Theory of Production Processes
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Lecture – 21
Concept of stress and strain, Elastic and plastic behavior

Welcome to the lecture on concept of stress and strain and elastic and plastic behavior. So, we have started the topic related to forming and in this lecture initially we will have the knowledge of some of the fundamentals about the stress and strain elastic and plastic behavior then strength of material and all that. So, we will have today in our lecture about the concept of stress and strain also the plastic and elastic behavior about the types of stresses and also type of strain. So, then how the failure occurs about these fundamentals we will have the discussion in this class. So, let us see that how I mean what happens in the case of forming you apply the forces and you try to deform the material.

So, you apply the external forces and these external forces try to do the changes inside the materials and that basically is responsible for formation now how it does that. So, basically if we think of the any system where you have the application of forces on anybody and if it is in equilibrium means the, you know you can use this equation of static equilibrium.

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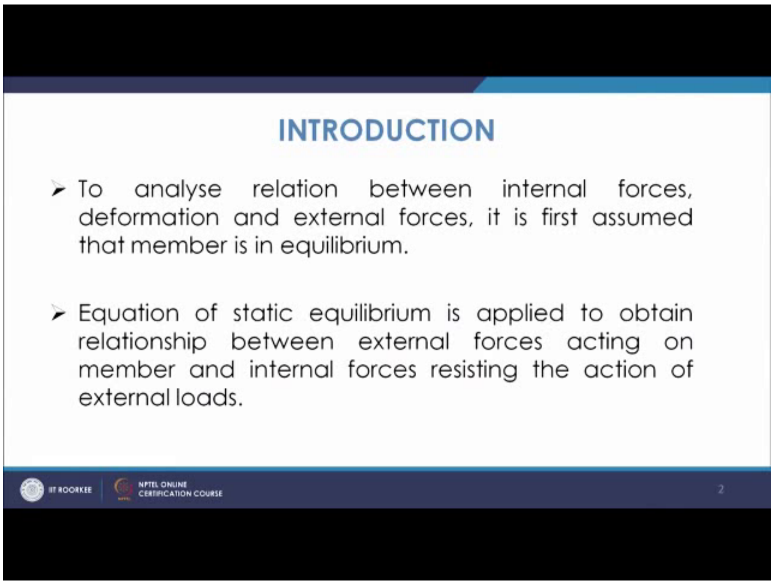

And then you can see by isolating certain portion that what are the internal resisting forces which are basically working to ensure that there is no change inside certainly once we apply the forces on any part that force will try to you know create some very certain deformation and that will depend upon in what range we are whether we are in elastic range or plastic range.

So, now, what we see that we have to analyze that and for analysis you have to have analysis of relation between the internal forces deformation and external forces the eternal force you apply and you have the resistance from the internal forces and that is also because before that you have the deformation of the elements. So, then you will have to see that the first assume that the member is in equilibrium. So, then only you can apply the equation of static equilibrium and then you can find the relation between external forces and the internal forces which are basically resisting that external forces. So, that is what the concept is that how can you see that how can you find that expression and you can find that from there you can find the internal resistance forces.

So, basically the stress what is stress. So, basically a stress is that internal resistive force acting over a certain area. So, that internal force is integral of the stress times the differential area over which it acts. So, suppose you have any member and you are applying any force in that if you take any section, now in that section if you are applying in this way on that very section and if it is in static equilibrium then on that section basically you will have the resistive force which is working against this force.

So, that is why that force is acting. So, this force which is acting that I mean stress will be acting at a point basically at a very small area and then stress if you take multiplied by that small area and if you integrate it in few take the integral of all these stress times the area over which it works then that basically is equal to that force. So, that is how the stress is computed you know that is how that stress is normally computed in normal case.

