

Operations Management
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Lecture – 54
Economic Order Quantity (EOQ): Problems

[FL] Friends welcome to session 54 in our course on operations management and as we are discussing the eleventh week, our topic is materials management and today is the fourth session of materials management, just to have a brief review of what we have covered in the last 3 sessions. We have covered the basic aspect of materials management, we have covered the scope the objectives, the responsibilities of the materials management section or department.

Then we have covered the classification of the materials based on A B C analysis, also we have seen the V E D analysis and then in the last session that we had, we have seen the economic order quantity that how we can make use of our scientific logic to find out the economic order quantity which will minimize the overall cost of inventory management. So, we have found out there is a simple equation, simple mathematical equation which can help us to calculate the economic order quantity that usually the students call as the EOQ.

So, we have seen the EOQ model in the previous session and today we will try to do certain mathematical calculations, try to solve certain numerical problems based on the economic order quantity model. So, basically what we are calculating is that, what is the economic order quantity that we must order, in order to ensure that the overall cost of inventory management is minimum. Also we can calculate based on the EOQ that how many times we must place the order based on the annual demand required. Then we can also see that after how many days we must place the order.

So, all the answers to these questions can be very easily calculated based on the economic order quantity. So, if we can first see that what is the input required, what type of information is required to calculate the economic order quantity, then how we can calculate the EOQ and then based on this EOQ what further calculations we can do in order to satisfy our, maybe the management that we are ordering as per scientific logic.

We are not wasting any money; we are making proper utilization of the money; that is available for materials or for managing the materials.

So, this number, if we are able to calculate will help us to find answers to all our queries. So, here we are not answering what we have to order. What we are definitely answering that how much we must order, when we must order, so that our overall cost is minimum. So, let us try to see mathematically how do, how we do these calculations.

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Problem 1

$C_c = \$0.75$ per yard $C_o = \$150$ $D = 10,000$ yards

$$Q_{opt} = \sqrt{\frac{2C_o D}{C_c}} \qquad TC_{min} = \frac{C_o D}{Q} + \frac{C_c Q}{2}$$
$$Q_{opt} = \sqrt{\frac{2(150)(10,000)}{(0.75)}} \qquad TC_{min} = \frac{(150)(10,000)}{2,000} + \frac{(0.75)(2,000)}{2}$$

$Q_{opt} = 2,000$ yards $TC_{min} = \$750 + \$750 = \$1,500$

Orders per year = D/Q_{opt} Order cycle time = $365 \text{ days}/(D/Q_{opt})$
= $10,000/2,000$ = $365/5$
= 5 orders/year = 73 days

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So, this is a problem 1 on your screen, you can see C_c is the carrying cost and the product here is the yards. So, the demand is 10,000 yards carrying cost as I have told earlier also, is always specified on you per unit basis. So, the carrying cost is dollar 0.75 per yard and the ordering cost is dollar 150. So, then we can calculate the Q_{opt} . These are the three things if you remember in the formula that we have derived in the previous session, the Q_{opt} is equal to square root of 2 into ordering cost into the annual demand divided by the carrying cost.

So, as is given in the formula for Q_{opt} , we have the ordering cost; that is dollar 150 per order, we have the carrying cost dollar 0.75 per yard and we have the demand annual demand; that is 10,000 yards. So, very easily we can calculate the Q_{opt} , because all the three values are known to us. So, 2 into ordering cost into the annual demand divided by the carrying cost, we put all the values here and our Q_{opt}

comes out to be 2,000 yards. So, this is our economic order quantity that if we order this quantity our overall total cost will be minimum.

Now, if we have calculated this quantity very easily, we can calculate that how many times we must order, how we will calculate that, we know the annual usage is 10,000 yards. So, now, 10,000 yards divided by whatever quantity we are going to order once, so that we have already calculated; that is Q optimum is equal to 2,000. So, 10,000 divided by 2,000 that comes out to be 5 orders per year.

