Operations Management
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Lecture - 55
Production Quantity Model

Friends, welcome to session 55 in our course on operations management. So, we are going to finish the discussion for the eleventh week with today's session and we are left with 1 more week of discussion on this important topic that is, operations management and in the next week, our focus primarily will be on the advanced techniques that are being developed for operations management.

In this week if you remember, we have covered different types of techniques, tools that we can use for materials management. In this particular week if you look back, we can see what we have covered. We have covered different types of topics starting from the fundamental aspects of operations sorry, materials management in which we have seen, why do we need to manage the materials or the flow of materials within an organization; starting from the procurement going through the process or the usage and finally, the accounting of the materials. Everything we have to manage.

Why because, this has a bearing on the overall efficiency and effectiveness of our operations. We have seen that we have to classify the materials into different categories. Two examples we have seen and for one of them we did some calculations also, that was the ABC analysis in which we have classified the materials as the high value and low usage and low usage based on the usage of materials in the company. So, A, B and C and B was the intermediate class. So, we had a class materials B class and C class, and C class materials were having low monetary value associated with them, but had high usage in the industry. Also, we have seen VED analysis, in which we have classified the materials as vital, essential and desirable.

Then we shifted our focus on the economic order quantity and we have seen that if we order our materials in a special manner, in a specific manner, in a scientific manner; we can save lot of money for the organization and we have seen that EOQ model helps us to calculate the economic order quantity, which will optimize or rather, I must say will minimize the total cost of materials or inventory management and in that we have seen the different examples that we have, to have an information about the carrying cost, the ordering cost and the order quantity based on which we can plan our materials management strategy or policy when to order, how much to order, how many times in a year to order.

All these answers can be got through scientific materials management. In the last session if you remember, we have taken different types of numerical problems related to the economic order quantity model and you have seen in the last problem that we undertook, that we solved, we have seen that if we follow the EOQ model, we are able to save the money instead of randomly selecting a number and ordering it. It is always better that we use a scientific method of managing our inventory or inventory policy must be based on scientific logic.

In today's class, our focus primarily would be to sum up the theory related to materials management. One or two examples we will see and we will try to wrap up the things, whatever we can cover related to materials management. As in the very beginning of this week I have told that materials management is a very huge large concept involving lot of tools and techniques, but the most common ones we have tried to address in the last 2 year. In the last 2 hours of discussion, today being the last half an hour of discussion, so, we will try to see production quantity model which is slightly different from the inventory management model of EOQ.

So, in production quantity model there are a few theoretical background characteristics that we will try to understand here. So, first we will need to define the production quantity model how it is different from the economic order quantity in production quantity model and inventory system in which an order is received gradually.

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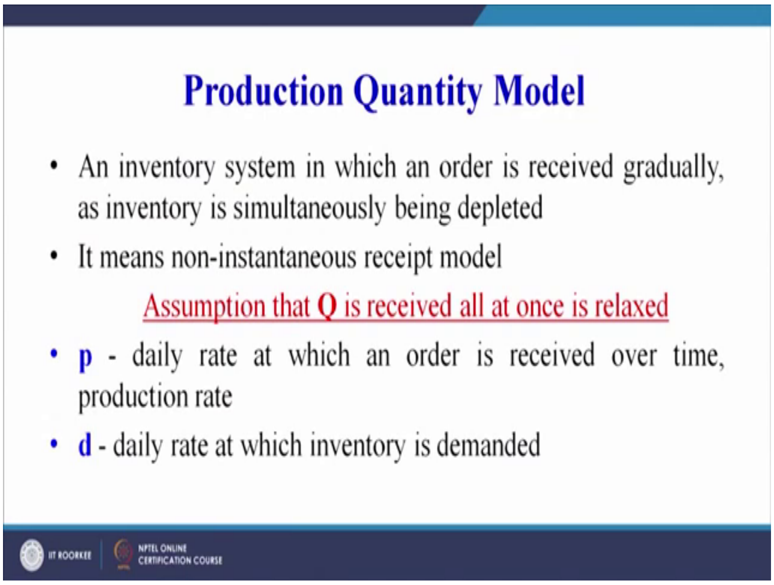

If you remember in the economic order quantity model that we discussed in week 3 during, sorry; not in a week 3 in lecture or session 3 during this week, if you remember we have seen that there is a order quantity which is there on the Y axis and on X axis, we have time. We are using the material that is the demand rate and as soon as we reach the reorder level, we reorder the amount that is equal to the order quantity and if within a specific lead time, that is on X axis, the material arrives at our destination or at our facility or at our factory and once that date or the contracted date is there, we receive all our order quantity at once and our inventory level again goes to the top.