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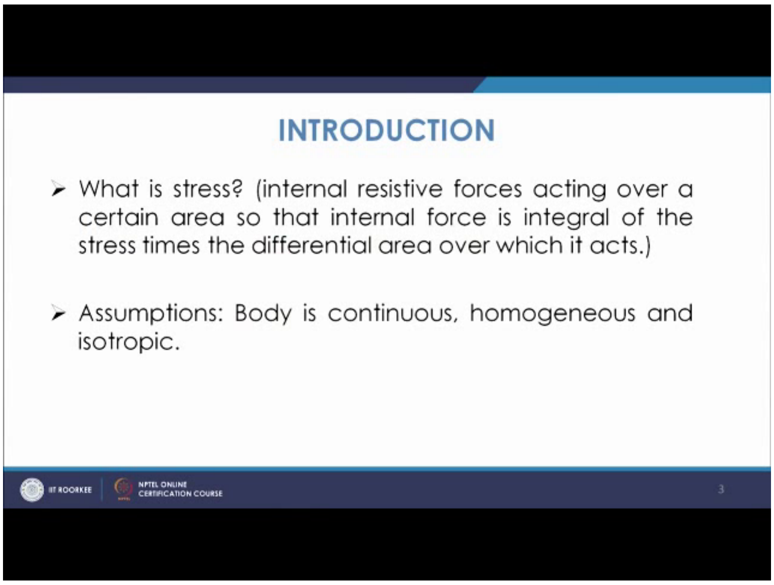

Now, what is the, you know the thing is the fact is that we know the deformation we cannot measure the stress, stress cannot be measured. So, for measuring the stress value first of all we find the strain. So, strain can be measured you have the instruments which can measure the strain and also in certain limit you have certain correlation between stress and strain also like the elastic limit how the stress and strain vary or so, based on that you can find the value of a stress.

So, you know that there are certain properties like you have been different like modulus of rigidity or, elasticity or, so that tells that stress will be proportional to strain in certain region. So, that basically based on that you can find the value of stresses. So, this is how the stress is computed the thing is that for analysis we have to have certain assumption and the assumption is that we have to assume that the body is continuous, homogeneous and isotropic.

So, what is continuous, continuous means the body does not have any void it is all full of materials and it does not have any void. So, that is that you assume that the material is continuous, then the material is homogeneous. So, homogeneous means the body has identical properties at all the points. So, at every point of the body it has identical properties. So, it is made of the same material in one sense. So, that is homogeneous and then the isotropic property. So, that is related to certain property and for certain property which does not vary with the, you know direction or orientation.

So, if the property does not vary with direction or orientation then we fail to say that the material is isotropic. Now if you look at the real sense in a macroscopic manner if you look at the microstructure of material none of the material you will find as continuous, homogeneous and also isotropic because if you look at the continuous means there is no void, but that is not the case you will have micro voids, you will have spaces which are vacant, you will have inclusions.

So, maybe the voids are present. So, it is not that way continuously if you look at the very micro scale, but then once you go to micro scale it is not the case. So, we feel that this is continuous. Similarly homogeneous means normally material will have different phases and every phase has different properties. So, if you look at the different places where the different phases are there it has different properties, but again this is true only for at the macro level if you analyze on macro level we assume that the property for certain mass, for certain you know region, which is quite of larger domain as compared to the macro level this homogeneous can this assumption also works.

Similarly, isotropic, there also we can see that the orientation of the grains they are in different direction. So, if you look at that way it is not ideally isotropic, but then again if you go on the macro level you can say that you have this isotropic property, but then we if the property is not isotropic it is known as anisotropic.

So, that can be seen if you look at certain kind of a processes like you have the forming process like rolling or forging where we you deform the grain very heavily in certain direction then this anisotropic property develops at a macro level you can say that. So, this is how these are normal assumptions which we take while analyzing and finding the relation for finding the stresses or so.

Again we should know something about the elastic and plastic behavior. So, what is elastic behavior, what is plastic behavior? So, as we know that if the material is regaining it is shape that is elastic behavior and if there is permanent deformation then in that case we tell that it is a plastic behavior shown by the body. Now when we talk about the engineering materials you have the Hooks law which is valid in the elastic region and that is that stress is proportional to strain. So, that and then the stress strain curve once we draw and where it is linear it is a linear variation in that case you will have the slope of that curve that talks about the modulus of elasticity of the material.

So, that is how we find you know the bulk modulus I have been murders of elasticity of the material. So, you have elastic zone and then when the if you apply the load and after that there is deformation and then it comes back to it is own position it means there is elastic deformation, but if it does not come to elastic deformation if you upper one deformation in that body then it is known as the plastic deformation. So, that is how we define the elastic and plastic deformation.

Now, what we try to define is the stress and strain, let us say what is the stress, suppose you have a member and this member is applied subjected to certain load.