So, that gives us an indication that we must place our order 5 times a year in order to satisfy the demand of 10,000 yards, and every time we must order 2,000 yards. Now we can easily calculate; what is the total cost which is the minimum total cost of inventory management based on the ordering cost as well as the carrying cost. So, we have this inventory cost TC minimum is equal to $C_o D$; that is the ordering cost multiplied by the annual usage demand divided by the Q ; that is the order quantity plus carrying cost multiplied by the order quantity and average of that divided by 2.

So, that is Q divided by 2 is the average value that we are taking. So, Q here is the order quantity. In general if we are not finding out the economic order quantity, this Q is may be a number for any company which has no idea about the economic order quantity or scientifically managing the inventory. So, what will happen? Intuitively a person will be ordering that, send 500 components today, maybe after 3 months he may send 450 components today.

So, this order quantity Q is a variable, if we are not using the mathematical relation that we are using here. Similar is the case, if you see for a shopkeeper if he is selling suppose the breads he will order randomly, because he will have a prior experience and he will say my order for next maybe 10 days is 20 breads of this size, 30 breads of this size and maybe 20 breads of this size. So, that is a random number selection based on his experience and many times, he may feel that some of the breads he has ordered have not been sold and are stale and he has to just put them into the bin.

So, the point is that, when we are using scientific logic, we will be able to find out this number Q , which will help us to minimize our total cost of inventory management. So, our TC if we remove this minimum here, the total cost of inventory management is the ordering cost multiplied by the annual usage divided by the order quantity plus the

carrying cost and the average order quantity; that is Q by 2, but since we are using mathematical logic. So, we are trying to find out the optimum Q value; that is we are calling it as the economic order quantity.

If we put Q optimum value here we are sure that the total cost that we are calculating is the minimum total cost of inventory management. So, in this formula we are putting it minimum. So, Q automatically becomes Q optimum and Q optimum we have already calculated is 2,000 and based on that we can do the calculation. So, TC minimum is 150; that is the ordering cost per order multiplied by the annual usage; that is D that is 10,000 yards divided by the Q optimum value that is 2,000 yards plus the cost of holding the part that is 0.75 dollars per yard carrying cost multiplied by the Q optimum value, and divide by 2 that is the average of that.

So, this summation will give us the TC minimum that is dollar 750 plus dollar 750 equal to dollar 1500. So, we have calculated Q optimum, it has helped us to minimize our total cost of inventory management. Now also we can calculate the order cycle time that after how many days we must order. We already know from this calculation that if we have a problem at hand where we have to order 10,000 yards per year, we know the carrying cost per yard, we know the ordering cost of placing a single order, we can very easily find out that; what is our Q optimum.

We can calculate how many times we must order and the next question can be that after how many days we must order. So, how many times we have to order; that is 5. So, very easily we can calculate the order cycle time also. So, suppose we take 365 days in a year. So, 365 days divided by the number of times we have to order or the orders per year that is 5 times, we have to order.

So, 365 divided by 5 we say that after every 73 days we have to place an order of 2,000 yards in order to have the minimum total inventory management cost or total cost of inventory management as being minimum. So, this way we can make use of the equations to solve the problem and to minimize the cost and in nutshell to save the money for the organization. So, let us see another problem, this was the most simplistic type of problem, let us see another problem.

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Problem 2

ABC corporation has got a demand of particular part as 10,000 units per year. The cost per unit is Rs. 2 and it costs Rs. 36 to place an order and to process the delivery. The inventory carrying cost is estimated at 9 percent of cost per unit.

Determine:

- (i) Economic order quantity
- (ii) Optimum number of orders to be placed per annum
- (iii) Minimum total cost of inventory per annum

Now an ABC Corporation has got a demand of a particular part at 10,000 units per year. So, again we are keeping our demand as 10,000 units per year, the cost per unit is rupees 2. So, per unit cost is given as rupees 2 and it costs rupees 36 to place an order and to process the delivery. So, the ordering cost is known to us, it is rupees 36 per order, the inventory carrying cost is estimated as 9 percent of cost per unit. Now cost per unit as per our problem is, as you can see on your screen it is rupees 2 per unit.

