Theory of Production Processes

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Lecture – 23
Description of strain at a point

Welcome to the lecture on description of strain at a point. So, in the last lecture, we discussed about the stresses. And we found that the stress which is a tensor in the case of three-dimensional stress, the stress at the point was seen that it was a tensor, so you have the nine components of stress. And in this lecture, we will talk about the strain and also we will see that how strain at a point is defined. Similar to the stress strain also has the components; in the case of strain also, you have normal component as well as shear component. So, you have normal strain and you have the shear strain. So, this way you have the strain defined.

Similarly you can have the strain definition for 3 D, so which is again a two-dimensional tensor. So, you have again it has nine component, you have the component of mean strain you have also component which is basically responsible for shear. So, you have shear strain components. So, this way we will define the different types of strains and also the strain tensors, stress tensors. Then you have later on we will see that how these you know normal components or shear components are basically interrelated, and how by relating the stress and strain components or stress and strain tensors, you can have the relationships or constitutive relationships so to define a by using the material property of the body.

So, in this lecture we are going to define about the strain at a point. Now, what happens that when we talk about the body, then you have the elastic deformation or you have the plastic deformation, so you know you have any point in the body. And when there will be deformation there the any point will be changing its position, so you will have the displacement of that point. So, suppose you have x, y, z, and it shifts to suppose a new point and that will be x plus u, y plus v and z plus w. So, basically if you have in the space you have the point x, y, z and then it shifts to x plus u y plus v z plus w that is displacement is u v and w in the x, y and z direction suppose you have the three orthogonal axis, then you say that yes there is strain. So, that because of the application of any force or stress if they change the point then you say that yes there is strain.

So, suppose you have a bar which has a length of suppose x and of a point it is x and then you have another point in that bar itself. So, a point is having the distance of x from the origin, and then between a and b you have so a and b, the distance is dx. So, basically when you strain it with the application of any force or so, now basically there will be change in between the points a and b. So, suppose a becomes a prime, and b becomes b prime. So, again so that in that case what we see we can define the strain and strain will be basically so a strain is defined as change in length divided by the original length.

So, if you take in the x direction, you will see that if there will be if the distance displacement is suppose u for the point a, and then b point will be suppose u plus dou u by dou x into dx because dx is the distance. So, that way you can have the change in length and divided by original length if you look at, then that way you can get the strain in x direction as dou u by dou x. So, this way a strain component may be defined in the x direction epsilon x which is defined that will be dou u by dou x. So, this way you define the strain in x direction. Similarly, you can define the strain in other directions. So, then once you define these strain components, then you will find the strain tensor for the three-dimensional state of stresses.

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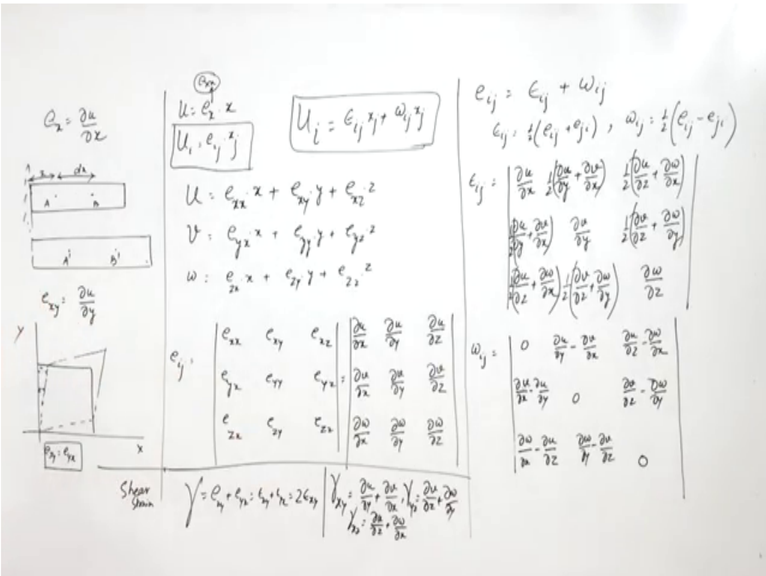

So, what we see is that we get the a strain in x direction as dou u by dou x that is u is the basically displacement in x direction and the dou x so because it has the length of dx it was. So, basically what we see is you get the strain in x direction as dou u by dou x. Now, how it comes so that we can see that if you have a bar which is having so any point a and b is there, and the a point is basically at distance x and then this is d x suppose. And then you have now this basically change to this length this a came to this point and b came to basically in this point.

