Operations Management
Dr. Inderdeep Singh
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture – 53
Economic Order Quantity (EOQ) Models

[FL] Friends, welcome to session 53 in our course on operations management. So, today we are going to discuss the economic order quantity model and we will try to see and understand this model with the help of an example. If you review the previous sessions that we already had in this topic of materials management, we have covered the basic aspects of materials management, why it is required, what are the objectives, what is a scope, what are the responsibilities of the material management department and then we have tried to understand the classification of the materials.

in the last session if you remember we have seen A B C analysis, we have seen V E D that vital essential and desirable materials, and if you remember we have already put some questions for understanding or for answering. Now these questions are what to order, when to order, how much to order, how many times in a year you must order.

And we need to find answers to all these questions, when we are managing the inflow of materials into the organization and then we are using those materials and finally, we are accounting the materials, we are accounting for the materials that have been procured by the organization. So, friends we need to find answers to all these questions.

Now what are the questions, again I am repeating that what we must order; that is what are the materials to be ordered; how much quantity we must order, then the next level is that when we must order. So, we need to find out the quantity that is a number. We need to find out the time where that when we must order. We need to set the reorder level that after this much of usage we are going to order the new consignment of materials. So, we need to find answers to all these questions and it is not that difficult, if we have some preliminary data available with us; based on that data very easily. We can calculate mathematically that; what is our economic order quantity.

Many times; we will see that we make use of the quantity discounts also, but here today we will take a simple problem without the quantity discount and try to understand that how do we do the calculations based on the economic order quantity. Now first thing we must understand is the three words; that is the economic, order and quantity. So, first thing is economic, means it is optimal that if we order this much quantity then we are going to save money.

So, first thing is economic, then the order; order means how much we must order, it should be economic. For example, it should give us the minimum overall material management cost. So, whatever we are ordering, whatever volume we are ordering, whatever number we are ordering must be economic and the last part is the quantity; that is in terms of number. So, economic order quantity is a number; that is the number of equipment, number of material, number of may be it can be the number in terms of the kgs number, in terms of whatever it is ah, may be a quantity that if we order our overall cost of materials management will be minimum.

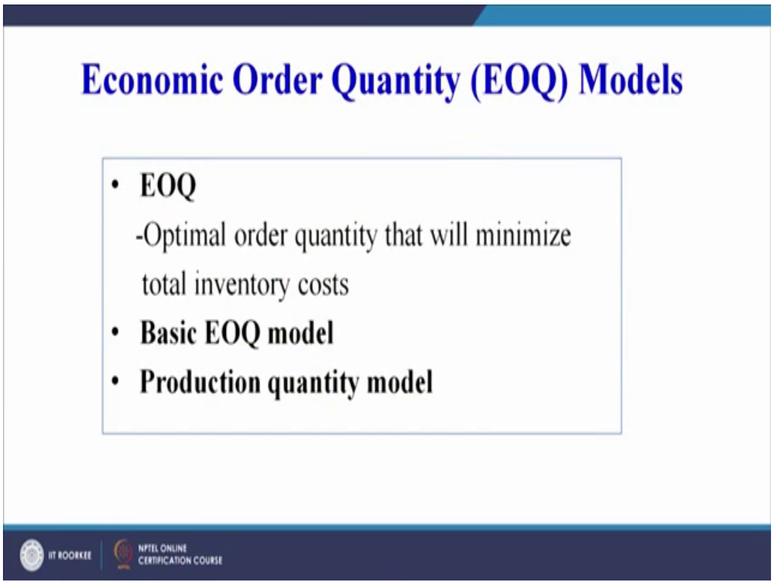
Now, what will make the cost of material; one will be the cost of material itself, another one is the cost of ordering or placing an order and the third cost is the cost of maintaining that inventory in our storehouse or the warehouse. So, three different costs are involved in materials management; the actual cost of the material, the cost of placing the order and the carrying cost. So, we will try to minimize this overall cost. Our major focus will be on the last two cost components; that is the ordering cost and the carrying cost. So, carrying cost will be dragged directly proportional to the period of time for which we are storing our material in the warehouse.