So, 9 percent of that is the carrying cost per unit. So, what we need to find out? We need to find out economic order quantity; that is EOQ, then we have to find out optimum number of orders to be placed per annum and the minimum total cost of inventory per annum. So, in the previous case we have seen the directly the values were given we had C_o , then we have D and then we have calculated $2 C_o D$ divided by C_c square root the economic order quantity, then we have calculated the number of orders, then we have calculated the ordering cycle after every 73 days.

But here we are, we have been given the same information, but in a, maybe form of a problem statement and then we have to deduce certain information from the statement. We have to find out that C_o C_c values, we have to find out D and then do the same calculation in order to find out the economic order quantity. So, let us again see whatever is already given to us.

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Given data:

Annual demand for parts (D) = 10,000 units/annum

Ordering cost = Rs. 36/order

Cost per unit = Rs. 2/unit

Inventory carrying cost = $0.09 \times 2 = \text{Rs. } 0.18/\text{unit}$



So, we can see here annual demand for parts is 10,000 units per annum which is given to us, ordering cost that is C_o is given to us; that is rupees 36 per order, also the cost per unit is given to us rupees 2 per unit and inventory carrying cost is 9 percent of the unit cost of the product. So, inventory carrying cost is rupees 0.18 per unit. Now based on this if you see, we know we have got the information we know D that is the annual usage or annual demand, we know the ordering cost and we have calculated with the simple calculation, the carrying cost per unit also.

Now if we know these three quantities; that is the demand, annual demand, the ordering cost per order and the carrying cost per unit very easily we can calculate the economic order quantity. Now economic order quantity; that is Q_{optimum} is given by twice of ordering cost multiplied by the annual demand divided by the carrying cost. So, square root of that.

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$$\begin{aligned}
 \text{Economic order quantity (EOQ)} = Q_{\text{opt}} &= \sqrt{\frac{2C_oD}{C_c}} \\
 &= \sqrt{\frac{2 \times 10000 \times 36}{2 \times 0.09}} \\
 &= 2000 \text{ units}
 \end{aligned}$$

So, if we put all these values here 2 into 10,000 into 36 divided by 2 into 0.09; that is 0.18 that we have already calculated. So, we get 2,000 units as the economic order quantity. Now once EOQ is known, annual demand is known to us, it is easy to calculate how many times we must order. So, here also annual demand is 10,000, economic order quantity is 2,000. So, without much difficulty we can say 5 times we have to order in order to meet this demand of 10,000.

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$$\begin{aligned}
 \text{Optimum number of orders} &= D/Q_{\text{opt}} \\
 &= 10000/2000 = 5 \\
 T_{\text{cm}} \text{ (minimum total cost of inventory per annum)} \\
 &= \frac{C_oD}{Q} + \frac{C_cQ}{2} \\
 &= (36 \times 10000)/2000 + (2 \times 0.09 \times 2000)/2 \\
 &= 180 + 180 = 360 \text{ Rs./annum}
 \end{aligned}$$

Now, optimum number of orders that is, as I as I have already told that our annual demand divided by Q optimum; that is 2,000. So, 10,000 divided by 2,000 is equal to 5. So; that means, that 5 times we must order in order to meet the annual demand. Now, the

third question was to calculate the T minimum. So, minimum total cost of inventory per annum. So, T minimum is equal to the ordering cost multiplied by the demand divided by the order quantity.

So, in our case when we are talking of minimum, we will take Q as Q optimum plus the carrying cost into the average order quantity; that is Q by 2. So, if we put all these values our ordering cost is rupees 36 per order, annual demand is 10,000, then our economic order quantity that we have calculated is 2,000. So, plus 2 into that is a unit cost multiplied by 9 percent of that that is 0.09.

So, that unit cost into the 9 percent of; that is our carrying cost that we have already calculated 2 into 0.09 into the economic order quantity; that is 2,000 divided average value of that divided by 2. So, if we add this, so we get 360 rupees per annum as the cost of managing the inventory. So, this is maybe two simple problems we have taken, based on the economic order quantity. There can be different types of problems that can be solved using the EOQ model.