So, you have this is u basically from here the displacement was u in this direction a has the displacement u. And if you look at this, this will be u plus dou u by dou x into dx. So, it is displacement from. So, you will have this prime a prime and this is b prime. Now, between a and a prime the distance will be u. So, between b and b prime the distance will be u plus dou u by dou x into dx. Now, the thing is that if you define the strain, so that will be dou u by d x dou x into dx is the change in length, and then the original length was in between the probe two point is dx. So, dou u by dou x into dx divided by dx, so that is why the strain is defined as dou u by dou x. So, this will you define the one-dimensional strain in a particular direction.

Now if you try to see you know other type of so this is strain, which is in one direction. Now, if you look at the two-dimensional state of stress, now in that case you will have this is the x-axis and this is the y-axis. Now, you may have the stress in such a way that you have the change of the phases in different manner. So, if suppose this is this is the original element and if suppose it changes to different shape, suppose it comes to this shape, so what will happen? This will go to that and then you will have this, and then it will move to that. Now, if we take such kind of deformation, now in this case basically you have this is basically the u, if you look at this is a displacement in the x direction and this is the displacement in the y direction. So, you can define e x y or e y x in that case. So, this is basically e x you can say this will be strain when we talk about the engineering strain, so you can have any way that also means somewhat similar, but in general we can have the meaning as E x.

So, now in this case have this is basically the representative, so this is a displacement in the x direction. So, this is displacement in that. Now, if you look at ah that way. So, you have the strain if you look at this is the strain, this is the strain which is occurring. And if you look at this value you will have the strain component suppose e x y or e y x. So, in this case you get the e x y or e y x as the same one. So, you will have e x y and ey x. Now, e x y that you will have the strain. So, in that case if suppose e x y you want to have now that will be dou u by dou y because in the y direction. So, is e x y will be dou u by dou y; e y x will be, so dou v by dou x like that you if you look at this. So, dou u by dou y the displacement is u this one and over this distance.

So, if you take this angle, so that is why you have dou u by dou y. Similarly, e y x will be dou v by dou x. So, this way we define these components e x y, e x z, e y x or you know e z x. Now, in this case it is so, but it can be if you take it in a different way than sometimes e x y may be negative of e y x. So, depending upon the nature of rotation, you may have the sign convention for these points. Now, let us see in this case what we get is we get e x y as dou u y dou u by dou y. So, we get e x y as dou u by dou y. So, because that multiplied with delta y that will give you the component u that is displacement in that. So, this way you define these strains.

Now, what we see is that if you look at you have u will be if you look from here u will be actually e x into x, so that is what for a particular value you can write u will be e x into x. So, you will have u will be e x into x. So, you can define the displacement components like u i will be e i j x j. So, basically e x is nothing but this is also as e xx we can write because this is once we are talking about the one dimension pure intention and this is the linear component. So, this is e xx. we are not going to that, so that will be and when we take for two dimensions or three dimensions in that case u i will be you can generalized way you can write u i as e ij into x j. The thing is that you can now define for the three-dimensional case you can define u as e xx into x plus e x y into y plus e x z into z.

So, you can define u and that too in the. So, once you define this U, now that will be one equation. So, you will be having u as e x x into x plus e x y into y plus e x z into z. Similarly, you will have v, so that will be again e y x into x plus e y y into y plus e y z into z. Then you have w as e z x into x plus e j y into y plus e z z into z. So, this is how the u, v and w are defined.

Now, the thing is that you know the strain component e x, basically is dou u by dou x. So, you will have the e i j if you define now e i j will be defined you can define it as a three you have the components. So, you have e x x, e x y and e x z. So, you will have e x x, e x y and e x z; similarly you have e y x, e y y and e y j; and e z x, e z y and e z z, that is how you define the strain and then further if you place these equations in its own terms. So, you will have e xx is dou u by dou x then e xy is dou u by dou y and this is dou u by dou z. Similarly, you have dou v by dou x dou v by dou y and dou v by dou z. Similarly, you have dou w by dou x dou w by dou y and dou w by dou z. So, this is how e i j will be defined such cases.

So, you will have some components as the normal components, another components are the shear components this side you will have these at this. So, along the diagonal you have the normal components and along the other at other places you have the shear components. Now, the thing is that this e i j now this as it is a strain, they say this is a strain tensor, so it has nine components this is a second rank tensor which has nine components. Now, as we know that once is a tensor then we can basically represent it in terms of so since a matrix you can represent it in terms of the addition of two separate matrices one will be symmetric matrix, another will be skew symmetric matrix.

So, you can divide it further, so that will be basically e ij you can further divide us one will be epsilon ij, another will be omega ij. So, this will be the symmetric matrix and the omega ij will be the skew symmetric matrix. Now, what will be epsilon ij? So, epsilon ij can be written as half of e ij plus e ji; and omega ij will be written as half of e ij minus e ji. So, this will have be, so this is strain tensor will be basically divided into two tensor components. So, one will be this part, another will be so one will be having the skew symmetric part, another will be having the skew symmetric part.