So, it was demand rate is constant; demand rate we have assumed and then, all the material is received at once and our inventory level goes back again to the top, that is, up to the order quantity and again we use the material. Start using the material as soon as we reach the reorder level, we reorder the consignment or we reorder the amount or the volume of materials required. Within a lead time we receive that material and again our inventory level goes to the top that is equivalent to the quantity economic, sorry, the order quantity.

In case of production quantity model if you see an inventory system in which an order is received not at once, but it is received gradually, so we can say that there will be a production or there will be a rate at which we will be receiving our inventory or receiving our order or receiving our components or parts or whatever we have ordered it is not received all at once as inventory is simultaneously being depleted. So, we are using the material as well as we are receiving the material also. So, both things happen simultaneously.

So, it means non instantaneous receipt model. So, whatever material we have ordered, we will receive it but we will receive it over a period of time. Maybe, we start receiving our material, suppose we have ordered 700 parts and we start receiving those 700 parts on Monday. So, on Monday, suppose we receive 100 parts, Tuesday 100, Wednesday 100, Thursday 100. So, in 7 days, we receive 100 parts every day. So, we receive 700 parts not all at once, we will receive 700 part all at once in economic order quantity model, but in production quantity model there will be a production rate that will be 100 parts per day; total 700. So, our production run will be for 7 days. So, we are receiving our 700 parts as 100 parts per day in 7 days.

So, we are simultaneously, whatever is available in our inventory we are using it, but we are also receiving other material. So, it means that it is a non instantaneous receipt model. That means that the assumption that Q or our order quantity is received all at once is relaxed in case of production quantity model. So, here we use two terms p that is the daily rate at which an order is received over time. So, as I have given an example, 700 parts ordered, 100 parts received per day in 7 days, we receive our 700 parts.

So, our production run is for 7 days and our production rate is 100 components or parts per day; that is p and d is the daily rate at which inventory is demanded and in the previous model that we have studied economic order quantity, the demand rate remains constant. So, we have p and d these two terms we are going to use when we are going to derive an expression for the optimum quantity, that is or the optimal quantity that we will call as the economy quarter quantity Now, whatever I have explained in the beginning of today’s session, here it is explained graphically.

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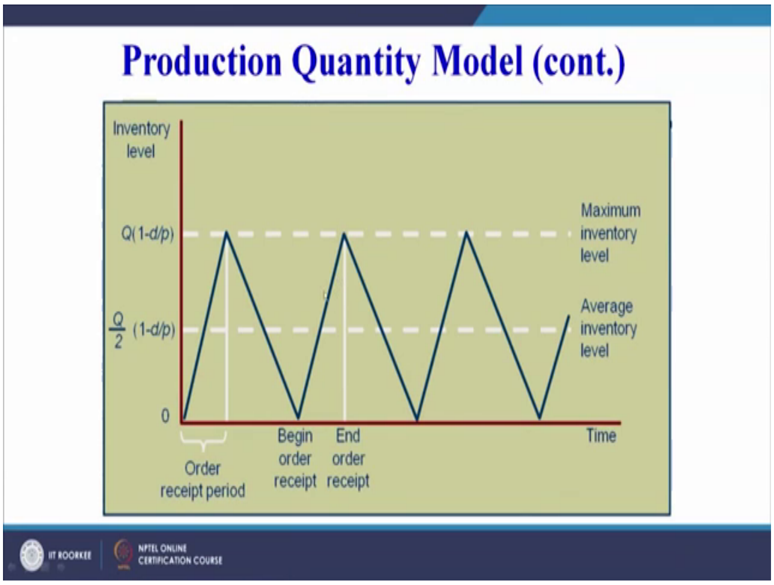

On Y axis, we have the inventory level or the quantity and on X axis we have time. So, we have time on X axis and we have quantity on Y axis. So, this is our top level, sorry, this is a top level that is q, that is Q into 1 minus d by p. Now, d is the demand rate and p is the production rate. In our case, d and p were 0 that we have covered in the economic order quantity and we have taken it as Q only. So, we start from q, we go to the reorder point, we order it with within a lead time we receive the material and again, our level rises to q, but now it will never rise to Q because d by p will definitely be some quantity.

So, d suppose is the demand rate, it can be suppose 50 components per day and production rate is suppose 100 components per day. So, 50 is demand rate p. So, 50 we are using 100 we are receiving. So, 50 by 100 it comes to 1 by 2; so, 1 minus 1 by 2 that is 1 by 2. So, it will be our maximum level can be Q by 2, because we are receiving also as well as we are using also. So, the average inventory level at any given time will be Q by 2, because if you remember in economic order quantity we have seen that when we multiply it by the carrying cost, we always multiply the average value of the quantity not the total value of the quantity.