So, we have seen that there are a few parameters; maybe sometimes some parameters may be given to us additional parameters we have to calculate. So, here in most of the cases we have seen three things are given to us, we are given the annual demand, we have been given the ordering cost for placing a single order, we have been given the carrying cost for carrying you any particular component or part of carrying cost per unit is given.

So, we can very easily calculate the economic order quantity and from there we can very easily calculate the total minimum, total cost required for managing the inventory. Now suppose some of you may be wondering that $T C$ minimum in our case is ordering cost multiplied by the demand divided by the order quantity plus carrying cost multiplied by the quantity and average of that; that is Q by 2. Suppose this Q value, we do not know and we do not know the formula also, we are ordering at our free will whatever requirement we see we place an order.

So, our Q is not fixed or it is not may be economic order quantity, it is some random quantity that we have decided and we are following that, we are still meeting the annual demand of 10,000, but we are not following the economic order quantity, we are using it at a random level only. So, if you see in the previous two problems, we have calculated

how many times we have to order based on the economic order quantity in both the cases.

Fortunately we have taken, we have seen that the annual demand is 10,000 and the economic order quantity comes out to be 2,000. So, number of orders becomes 5. So, the materials manager know that we have to order 5 times in an year to satisfy this annual demand of 10,000 components and our total cost will also be minimum.

Now, my question is that, this Q, suppose we do not know and that number of orders also is not fixed. So, we are or Q is variable, we are changing Q today, we have ordered 5,000, maybe next month we are ordering 400 then for next 4 months we are not at all ordering. Then again we are ordering maybe 5,000 components together. So, we do not follow any specific sequence number of orders, number of components to be ordered in a single order. So, we are not using any scientific method of inventory management, we are doing it randomly.

So that may lead to; in most of the cases, may lead to spending extra money for managing the inventory in, maybe very few cases our intuition, may be such that we are successful in spending less money as per the scientific method that may be, maybe 1 percent or 2 percent cases. Only in most of the cases when we are doing random ordering we are not following the scientific system of materials management.

We will be overspending only and that thing we want to explain with the help of an example and that example is now we are going to cover.

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Problem 3

XYZ corporation currently practices the following system for the procurement of an item:

Number of orders placed in a year = 8

Ordering cost = Rs. 750/order

Each time order quantity = 250

Carrying cost = 40 % of cost per unit

Cost per unit = Rs. 40

Comment on ordering policy of the company and estimate the loss to the company in not practicing scientific inventory policy.



So, this we are trying to understand now with the help of a problem and on your screen you see the problem 3 for today. So, whatever I have explained that if we are not following the scientific method of inventory management, we may be overspending on our materials management. So, that overspending sometimes may not be good for the organization. I must say not sometimes, almost all times, it is not good for the organization to spend extra money or the money that could have been easily saved by following a scientific method of materials management.

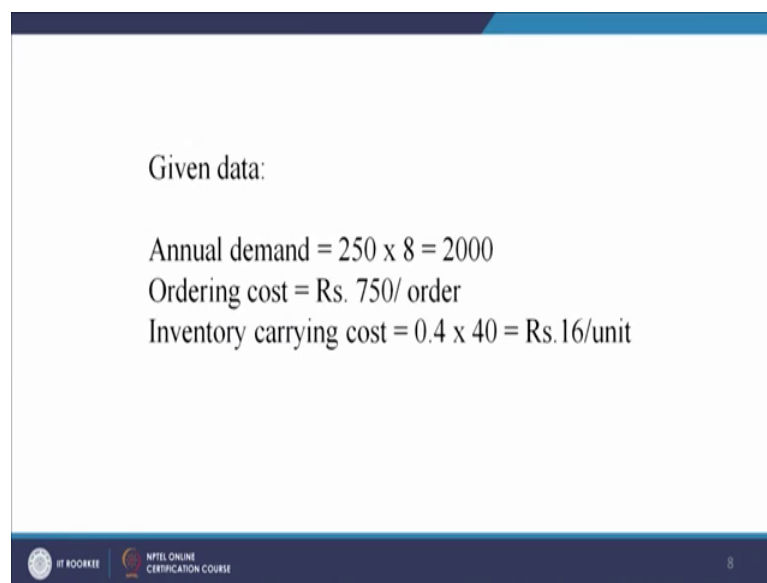
So, this we will try to help explain with the help of this example. So, this x y z corporation currently practices the following system for procurement of an item. So, this is the current method of materials procurement. So, you can see number of orders placed in a year. So, they are placing 8 orders in an year, ordering cost is rupees 750 per order. Each time they order 250 units carrying cost is 40 percent of the cost per unit and the cost per unit is given as rupees 40.