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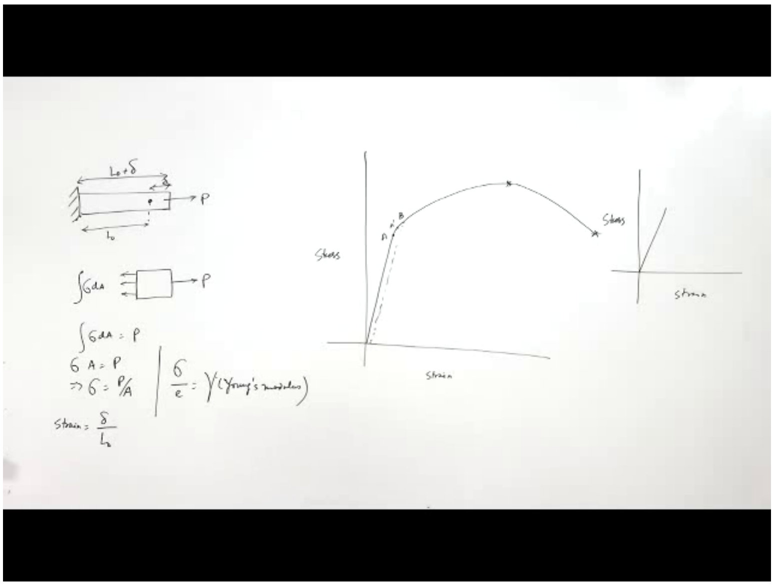

So, if you have a member and you apply a load P. So, it was the earlier length L naught and then once you have pulled it with that force in P and then the length becomes suppose is increased by delta. So, you the application of the pressure I mean force P this becomes L naught plus delta. So, this is delta, because of the application of this force the length is increased to delta. So, P is the external load, now if you look at the free body diagram if you draw at any cross section. So, what we see is you have the pressure P and then you will have a resistive force here acting this is acting over this whole area and this is basically.

So, you will have a stresses acting at all these points of at very small areas and then that if the stress is sigma and then if it is applied to very small area dA very small area and then you have a sum it and then you take it is differential. So, sigma integral of sigma dA will be equal to. So, you equate that as P. So, this way you have to equate this external force to that external I mean to the stress multiplied by the area over which this is stress works.

Now, if the stress is uniformly distributed over the area in that case it will be sigma if it is uniformly distributed over the area and then being as epsilon dA will be A. So, this will be P. So, sigma will be P by A. So, that we call it as average stress this because we assume that this is uniformly distributed over the area. So, we call it as the average stress on this surface; however, a stress is there the point which is defined and depending upon this orientation also of this plane this stress will be changing at those points. So, that we will see later now the thing is that the as we know that once we try to deform the material that in that case that you have up to the elastic limit or in the elastic range you have the stress is proportional to strain and the slope of that is basically the young's modulus.

So, stress by strain will be basically the Young's modulus. So, you say we the strength then that is the young's modulus of elasticity. So, that is how you find the elastic you know this young's modulus of elasticity once you have that curve. So, if you look at the tensile deformation of ductile material let us say if you apply the force. So, what happens once you apply the force then in that case there will be zone up to which you will have the linear deformation. So, this is your strain and this is your stress.

So, what happens in the initial condition, initial situation when you are trying to apply the load a point is reached up to which there is linear variation the there is straight line. So, that point is known as the proportional limit, because stress is complete directly proportional to strain. So, first point will be the proportional limit then you will have then the linearity is basically deviated you do not have no longer the linear relationship completely but then there is still the elastic property intact.

So, up to this, even if you remove the load from here the material will come again to this state you do not have any strain in the material permanent strain is not there. So, that is known as and, but the linearity does not hold Hooks law will not be applied because it is not directly. So, you will have up to this point if you take the slope of this line that will be the modulus of elasticity. So, this point is known as elastic limit, then after this the plastic deformation starts now if at this point if you leave if you leave if you unload then you will have some amount of plastic deformation some strain is left. So, that is basically known as some offset strain offset value and this point basically depending upon this value you call it as the yield points depending upon certain specified value. So, this point basically is known as the yield point.

So, B will be your yield point and then after that what happens the deformation starts and this goes away and this way it fluctuates here now how it comes. So, let us say this point is B. So, B is the yield point and the load which is divided by the original cross sectional area that will be the yield strength of the material. So, here you will have a strain of normally 0.002. So, normally 0.2 percent of strain at this point we the amount of stress generated that will be basically the yield strength of the material. So, that is how we calculate the yield strength.

