric strain and then that will be 1 minus 2 nu by E into this is the 3 times the mean stress that is what mean stress is sigma x plus sigma y plus sigma z by 3. Now, K is basically, so K what you get as sigma m by epsilon this volumetric strain, now this volumetric strain. So, this divided one should divide this, so that will be your E term will go that side. So, you will have E divided by 3 into 1 minus 2 nu. This is one of the relationship which you get by having these placing the equations and seeing the addition of these equations you get this as another.

So, you will have K as E upon 3 into 1 minus 2 nu. This is one of the equation. So, basically in this fashion we can have the equations for the different elastic constants. Now, you can manipulate these equations, and you can have the relationship between these constants in a different manner. Suppose you want to have the relationship between G, E and nu, so G is found to be as E by 2 into 1 by in 2 by.

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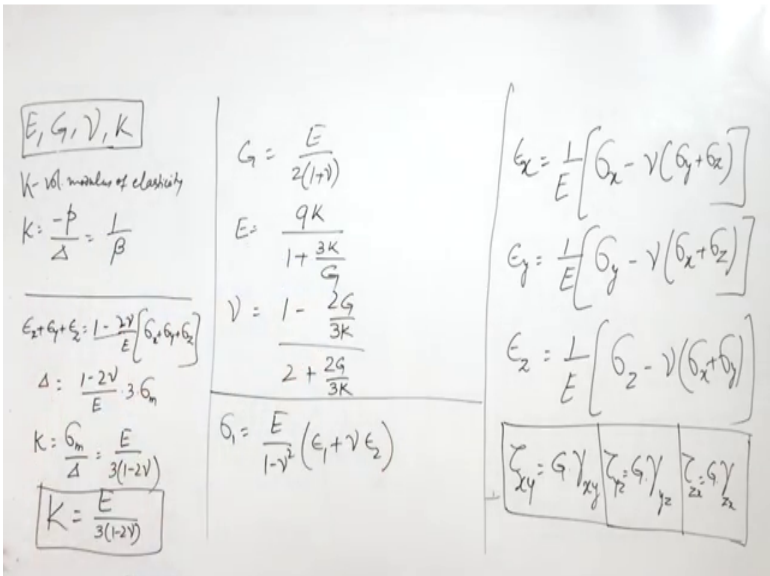

So, G will be E by 2 into 1 plus nu. So, this is how this is one of the other constitutive relationship by which if you know the G, E or G and nu you can find E; or if you know E and nu, you can have the value of G. Similarly, if you want to have the relationship between E, K and G. So, E will be 9 K plus by 1 plus 3 K by G. So, basically E will be 9 K by 1 plus 3 K by G. So, this will be another equation. So, in that if you know one of the constant you can find the constant for others.

You will have another expressions like nu will be equal to 1 minus 2 G by 3 K divided by 2 plus 2 G by 3 K. So, 1 minus 2 G by 3 K divided by 2 and then 2 plus 2 G by 3 K. So, these are the expressions which can be used for. And as you can further you know take these values from one equation to other, and find the relationship between these constants. Now, the thing is the purpose is that you have to calculate the value of stresses from the elastic strains. So, once you have the value of elastic strains, then you can calculate the value of stresses.

So, basically in case of plane stress, you can find, so if you try to calculate the value of elastic, and the value of a stresses from these elastic strains, suppose you want to find sigma 1. So, sigma 1 will be E by 1 minus nu square into epsilon 1 plus. So, sigma one will be, so this way if you know the epsilon 1 and epsilon 2, and nu and E, so this way you can find the value of elastic I mean strains for the stresses for the given value of elastic strains. Similarly, you have sigma 2, you can find E by 1 minus nu square epsilon 2 plus nu into epsilon 1 in the case of plane stress conditions. In case of plane strain condition, you will have strain in one of the direction as zero because one of the direction may be quite large. So, in that case you epsilon 3 expression which we have got earlier that can be equated to 0 and you can get the values. So, such you know you can get these situations where you have to find the value of these stresses from the elastic strain values, and you can use these relationships for finding that.

Thank you very much.