Now, you can have further these points. So, if you take epsilon iz, so epsilon ij, if you look at in that case now epsilon ij will be, so these diagonal terms will be the same sigma u by sigma x. Then if you look at the all other terms, they will be changing but it will be a symmetric matrix. So, you will have dou u by dou x, then this term will be dou u by dou y plus dou v by dou x. So, similarly it will be here dou u by dou y plus dou v by dou x. So, this is a symmetric matrix. Then this will be dou u by dou z plus dou w by dou x. So, similarly, here dou u by dou z plus dou w by dou x.

So, further this term will be dou v by dou y and then it will be dou v by dou you know so this will be dou v and by dou z plus dou w by dou y, so that term will come here dou v by dou z plus dou w by dou y. And this will be dou w by dou z. Now, this will be the matrix which you get and then it will be all half. So, all these components will be basically half, then all will be divided by half because you have division by half here. So, in those cases they remain the same along the diagonal and at other places you have this value.

Now, if you let look at the other component that component will be further seen, so that is omega ij. So, this is basically again now in this case, these diagonal components will become 0, because you are subtracting. And then once you start subtracting these, all these components will be 0. So, this will be 0, and this will be dou u by dou y minus dou v by dou x. Similarly, you will have dou u by dou z minus it will be again dou w by dou x. So, this way you will have the components here and here. Now, in this case, you will have again dou v by dou x minus dou u by dou y and this will be dou w by dou x minus dou u by dou z.

So, then further you will have these two components, here you add the term dou v by dou z minus dou w by dou y. So, it will be dou w by dou y minus dou v by dou z. So, this way you get the two components, and this component this omega ij this is basically talking about the rotational component dou u by dou y minus dou v by dou x basically is the rotational component. So, if it is 0, then you have basically the irrotational deformation. So, if w ij will be zero then you can say that there irrotational deformation.

So, these two components which we get basically this is known as a strain tensor, and this is known as the rotation tensor rotational tensor. And if that is 0, it means the deformation is basically 0, rotational deformation is 0. Now, the thing is that you have the shear strain definition and now here what we see that u i you can further define. So, this e ij basically is defined as epsilon ij plus omega ij epsilon ij becomes the strain tensor and this becomes the rotation tensor. So, you can write u i as e ij epsilon. So, you have epsilon ij into x j plus w omega ij into x j. So, you can write that epsilon ij xj plus omega ij x j, so that is what you get now from here now that is that we have to define the shear strain.

So, what is the shear strain? So, basically if you look at this figure you have this as the you know x y gamma x y, and then you have you have the strain here as e yx. Now, if you have that means, shear strain will be defined as the total angular change for a right angle, so that will be e xy plus e yx that is dou u by dou y plus dou v by dou x. In the case of pure shear, so you have this and this coming together, so that will be gamma. So, basically that way you define the shear strain as total angular change from a right angle.

So, if you define the shear strain, now if you define that shear strain that will be dou u by so e xy plus e yx. So, that shear strain gamma that is shear strain so that will be basically xy plus e yx and that will be basically you know so now that will be as you know it will be dou u by, so this becomes now in this case, you have two times epsilon xy, so e xy e yx being same. So, it becomes two times epsilon xy. So, it will become two times. So, in that case you will have epsilon xy as epsilon yx, because you see in that case. So, become two times epsilon xy. So, now, you can write the shear strain in x y as dou u by dou y plus dou v by dou x.

Similarly, though shear strain in y z will be dou v by dou w plus dou dou v by dou z plus dou w by dou y. Similarly, you have shear strain in the x z direction will be dou u by dou z plus dou w by dou x like that you can define the shear strain in such cases. Now, as you know that this strain tensor of being a tensor part, we can have the transformation. So, the best part of the tensorial aspect of stress or a strain is that you can go for transformation of axis. So, you can have the components along the axis which are at certain angle with respect to the original axis. So, you can have those components. And then for that particular axis again for a plane which is inclined at certain you know at angle which has certain direction cosines then you can have the expressions for finding the principle strains. And then the principle shear strain you know principal planes at which there is no shear strain like that, so that can be found out.

Now, the thing is that now we come to the aspect of deformation. So, the deformation we can define as one is the change in volume and another is the change in shape. So, if you talked about the change in volume, then that leads to the volumetric strain. And then change in shape, so the change in shape is because of the strain you which you are putting in, so that is basically because of the you know mean strain or that is because of the tension or compression. And then you have the other component that is the deformation. So, one you have change in volume and another is change in shape. So, basically one of the component is responsible for the change in volume, another of the component is which is responsible for changing the shape, so that is how the deformation is defined.

So, if you try to define the deformation in that case, now let us say you consider a parallelepiped. So, if you talk about the deformation as we discussed you have either because of the change in volume or change in shape.