So, you can see whatever information is required for calculating the economic order quantity, they already have, but they are not finding the economic order quantity, because they are not using the scientific method of materials management. So, what they are doing? They are following this policy, they have framed this policy and they are following it. We will try to see that if they follow a scientific method of managing the materials by using the economic order quantity model what will be the total cost, and following the current method what is the total cost and what is the difference of materials? What is the difference in the cost of materials management?

So, what is the question here? The question is, we have to comment on the ordering policy of the company and estimate the loss to the company in not practicing the scientific inventory policy. We have to comment on this policy that the company is following and then we have to compare it with the scientific policy, and see that; what is the loss to the organization by not following the scientific policy of inventory management.

So, whatever information is given here, you can see number of orders placed in a year is 8 ordering cost is given. Each time order quantity is given carrying cost is given in terms of the percentage of the cost per unit and cost per unit is also given the information; that is available is as per the previous problem only. Now let us see what is a given data? Given data is already on your screen.

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Given data:

Annual demand = $250 \times 8 = 2000$
Ordering cost = Rs. 750/ order
Inventory carrying cost = $0.4 \times 40 = \text{Rs. } 16/\text{unit}$

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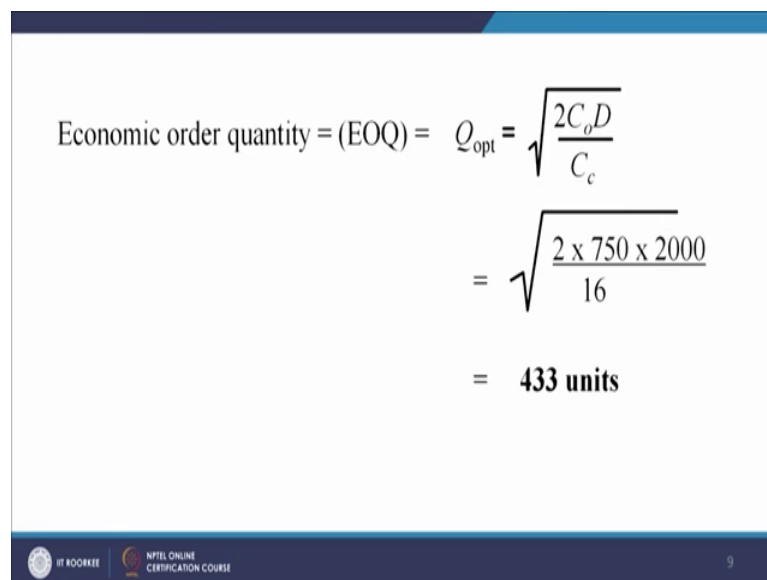
Annual demand is given, annual demand we can calculate very easily, because 8 times they are ordering 250, they are ordering or 250 units they are ordering every order. So, 8 orders of 250 units each; that means, that the annual demand is 2,000 units ordering cost is rupees 750 per order and the inventory carrying cost is 40 percent per unit cost; so, 40 percent of rupees 40; that is rupees 16 per unit.

So, in the previous slide we can again see carrying cost is 40 percent of the cost per unit and cost per unit is rupees 40. So, from here very easily, we can calculate the carrying cost. Now you can see that these three quantities are only required for calculating the

economic order quantity and the equation is. I think we have solved 2 3 problems. We have seen the derivation also, simple derivation in the previous session.

All of us know; it is square root of 2 into the ordering cost multiplied by the annual demand divided by the carrying cost, everything is available with us, but still the company is not following the scientific policy of inventory economic order quantity.