Ordering cost is directly proportional to the number of times we are placing an order. For example, if every month we are placing an order and cost of ordering is rupees 10, so 12 into 10 will be that annual ordering cost, because we are ordering 12 times and each time we have to spend rupees 10. Similarly, the carrying cost is usually specified per unit. So, if we have 100 units that are stored for 12 months and for every month the cost is rupees 10.

So, for every unit we will see that for one particular product or one particular raw material component, the cost will be the unit cost of holding it per month multiplied by the number of months for which we are holding that part in the factory or in the warehouse. So, if we are talking of 12 months period, it is stored for 12 months 12 multiplied by the unit cost of per month. So, that is rupees 10 per month. So, 120 rupees will be the carrying cost per unit for that part, which has been stored for 12 months, 12 months and unit cost is rupees 10 per month.

So, therefore, the addition of these two cost will be the total cost of managing the inventory and the cost of material will then be added into this total inventory cost to get the total cost of the material. So, we have to see that what is the economic order quantity that we must order. In order to minimize the total cost that we are spending on inventory management and this is our topic today, how to calculate this economic order quantity.

(Refer Slide Time: 06:27)

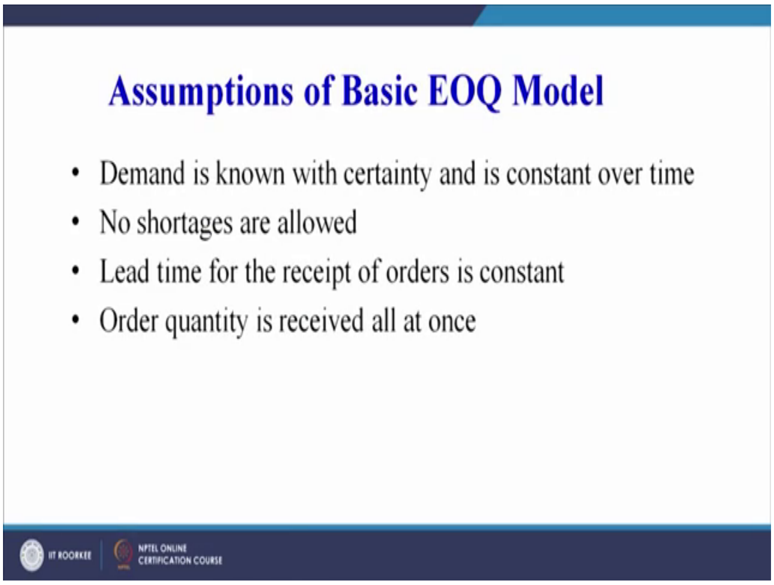


Now, let us start our presentation with this brief introduction about economic order quantity. EOQ is optimal order quantity that will minimize the total inventory cost, as I have already explained, it is the optimal or the economic order quantity that will minimize the total inventory cost. So, we are not focusing on the actual cost of the raw material or the component or the part for which we are doing the economic order analysis or economic order quantity analysis. We are focusing on the inventory costs only.

An inventory costs are two types of costs, only the carrying cost and the second one is the ordering cost. So, sometimes the carrying cost is also used by many authors as the holding cost. So, we are using carrying cost for keeping the inventory in the warehouse and the other one is the ordering cost that we have to pay while placing the order of the cost spent in ordering the material.

So, basic EOQ model we will try to understand with the help of a diagram and then there is another model which is a production quantity model, which also helps us to calculate the order quantity which we must order in order to minimize the total inventory costs. Now before this model is explained with the help of a diagram, let us see; what are the assumptions of the economic order quantity model. And these assumptions have to be kept in mind, because these are the assumptions based on which the model has been developed or the equations have been developed.