So, it will we always multiply it by Q by 2C, C into Q by 2 and for order quantity, we multiply CO, that is the ordering cost into the annual demand divided by C divided by; we can see in the next slide we calculate it CO into d divided by the overall quantity.

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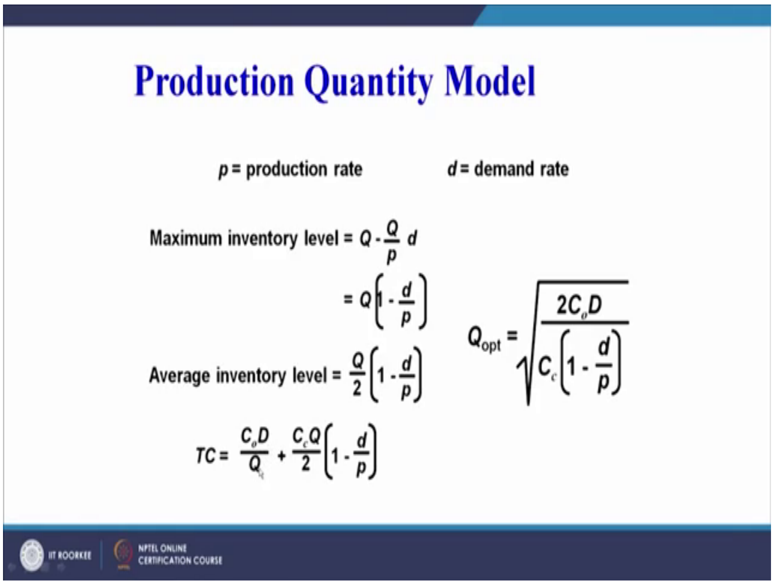

So, CO into d divided by Q; so, here again coming back to the same slide, we can see Q minus 1 by d by 2 d by p, sorry, and Q by 2 into this is average inventory level in case of the production quantity model. So, this is you can see here, we are receiving the order from this point to this point. So, order receipt we are not receiving the order all at once. So, if we receive the order all at once from zero, the value should rise directly to Q, but it is not happening. We are receiving the material here as well as the demand is also constant. So, begin order receipt here and end of order receipt here.

So, there is a period of time in which we are receiving our order that we have already placed. So, in case of, now, you can superimpose these two graphs together and you can try to understand that what can be the difference between the economic order quantity model and the production quantity model. The only difference the only major difference is that here, the order is not received at once or instantaneously. It is received over a period of time. Now, we have to calculate the Q optimum that is what will be the optimal quantity which will minimize the total inventory cost.

So, here we can see here we have p that is a production rate the rate at which we are receiving our material that we have ordered, d is the demand rate already I have explained. So, maximum inventory level as I have already told, it is Q can be the maximum, but we are using also we are using that is a demand rate and p is the production rate at which we are receiving.

So, Q minus Q p by d q into 1 minus d by p, which was there in the previous slide, which was the maximum inventory level which is shown. Here, Q into 1 minus d by p, this will be the maximum inventory on any given day according to the production quantity model. So, Q into 1 minus d by p this is a maximum inventory level average inventory level. Obviously it is Q by 2 1 minus d by p.

So, the total cost now as in case of economic order quantity we have written an equation, similarly we can write an equation in case of production quantity model also. So, total cost is equal to the ordering cost into the annual demand divided by Q that is the maximum inventory level plus CC into Q that is our rate at which we are using minus 1 minus d by p divided by 2.

So, CO into d divided by Q that is our ordering cost and CC into Q by 2 that is average value into 1 minus d by p that is a quantity which has been added because we are continuously receiving the model, receiving the consignment and as well as we are using the material or the inventory also. So, some of you may be wondering and this has come to my mind that why we are having Q here and Q into 1 minus d by p here.

So, what I believe that here whatever we are ordering, we are ordering, our ordering procedure is not changing, the only thing that is changing is that we are receiving the material over a period of time. So, our ordering cost as in case of economic order quantity remains the same and the quantity that we are ordering is also remaining same. So, Q remains same here and old demand remains same, ordering cost remains same.

So, ordering cost multiplied by that demand divided by the maximum inventory level or the order quantity that remains the same, but what is changing here the changing phenomenal or the changing thing here is the Q into 1 minus d, because now continuously we are using the material as well as we are receiving the material over a period of time and that period of time is called as the production run.

So, similarly equating these two things, we can calculate Q optimum, which will be twice the ordering cost multiplied by the annual demand divided by the carrying cost into 1 minus d by p; d is the demand rate p is the production rate. So, as per this, now we can calculate, try to understand with the help of a numerical problem.