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Economic order quantity = (EOQ) = $Q_{opt} = \sqrt{\frac{2C_oD}{C_c}}$

$$= \sqrt{\frac{2 \times 750 \times 2000}{16}}$$
$$= 433 \text{ units}$$

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If they calculate, this will be the economic order quantity; that is 2 into the ordering cost multiplied by the annual usage or annual demand divided by the carrying cost; this comes out to be 433 units. Now you can see that EOQ is 433 units, but the company is ordering only 250 as the single order.

So, therefore, we can see that the economic order policy is giving us some other value; that is 433 units, but they are following their own policy of ordering 250 units in a single go. So, how these will, this is going to affect financially, we can see, because this formula has been calculated for total cost being minimum. So, we can calculate the total cost; that is this minimum, minimum total cost for inventory per annum.

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$$\begin{aligned}
T_{cm} \text{ (minimum total cost of inventory per annum)} \\
&= \frac{C_o D}{Q} + \frac{C_c Q}{2} \\
&= (750 \times 2000)/433 + (16 \times 433)/2 \\
&= 3464.2 + 3464 = \mathbf{6928.2 \text{ Rs/annum}}
\end{aligned}$$

So, this we can see as per standard formula we have been using it today quite often. So, C_o into D divided by Q plus C_c into average order quantity. So, our Q coming here is 433. So, with this we know that ordering cost is 750 per order multiplied by the demand is 2,000 per annum divided by economic order quantities 433 plus carrying cost is 40 percent of the per unit cost; that is 40. So, 40 percent of 40 comes out to be 16 multiplied by the economic order quantity 433 average of that divided by 2.

So, our inventory cost or total minimum cost is coming out to be rupees 6928 per annum. This is as per the economic order quantity model. Since we had all the values we are studying EOQ, we know how to calculate EOQ and how to calculate the total cost based on EOQ and this is the minimum total cost, but as per the company policy, they are using Q as 250 and number of times they are ordering is 8.

So, if they use Q other things remaining same, the ordering cost remains same, 750 per order annual usage is 2,000 carrying cost again is rupees 16; that is 40 percent of the unit cost; that is rupees 40 only Q is changing. So, Q we are using a scientific method as 433, but they are using it as 250. So, let us see if we use it as 250, what is the cost.

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$$\begin{aligned}
&\text{Total inventory cost under present system} \\
&= \text{Annual ordering cost} + \text{Annual inventory carrying cost} \\
&= 8 \times 750 + (250/2) \times 16 \\
&= 6000 + 2000 = \mathbf{8000 \text{ Rs.}}
\end{aligned}$$

$$\begin{aligned}
&\text{Loss to the company in not operating scientific inventory policy} \\
&= 8000 - 6920.2 = \mathbf{1071.72 \text{ Rs.}}
\end{aligned}$$



So, total inventory cost under present system, what they are using annual ordering cost plus annual inventory carrying cost. So, annual ordering cost we can see 8 times. They are placing the order multiplied by 750 is the ordering cost per order. So, that is 6000 plus the average value of the order quantity; that is 250. Their ordering was average value 250 by 2 multiplied by the carrying cost that is 16; that is 40 into 40 percent of the per unit cost; that is 40 percent of 40 that is rupees 16.

So, this value comes out to be rupees 800. So, they are; sorry, 8,000. So, they are following a policy in which the total inventory cost is coming out to be rupees 8,000, but as per the economic order quantity, the cost is coming out to be somewhere near 6,900 rupees. So, what is the loss?

So, the loss to the company is in not operating. The scientific way is 8,000. They are currently spending following the current inventory policy, but as per EOQ, it is 6,920. So, at least 1,100 rupees approximately or 1,071 rupees approximately, they are losing by not following the scientific policy of inventory management. So, with this we conclude the today's session with a moral that scientific methods definitely give us an edge over the intuitive methods.

And in this case, we have seen that if the company follows the scientific method of inventory management, they will definitely save certain money and that money can be used for any other useful purpose. In our last session on materials management, we will

focus on other problems related to materials management and then we will enter into the last week of our discussion on this course on operations management.

Thank you.