(Refer Slide Time: 07:51)



So, first is that the demand is known with certainty and is constant over time. So, it is very easy to understand that whatever we are producing, the materials are being used for that production process, the demand for that product is constant. So, constantly the materials will be used and we have to, maybe have a relationship of the use of materials which is linear, because there is a constant demand for that product in which we are using the material.

So, the demand is known with certainty and is constant over time. So, for variable demand or variable consumption of materials, this type of model is not valid, no shortages are allowed. So, it is now never going to happen that there is shortage and the process has been stopped. So, the model is valid only when there are no shortages happening. During the production process lead time for the receipt of orders is constant.

Suppose the lead time is 7 days and on September 21st I am ordering. So, it is bound that after 7 days I will get the order.

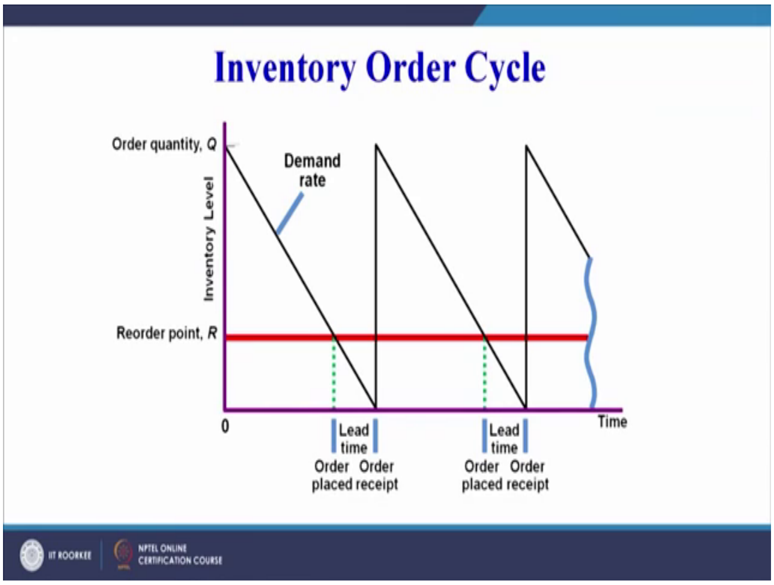
Again I am ordering on January 21st, after 7 days the order must be at my premises that is the lead time between placing the order and receiving the quantity is fixed and is not variable, it will not vary; otherwise the model is not valid and last is order quantity is received all at once. So, we are not getting our order, may be after may be at a rate. At a rate means that suppose we have ordered 100 components. On September 21st we have ordered 100 components.

So, it is assumed in EOQ model that if the lead time is 7 days. After 7 days we will get all 100 component in a single consignment or in a single packet. It is not going to happen that after 7 days we are receiving 20 components first and then after 2 days and other 20 components and over the next 5 days we are getting [to/twenty] 20 components every day and then multiplied by 5 day. So, we are getting our order as 5 different packets of 20 each adding up to 100. No all 100 will come together and will be received in our factory or in our premises all at once.

So, under these assumptions the economic order quantity model has been developed. So, if this type of scenario exists in the organization, it is easy for the organization to calculate that what is going to be the economic order quantity for the company. Again I will read all these points for you; demand is known with certainty and is constant over time, no shortages of material are allowed, lead time for the receipt of orders is constant and order quantity is received all at once.

So, we will see that when the model changes some of the assumptions also change. Now this is the inventory order cycle on your screen, on y axis we have the order quantity or we can also call it as the inventory level or the number of components that are available in the warehouse or the storehouse.

(Refer Slide Time: 11:35)



So, it is being represented by Q on y axis, we have the quantity on x axis, we have the time, you can see here it is time. So, it is a plot between the quantity where that is available or the inventory that is available on day zero with time on x axis. So, we can see here this is the demand rate as we have assumed that demand rate is constant over time. So, the rate of usage of material is constant over time and that will depend upon the product that we are producing. If continuously we are producing the product our demand rate will be constant over time. So, the demand of the product is constant and therefore, the demand of the material requirement is also constant.