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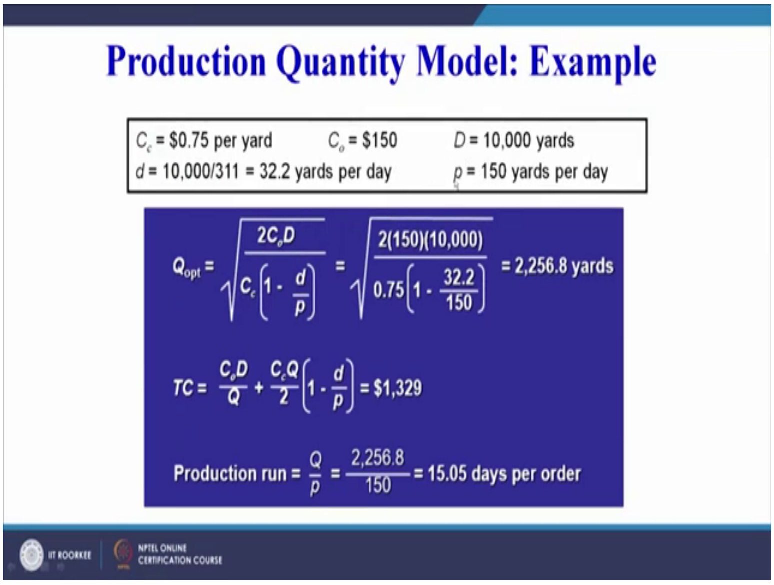

Now, here we can see similar problem is given here the production quantity model. Carrying cost is dollar 0.75 per yard, the demand is 10000 yards. The similar problem we have taken for EOQ also, the ordering cost is dollar 150, the demand rate is 10,000 divided by 311, that is we are working; suppose 311 days in a year, so, 10,000 is the annual demand as given in the proble; d is equal to 1000 yards.

So, 10,000 is our annual demand divided by 311. So, per day we have calculated the demand rate with small d. So, small d is 32.2 yards per day and production rate is 150 yards per day. So, according to that we can calculate the optimum value 2 into CO into d as per the formula. So, if we put all the values here, we can see that we get the Q optimum as 2256.8. So, we can safely assume 2,257 yards.

So, TC we can see here, the total cost we can calculate, it is coming out to be as per the Q optimum value. We will put Q optimum value here, we will put Q optimum value here, putting those this 2,257 yards at this point and this point annual demand, we know ordering cost, we know demand rate, we know production rate, we know if we put all these values here, we get the total cost as dollar 1329.

So, we can see the total cost can be calculated. Now, suppose instead of 2,257 yards, the company takes any other value random value or any value based on the intuition, we may find out that the total cost will certainly be more than this cost. In rare of rarest example it can be that intuitively somebody has or decided on a particular quantity and has been able to achieve the minimum total cost as compared to this scientific method of total cost, then we can calculate production run that for how many days, this material we will be receiving.

So, we can calculate that. So, 2,257 we have to receive; this is a total material that we have ordered and then, our production rate is already given 150 yards per day. So, 150 we will divide it by 150 yards per day. So, we will get that 15.05 days per order. So, if I have ordered today and the lead time is suppose 10 days, after 10 days I will start receiving the order, but the production run will not be maybe single day; that all 2,257 yards are received the same day. No, they will be received over a period of time and that time is given as 15 days per order.

So, we once we have ordered our production run will continue for 15 days. If you see this figure begin order receipt and order receipt. So, there is a difference between the two in case of economic order quantity EOQ model. This line is straight when we receive the model, sorry, when we receive our order everything is received on the same day. Now, this is something number of production runs how many times we need to run our production? So, we can see annual demand is 10,000.

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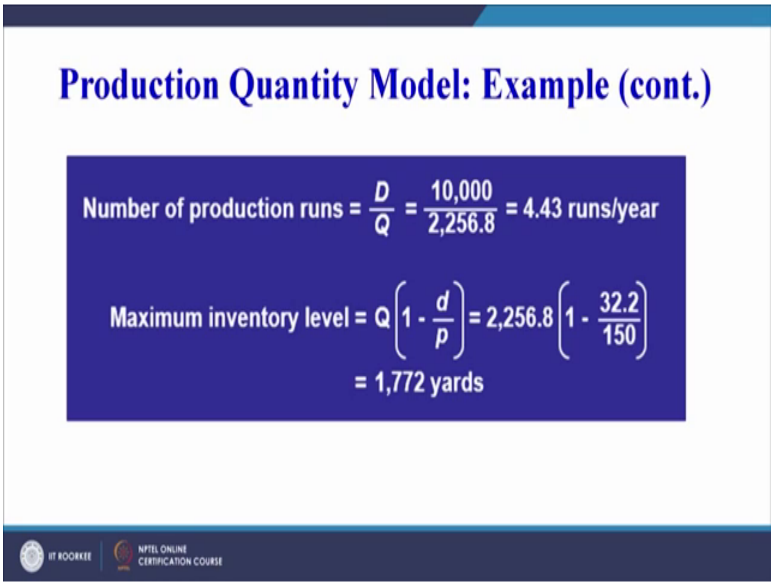