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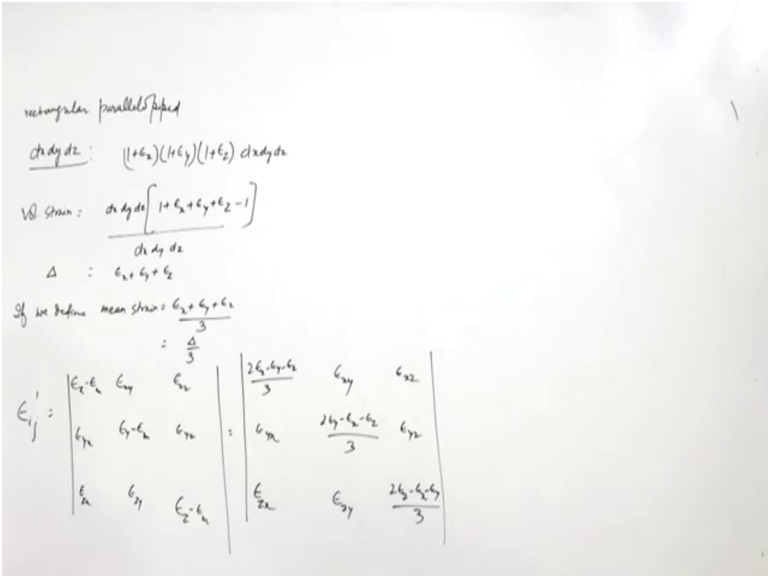

Now, if you talk about the volumetric strain. So, now, suppose if you have a rectangular parallelepiped. Now, in that case, if you have the parallelepiped with the sides basically dx, dy and dz. And if suppose it is strained, so the d x side becomes 1 plus epsilon x into dx. Similarly, the dy side becomes you know 1 plus epsilon y into dy, so like that. So, if you take that, so dx dy dz the volume now that comes as 1 plus epsilon x 1 plus epsilon y 1 plus epsilon z into dx dy dz. So, this was the initial volume and this is the final volume because of the strain. So, if you take at the volumetric strain, so change in the volume by original volume. Now, change in volume means this minus this, so that will be dx dy dz into this minus 1.

So, if you look at this, it will be 1 plus epsilon x plus epsilon y plus epsilon z, then if you neglect the higher order terms like epsilon x epsilon y be assuming that is very small. So, you can neglect and then minus 1 and then divided by dx, dy, dz, so that becomes as epsilon x plus epsilon y plus epsilon z. Now, this is nothing, but the first strain deviator if you try to find the tensorial analysis. Then once for finding the routes at that time you will see that this is the first strain deviator just you got as in the case of stress deviator.

Stress tensor you got the first stress invariant this is also the first strain invariant that is known as the first strain invariant.

And that basically if you take so this is also defined as basically if you take the mean strain in that case if you take it the volumetric strain. So, if you define the mean volumetric strain mean strain as if we define mean strain as epsilon x plus epsilon y plus epsilon z by 3. So, we define this as, so if this is volumetric strain is this, so mean strain will be delta by 3. So, that is basically so that is how mean strain becomes equal to delta by 3, change in volume that is a volumetric strain divided by 3.

Now, the thing is that you will have two components of the strain, you will have the strain deviator. And this strain deviator, so for that you have to subtract this mean deviator part from the normal strain part, so that way you can get the strain deviator. So, strain deviator epsilon ij prime, so this epsilon ij prime for that you have to this epsilon ij prime. What you have to do is the strain components which are there along the diagonal you have to subtract this mean strain component, so that basically you have to subtract from here.

So, all the sigma epsilon x, epsilon y and epsilon z all that is to be subtracted with mean stress. And then you have all other components. So, you have x y, you have x z, you have y x, you have y z, you have z x and you have z y. So, this is basically this becomes the strain deviator. And that becomes equal to if you look at it will be 2 times, so if you this is epsilon x plus epsilon y plus epsilon z by 3, so that becomes basically two times epsilon x minus epsilon y minus epsilon z by 3. Similarly, you have epsilon x y epsilon x z then you have epsilon y x, it will be 2 epsilon y minus x minus z by 3 then epsilon y z. Similarly, you have z x z y and 2 times epsilon z minus epsilon x minus of y by 3. So, this is the known as strain deviator which is basically responsible for the other kind of a strain.

So, one is for the volumetric strain that is mean strain. And from the strain part, you have removed that. So, this is for responsible for the shape change. So, basically the hydrostatic part what we call at is mean part that is how the mean or hydrostatic component comes into picture. So, the mean component part is responsible for any kind of volumetric change or volumetric strain, and then this is known as the deviatoric strain part, this is responsible for basically any kind of deformation, so that is how I mean shape change. So, the shape change is because of the such strain component. So, these are the different aspects of the strain in case of the study of these metalworking processes.

Thank you very much.