Why, because suppose we are producing 100 cars per week and this demand is constant. So, 100 cars per week means that we may require 400 tires per week or maybe we may require 100 steering wheel systems per week. So, depending upon that the 100 that demand rate is constant for the steering wheel systems or the tires for the automobile. So, if the production is continuous, demand is constant. So, this demand rate will automatically be constant. So, here the demand rate as per assumption is constant and you can see that the quantity is being used continuously over a period of time, as soon as you reach the reorder point that is depicted by the red line here.

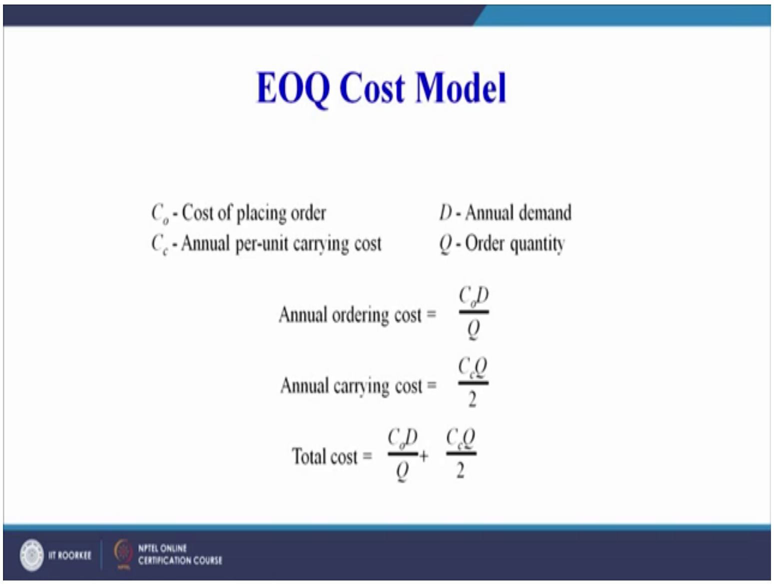
So, as soon as we reach the reorder point we have to place a new order. So, here we place the new order and this is the lead time on time scale. This dotted green line depicts the time when we have placed the order and when we are receiving our order this is the lead time. So, we receive our order here and as is the assumption that all the volume of material or all the number of material that we have ordered, supposed 100 components or 100 parts we have ordered, all will be received all at once.

So, we receive 100 here and our quantity in the inventory again is replenished and comes to the original queue level of our maybe inventory. So, this is the order quantity, initially we had queue. So, suppose we had 100 we are using it at a constant rate, maybe after we have used 80, this is maybe representing 20. So, when 20 are left with us we reorder. So, we use those 20 and by the time we are at zero level our 100 components have again arrived, because we have ordered when we were left with 20. So, again we have 100. So, 100 is replenished here. Again we start using that those 100 components at a constant demand rate.

And once we have reached this reorder level of 20 we reorder and then we have finished the usage of this 20, again it is replenished by 100 components and this cycle continues. And to add to this cycle we also have a safety stock which is below this zero level. So, in many cases we will keep a safety stock also, suppose because of any problem this lead time is extended or we are not able to maybe get our replenishment of material here.

this may further go down and take the material from the safety stock and again as soon as the order is received we replenish it to the order quantity level. So, this is our maybe economic order quantity model based on this now we have to find out the order quantity that; what is the order quantity which will give us the minimum total cost of inventory management? So, here we can see in economic order quantity model, we see that there are two types of costs involved.

(Refer Slide Time: 16:09)



As I have already explained in the beginning of this session that we have a cost of placing an order and we have a cost annual per unit carrying cost which I have explained in the beginning today, that suppose the annual cost we have to calculate for inventory management. So, the carrying cost is, suppose I have taken an example rupees 10 per month every, for every component; so, the 10 per component per month multiplied by 12 months. So, annual cost will be 12 into 10 for one component.