So, economic order quantity that we have calculated is 2,257. So, 4.43 runs per year. So, approximately we can say, if we round it off 5 runs per year are required to meet this annual demand rate demand of 10,000 units or 10,000 yards. So, maximum inventory level also, we can calculate the maximum inventory level is Q into 1 minus d by p which we have seen in the graph also; graphically we have represented this thing.

So, that will be 2 into sorry, 2,256.8 multiplied by 1 minus. What is this d? d is the demand rate that is, 32.2 yards per day; from where we have calculated this? 32.2 came when we have divided the annual demand. The annual demand was 10,000 and we have divided it by 311 working days. So, 10,000 divided by 311 working days, we can easily calculate that what is the demand rate per day. So, the demand rate per day is 32.2 and whenever we are ordering, so, the everyday we are receiving 150 yards per day. So, 150 that is a production rate; so, demand rate divided by the production rate. So, that value comes out to be 1772 yards.

So, on any given day, whenever we have the maximum inventory level that will be 1772 yards; now, maybe we have another important topic which we need to cover. All of us must understand that that is the concept of quantity discounts. Now, in quantity discounts we know that whenever if we order more, we get a discount. If we order, maybe you can relate it to your daily life also.

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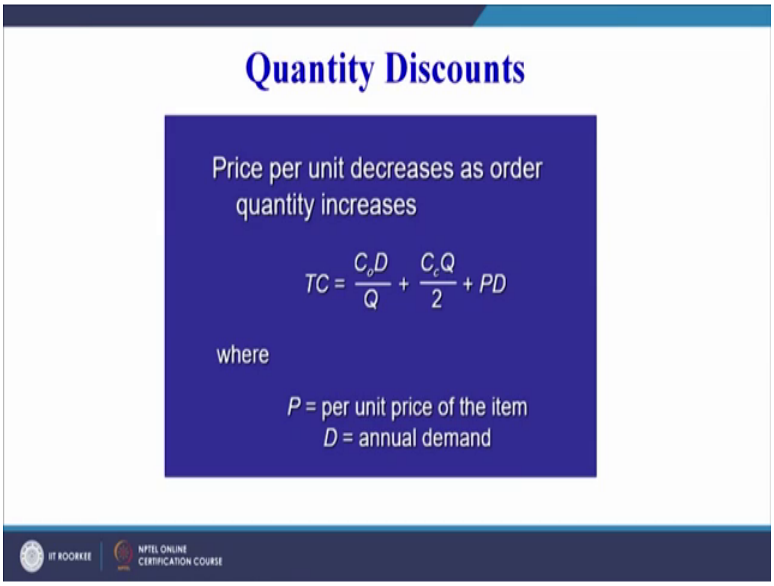

Suppose, you order something- if you buy maybe 3 chocolates, you may get it at a higher price, but if you buy 30 chocolates, a pack of 30 chocolates you may get certain discount on that. So, the price per chocolate may reduce if you are buying in bulk or if you are buying in a large number. So, the quantity discounts can be made use of when we are managing our materials. So, if we are a continuously manufacturing company, there is a continuous manufacturing or mass manufacturing happening in our organization.

We know that every year this much quantity we need to procure, so, instead of ordering too many number of times, we can take advantage of the quantity discounts that are available by the material with the discount and we can store it, but we have to do a trade of that the storing cost or the carrying cost of that inventory must not outweigh the advantage that we are getting from the discount.

So, we have to do a trade off, but certainly there are circumstances where if we take advantage of the quantity discounts, we will always be able to save a good amount of money for our organization. So, here we can see the price per unit decreases as order quantity increases. So, we will try to understand this with the help of an example. The total cost you can see we know this as per the economic order quantity model, ordering cost multiplied by the annual usage or annual demand divided by order quantity plus the carrying cost multiplied by the order quantity divided by 2 or the average order quantity Q by 2 plus p into d. Now, d is the annual demand and p is the per unit price of the item.