Now after that the deformation starts increasing you have the if you go you will have the this strain go on increasing, now what happens during this stays there is a strain hardening going on due to the plastic deformation, at the plastic deformation goes on there will be strain hardening taking place. So, because of that the stress value goes on increasing. So, load is increasing and you have the area so you will have the. So, you come up to this point, where this is maximum.

Now, at this point basically, this maximum point is known as the UTS point or the tensile strength point. So, this point we call it as the strength which we get there that is known as the tensile strength of the material now after the tensile strength of the material after that basically the material and then next down. So, material basically at that point the necking has formed, the materials cross section has decreased, now what happens since this calculate. So, this is engineering strain and engineering stress, engineering strain is basically the change in length by original length and engineering stress is basically the force which we apply by original area. So, strain basically will be change in length by original length. So, you have change in length as delta and original length is L naught. So, that is your engineering strain and that is what has been computed there.

So, this is strain now at this point basically after that the necking takes place now after that what happens that the for the further deformation the load is not required basically to increase the load basically decreases. So, that is why this curve comes down and then ultimately it fractures here because after this point the area is completely decreasing at a very high rate and that is why the load basically comes down and, the stress value goes down and here is the specimen fracture. So, that is how it occurs this, the tensile behavior of material is there in the case of ductile materials.

Now you may have different types of materials and if you look at the other kind of material, this is for ductile material now you may have a brittle material; in the case of brittle material this stress strain curve is in different way here actually if it is completely brittle this will be the curve. So, for ideally brittle material where there is no appreciable you know deformation which is there or strain which is remaining and the material will fail without any appreciable deformation.

So, this is typically happening in the case of brittle material whereas, in some cases you may have some amount of also the plastic deformation. So, if you have the material with slight amount of ductility then the curve may be going like here it will go and then some amount it will go and then it will be fracture here.

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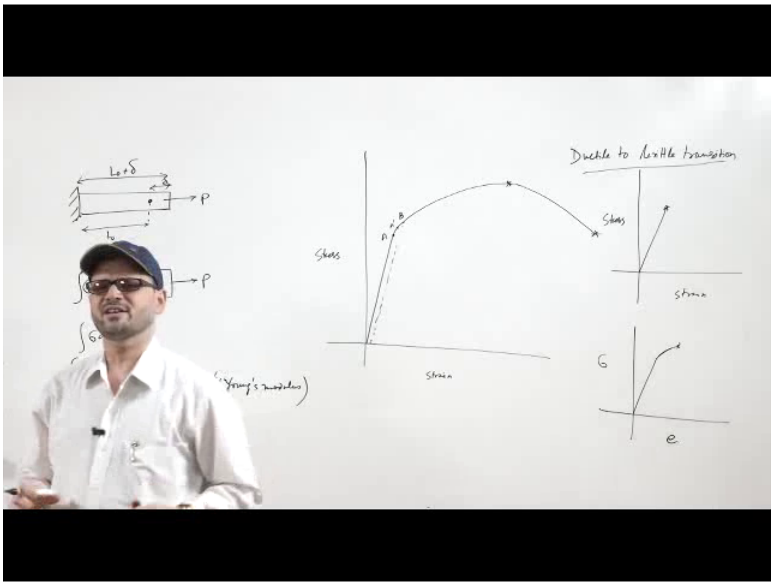

So, in this case it is fracturing here. So, it is completely ideal material whereas, with some ductility it will. So, here the stress strain curve for certain ductile material, but the thing is that many a times now what happens in the case of basically pure brittle materials in those cases the cracks which are there which generate and these cracks basically then spread over the area and then you have without any indication you are getting this fracture. Now in these cases you have appreciable deformation you know it is not desirable to have such kind of fracture because does not give any indication of when the fracture will occur.

In this case you are having that indication that yes material is going to have you know fail, but in this case you do not have any such indication. So, it is not desirable in the sense that there will be catastrophic failure of the material there may be injury to a person. Now the thing is that there is a phenomena known as ductile to brittle transition, now the thing is that there are many conditions in which the ductile material will behave as the brittle one or the brittle materials also may behave as the ductile one.

So, there are conditions like the type of loading or the temperature conditions because of which. So, if there is rapid loading rate of loading or if there is very small temperature in that case at very low temperature even the ductile materials like plain carbon steel may behave in a brittle way so, they may fail in a brittle manner, whereas, brittle materials also if you to take at higher temperature if the temperature is increased they may fail in a ductile manner. So, that is how, it depends upon these conditions like temperature or the rate of loading or so in which the even the ductile materials may behave as the brittle may fail in a brittle fashion or brittle materials may fail even in the ductile fashion like you know tungsten also which is brittle at room temperature may be ductile even at the elevated temperature.