Suppose that component is stored in inventory for 1 complete year, so it will be 120 rupees. So, that is the carrying cost for that component, an ordering cost will be based on the number of orders. So, suppose we order every month and the cost of placing the order is rupees 10. So, we are placing 12 orders. So, 12 into 10 120 will become the ordering cost. So, number of orders into the cost of placing a one order or a single order will give us the annual ordering cost and similarly the annual carrying cost per component.

So, sometimes we may have the carrying cost on daily basis also, that carrying cost for the component is rupees, maybe 50 paise or rupees 1 per day, so that is also possible. So, these are the two types of costs involved in managing the inventory. Now in order to do the calculation we must have the annual demand; that is what is going to be the requirement of the material for which we want to identify or calculate the economic order quantity, what is going to be the annual demand of that material also.

The Q is the order quantity which is our variable for which we want to calculate that what is, what must be the optimum order quantity. Now the annual ordering cost, annual ordering cost as I have already explained will be the cost of placing the order; single order and multiplied by the number of orders. Now number of orders; how we can get or how we can calculate, it will depend upon the total annual demand as well as the quantity that we are ordering once. So, suppose the annual demand is 10,000 parts or components.

So, annual demand is 10,000 components order quantity. Suppose we decide that we are ordering 200 parts in one order. Now 10,000 divided by 20 or maybe sorry 200 we will get the number of orders and this number of orders multiplied by the cost of placing an order will give us the annual ordering cost. So, I think I have made it clear that we have taken an example that the annual demand is 10,000 and Q we are assuming. For example, 200 per order we are doing.

So, 10,000 divided by 200 will give us the number of orders and multiplied by the ordering cost per order will give us the annual ordering cost. Similarly we have to calculate the annual carrying cost also and that can be calculated by the carrying cost per component into Q, Q is our order or the order quantity. So, carrying cost, but usually we divide it by 2 why we have divided it by 2, because we take the average quantity for doing the carrying cost analysis, why, because suppose today in the beginning of the month, I have 100 components in my inventory over the month, I am using it and towards the end of the month I may be left with 0 component.

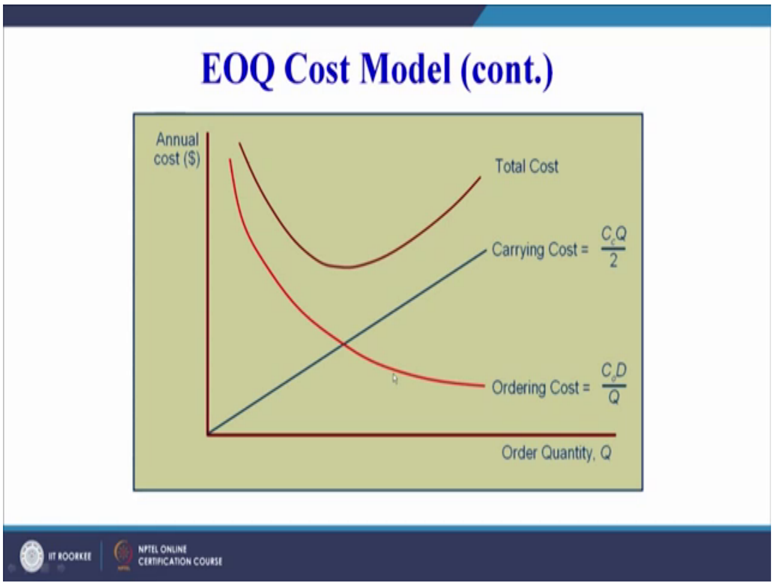
So, if the components are being used continuously and therefore, we take a average value of the quantity and many times it may so happen that we receive the order in the middle of the month and the parts are used, maybe in the second week of the month when and then the new order is coming. So, whatever we are getting towards the middle of the month again we have, we have started to use it for the next 15 days. So, therefore, we take the average value of the quantity that is Q by 2. So, we know that; what is the carrying cost per component multiplied by the number of components; we can calculate the annual carrying cost.