So, you can see that, this all other things remaining same this p will change when we are ordering more number of parts or components or materials. So, p is price per unit of the item. All other things remaining same; annual demand remains same; this p will change per unit price if we order more. Suppose, we order 100 or less than 100, this is the price if you order more maybe, if you order 200 and more, this is the price. So, price will change with the order that we are placing and accordingly, we can take use of or make use of or take advantage of that quantity discount possible. This is a, you can say very clear depiction of the quantity discount model.

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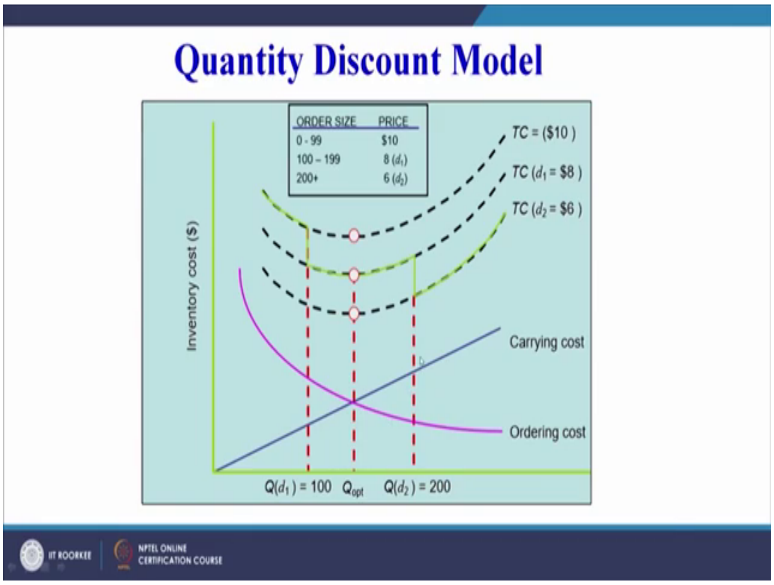

You can see the order size if it is visible to you or I will read it for you. Order size 0 to 99; so, if you less, if you order less than 100, you get a price of dollar 10 per part; if you order 100 to 199, you get dollar 8, if you order 200 plus in a single go, you get a price of dollar 6. So, you see if you increase the number of parts you are ordering, your price comes down. So, 200 and more parts you order your prices number sorry your price is dollar 6 only, but if you order less than 100 your price is dollar 10. So, we can take advantage of this quantity discount and this bottom graph, bottom part of the graph is almost the same.

So, if the quantity is increasing, you know, the ordering cost will reduce. So, as I have explained the, if for the whole year you are ordering once, your order, your ordering cost will be less, but if you order 3 times, your ordering cost will be more. So, if the ordering quantity is increasing, your ordering cost comes down, but your carrying cost increases. Why carrying cost will increase because, suppose in January only you order for the whole year, you have to maintain that inventory throughout the year.

So, your carrying cost will increase with the order quantity, but your ordering cost will reduce with the increase in the ordering quantity and this is, these three curves represent the total cost curve. So, here we can see, therefore your quantity is greater than 200, this red line, the last red line it represents that quantity is 200 or more. So, if your quantity is 200 or more, this is your total cost curve if quantity is between 100 and 200. This is your total cost curve and if it is less than 100, this is your total cost curve. So, we can see that if we order 200 or more, we definitely can get advantage of a total cost will be less. So, here we can try to understand this with the help of an example.

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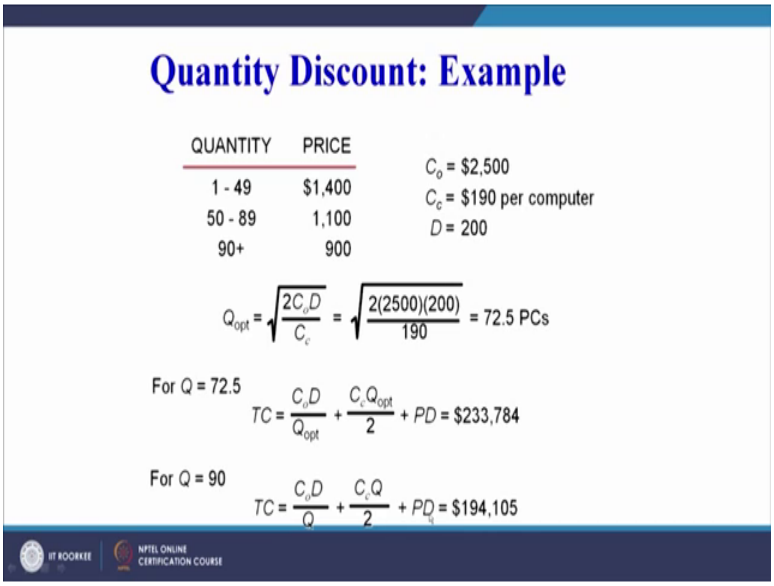

Quantity 1 to 50 or 1 to 49 means less than 50 dollar 40; 1,400, from 50 to 89; 1,100 US dollars in 90 plus 900 dollars. So, you can see that the price is reducing when the quantity that you are ordering is increasing. So, ordering cost is dollar 2500, carrying cost is dollar 190 per computer and d or the annual demand is 200.