So, a many a times the material may be brittle in tension, but it may be ductile when it is subjected to hydrostatic compression. So, there may be different cases and that all depends upon the kind of operating parameters under which the material is subjected to. Now we will discuss about the methods of why the failure occurs. So, what is the point what are the conditions, which constitutes the failure of the material, now the failure may be because of excessive elastic deformation or it may be yielding or excessive plastic deformation or it may be because of fracture.

So, if you look at the deformation or failure it has only 3 modes either because of the excessive elastic deformation or because of the excessive plastic deformation or because of the fracture. Now if you look at the excessive elastic deformation now you can take the example of certain you know bars or externally you may have the forces in such a manner that because of the excessive elastic deformation it has failed because what happens that you have deflection of beam which is maximum under certain load.

So, that way it will not be you know suitable for further use or you may have sometimes the buckling of certain slender columns where the load which is their which it actually surpasses the Euler critical load. So, in that case we say that it is failed. So, if the deformation is going beyond certain value or if the load which is reached more than that is a critical load in that case we say that this is basically failed. So, in the elastic region basically it is controlled by the modulus of elasticity.

Now, modulus of elasticity is basically the property of the material which cannot be changed. So, what you can change is you can change the rigidity of the structure you can make it stronger by having make by giving certain rigidity. So, that way you can have the improvement in those materials. Now coming to the yielding or excessive plastic deformation, in that basically there will be shape change if you talk about the plastic deformation normally you have the shape change because of the deformation.

So, again you will have the special formulas you will have the relationships and then that using the failure criterion you can model these phenomena and also there are conditions when you as we discussed that you may have the conditions like you may have temperature as a parameter. So, if you have the lower temperature, if you have the higher temperature so, many a times the temperature is higher under that condition at very high temperature than the normal temperature. So, the strain hardening is not exhibited.

So, as we have seen that in such cases the strain hardening is observed, but when the temperature is higher this strain hardening is not observed and the metal will be continuously deforming at constant stress. So, in a time dependent manner and that is a type of failure known as the creep. So, this kind of failure occurs in the case of these excessive plastic deformations or yielding. Then you have the also fracture like you may have the sudden brittle fracture you may have the fatigue or you know the progressive type of a delayed type of progressive fracture and then you have a delayed fracture.

So, you may have sudden fracture that we have seen in the pure brittle kind of material where they do not give you any kind of indication you have a formation of small crack that basically propagates and then ultimately the idea becomes. So, small and the stress level increases. So, high that the material fails over there then you have sometimes a progressive fracture and that is because of the fatigue. So, that is also known as fatigue fracture.

Now, this fatigue fracture is what basically in that you have the alternating type of load or fluctuating stresses are developed. So, in this case basically material fails even at the nominal values at which there will not be deformation, but then if the material is subjected to these fluctuating stresses for a long period of cycles in those cases the failure is because of the smaller value of stresses at which in normal case it will not fail. So, that is known as fatigue type of a failure and you have to design the material against this fatigue and you have some empirical relationships which work for such type of failure, then you have also the delayed fracture which occurs in the case of a stress rupture failure.

So, normally if the material is statically loaded for at a very high temperature for a long period of time then it this type of fracture, known as delayed fracture that occurs in the case of materials. So, depending upon the stress and temperature you know conditions there may be no yielding prior to fracture and then material will be basically failing. So, in presence of hydrogen it has been seen that steel fails as an example of the delayed fracture. So, that way you have in this way the failure occurs in the different kinds of materials, now let us discuss something about the types of stresses now once we decided we had seen that how we find the stress.