So, we have I think got all the information available. So, as per the economic order quantity, Q optimum is 2 into CO that is ordering cost into the annual demand divided by the carrying cost. So, if we calculate we get 72.5 number or 72.5 PCS, we must order in order to take advantage of the economic order quantity. This Q optimum will lead to the minimum total cost of inventory management. So, for Q is equal to 72.5, we can now calculate the total cost. We know the price also the price from here. We can calculate 72.5 PCS, means it is lying between 50 and 89. So, 72.5 is there, so we can calculate. CO is known to us ordering cost, annual demand is 200 known to us, Q optimize have 72.5. Carrying cost is also known to us, that is, dollar 190 Q optimum we have calculated.

So, we we calculate it is dollar 233,784. So, this is our, we can say that cost that is the total cost, but if we do the calculation, we can see that Q is 72.5 the price, p here what we, what p will, we will take p, will be 1,100 dollars; 1,100. I think I am correct; p will be 1,100 because our Q is 72.5, but if we know that if we orders 90 and plus our price will come down to 900 USD or the or the dollar.

So, we can see that if we order Q equal to 90 and do the calculation for price 900 p is 900 d is 200 only, that we have already know Q optimum, we have already calculated. So, if we calculate for Q equal to 90 Q optimum is 72.5, but here we are using Q as 90 because if we order 90 plus our price comes down to 900, so, our p here, that is our price,, is 900. Now, because we are ordering 90, so 900 and d 200.

So, p is 900, d is 200, rest all other things Q will change only. Q will instead of 72.5, now, we will be using Q as 90 and we can see our total cost has reduced substantially. So, this is the importance of the quantity discounts that we can take advantage of. So, very quickly, we can just understand the concept of safety stock or under maybe just revise these words safety stock is the buffer added to and on hand inventory during late time?

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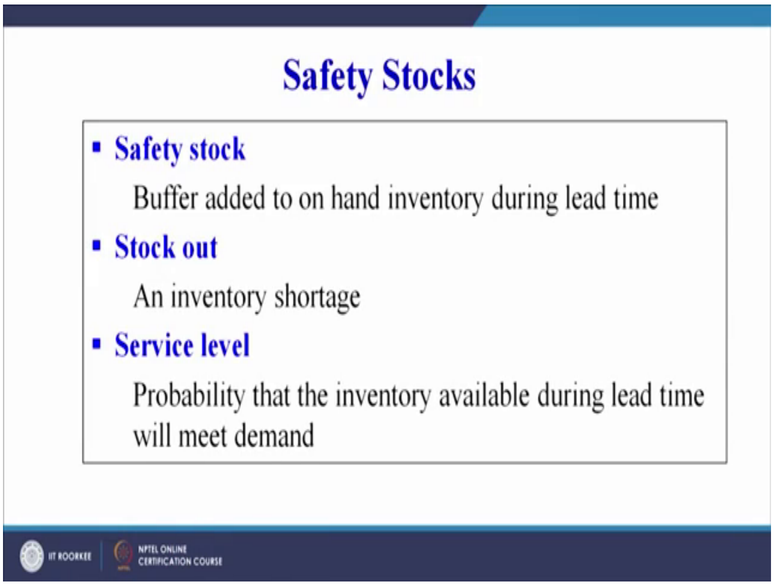

So, we need to keep some safety stock in order to keep our operations up and moving or operations going on continuously. Stock out is an inventory shortage which is a problem or which will lead to the break in the continuity of the operations or the manufacturing process service level probability, that the inventory available during lead time will meet the demand.

So, we have a element of probability also that during lead time, lead time is the time when we have placed the order and the order is yet to arrive and we are using them, whatever available whatever material is available in the inventory. So, service level is the probability that the inventory available during lead time will be able to meet the demand.

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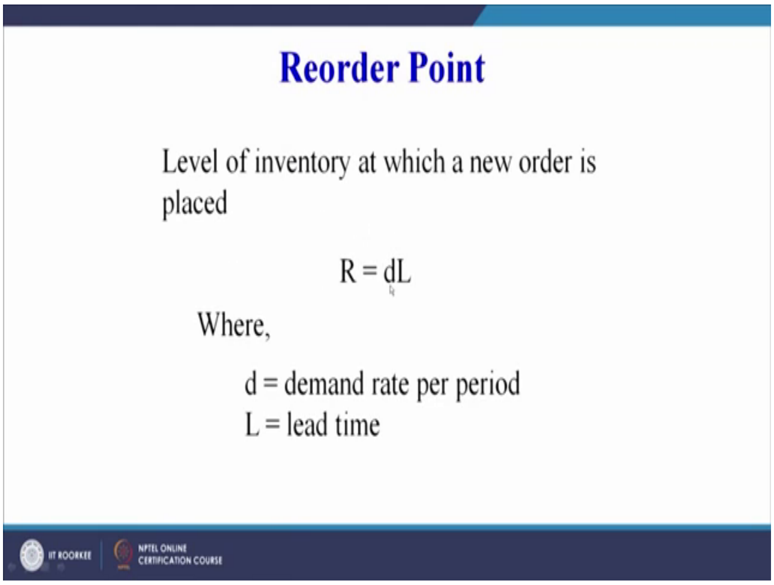

So, reorder point we can calculate R is equal to d; d is the demand rate which we have seen in the production quantity model also and L is the lead time. So, reorder point we can see it. Suppose, the demand rate is 5 components per day and lead time is also 5. So, our reorder point will be 25. So, whenever we reach that, only 25 components are left. We must place an order why because demand rate is 5 and we will get our order after 5 days. So, 5 into 5 that is 25. So, 25 components means that we have to reorder; otherwise, we will reach a situation of stock out if we do not have any safety stock.

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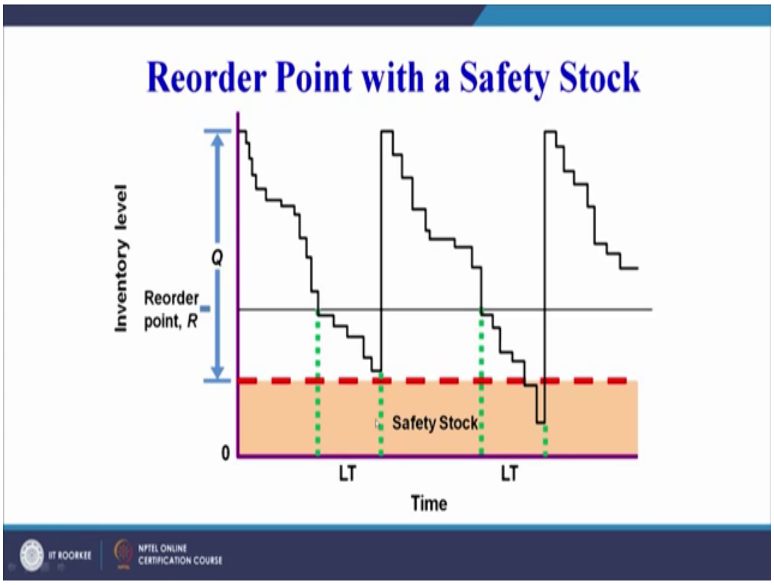

So, here we can see this is our reorder point R variable demand with a reorder point. So, here we can see in all our previous examples, economic order quantity production, quantity model we have seen that the demand rate has been linear or continuous, this line has been linear that we have assumed, but if it is non-linear we have a problem. We do not know that when that demand can be more, when the demand can be less.

And in that situation, many times this situation on your graph represents a stock out we do not have any material. This is our zero level of the material. So, we do not have any material here. So, in order to overcome this thing, we usually keep a safety stock. So, in order to absorb this demand, because this is a non-linear demand, here we do not know that if the demand is linear, it is easy to calculate the reorder point as well as we will mostly not be going to a situation of a stock out, but if it is a non-linear demand there can be more demand. We do not have the material.

So, it in those situations we keep a safety stock and this colored stock. Here, colored portion represents the safety stock and here we can see in case of a non-linear demand rate, even if we have the safety stock, we can manage it. Even if we have level reach the zero level inventory, our safety stock and absorb this demand because of the non-linearity in the demand rate. So, this is the way we can manage our materials and can keep our operations up and going.

So, with this I conclude the today's session and we have covered today the production quantity model, we have covered a numerical problem associated with the production quantity model, we have tried to understand the concept of reorder point, we have tried to understand the concept of quantity discounts; that if we order our material beyond a particular value our price comes down and we must take advantage of those quantity discounts. And finally, we have understood the concept of safety stock, that if our demand rate is non-linear, there is a tendency of a stock out and in order to avoid that stock out and to ensure healthy service level, we must always have a safety stock.

Thank you.