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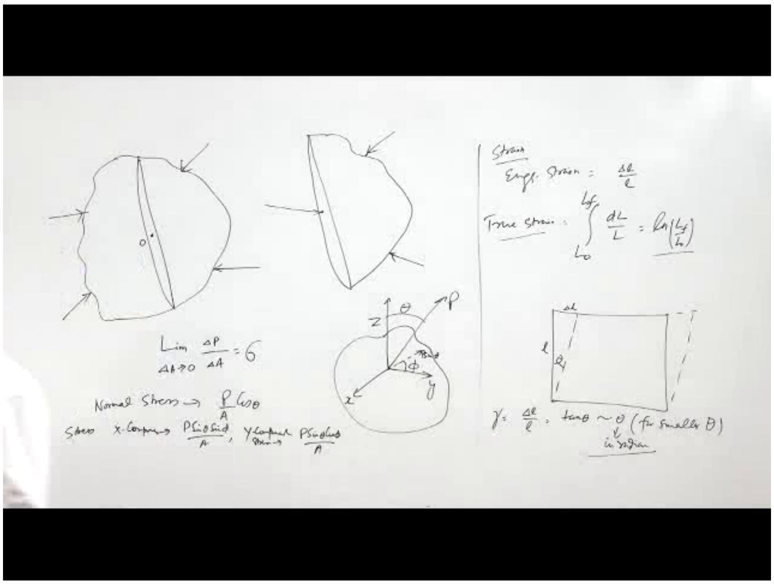

So, suppose you have a body which is being acted upon by different forces from all the sides and we want to find the stress. So, we have we can isolate some part and suppose this is the plane and we take. So, this is part 1 and part 2 suppose.

So, once we take this portion out. So, you will have this as the portion. So, suppose this is the portion now in this what you see is you will have these 2 forces and then you assuming that it is in equilibrium you will have at this point you will have suppose this point is O and this is a plane in that case what you see is you will have a resultant force which is keeping this in equilibrium. So, suppose this is the resultant force which is keeping that in equilibrium.

So, that way what you see is this force basically this is the internal resisting force and this is acting on a very small area then it is known as. So, that the stress multiplied by that small area and that will be integrated over that whole area and that way the stress is computed. So, stress will be defined when you know that the stress. So, if you have the P acting at that particular. So, if suppose this is the P and if delta P is actually one of a small area, then if the, you take the limit delta approaching to 0 this is what a stress is. So, that is what stress is defined at any point.

So, you will have the P acting over the complete area suppose and delta P will be acting at a very small area when this area small area tending towards 0. So, in the limiting case what you define is this is known as the stress. So, this is how the stress is defined; now the thing is that if this plane is will be different then this will be a different case. So, this way that the stress will be calculated and you can understand the concept of a stress in such situations.

Now again the stress suppose on any plane you have the stress if there is a stress now if this stress is making. So, this is basically if suppose this is a P and if on this plane this is the z and you have 2 directions x and y and if this is basically and if the area is A in that case and if this is making theta. So, this P will have 2 components P cos theta and P sin theta. So, P cos theta will be here and P sin theta will be here, P cos theta will be the component which is normal to this plane and that force which is the vent divided by area that will be giving you the normal stress.

So, normal stress will be P by A and cos theta. So, because this is angle making angle theta with that, now for this suppose this will be P sin theta and suppose this is making angle phi with y axis. So, again these will have 2 components in y and x direction and they are known as the shear stresses and in this case you will have cos P sin theta into cos phi. So, that will be in the y direction and P sin theta into sin phi will be in the x direction. So, that will be stress in X component. So, as the X component is stress that will be P sin theta sin phi by A and Y component stress that is shear stress basically that will be P sin theta and then cos pi by A. So, this is how you find these 2 kinds. So, anyway on a plane basically you have one normal stress and you have 2 shear stresses which are acting, as far as strains are concerned you know that you have the engineering strain. So, if you look at the strain the engineering strain will be change in length by original length.

So, you will have delta l by l. So, that is engineering strain and when you talk about the true strain basically here what you will assume that you find the stress when you assume that the all these stresses are calculated based on the original cross sectional area, but when you calculate based on the actual cross sectional area which is there at instantaneous time in that case it is true strain. So, that will be basically if suppose dl is the change in length and L is basically the actual length. So, you will have dL upon L and for that you will have the variation from if the L naught is the original length and L f is the basically final length, in that case ln of L and then you will have ln of Lf by L naught that will be basically the true strain that you get. So, ln L will be varying from L naught to Lf so, ln Lf by L naught.

So, this will be the true strain that is being used mostly for the plastic deformations it has a practical meaning of that that we will see in our later work there is another kind of strain that is shear strain. So, if you look at any cube and if you apply pure shear strain pure shear and in that case if the sides are deformed like this with angle theta then if suppose this is the change in length and this is the length in that case shear strain will be d delta l by l and in that case this is basically equal to 10 theta and that is equal to theta for a smaller theta, theta is in radians. So, this is known as shear strain. So, these are the different terminologies about the stresses and strains which are going to be used in our subsequent lectures.

Thank you very much.