But the number of components will be taken as an average. So, then the total cost will be the, total inventory cost will be the ordering cost plus the carrying cost. So, again just to, because the formula will be based on these two quantities only; so, let us again maybe try to understand the annual ordering cost. So, annual ordering cost is the cost of placing a single order multiplied by the number of orders.

The number of orders we can calculate by the annual demand divided by the quantity that we are ordering in a single order. So, that will give us the number of orders. So, the number of orders when we will multiply by the cost of a single order, we will get the annual ordering cost and similarly annual carrying cost will be the cost of [ca/cost] cost of carrying one particular part or equipment multiplied by the number of parts or components.

So, that is, that will give us an average value of the number of parts or component when we add the two carrying and the ordering cost we will get the total cost. Now let us try to find out how these will vary with the quantity with the parameter Q.

(Refer Slide Time: 27:11)



Now, this is the annual cost on y axis on x axis we have the order quantity. So, we can see if the order quantity is increasing the ordering cost is decreasing. So, now, you can very easily see this is d by Q. So, D remains constant. So, D means the annual usage. So, annual usage if you say 10,000 and now suppose you are ordering 5000 components in a single go. So, 10,000 divided by 5,000. So, only twice you are ordering in a year. So, your carry ordering cost for a single order multiplied by twice; only here are ordering.

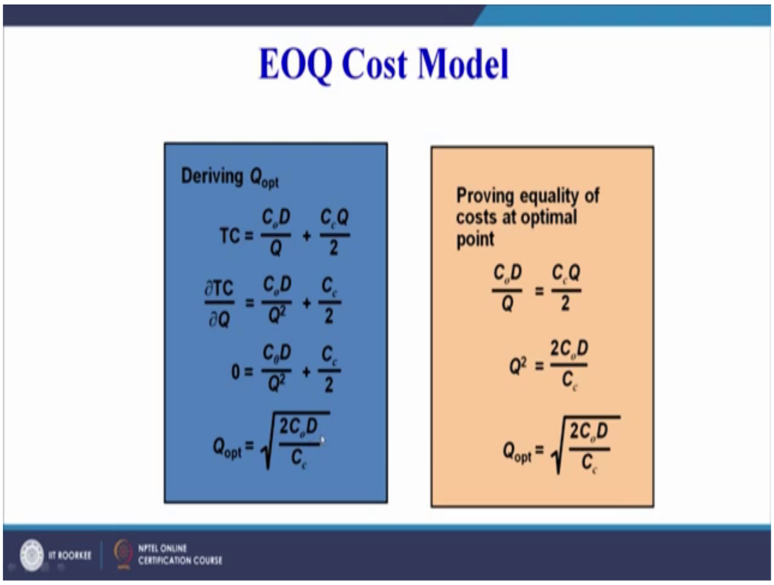
So, your total ordering cost is less; suppose you are ordering 12 times a month or maybe 10 times a month easier for calculation. So, suppose you are ordering 10 times a month. Sorry, not a month 10 times a year, in 10 times a year. Now, you can see what is your annual use over annual demand 10,000 you are ordering 10 times. So, when you are ordering 10 times, means that it was single go you are ordering only 1,000 components or parts only. So, 10,000 is the annual usage every, maybe after every specified interval of time you are ordering 1,000 components only. What is the number of orders 10,000 divided by 1,000; that is 10 times your ordering.

So, if you are ordering 10 times multiplied by the cost of placing a single order your cost of ordering will be more. So, you can see that when the order quantity is increasing your ordering cost will decrease, and just to explain if all the 10,000 components or parts we are ordering in a single order only. So, your C o or ordering cost is multiplied by 1 in single order only. So, your overall ordering cost is less and if you are making maybe 100 orders 100 multiplied by the ordering cost of placing single order your cost will be more.

So, if the ordering cost, if the order quantity is increasing your ordering cost will decrease. Now the carrying cost. So, if your order quantity is increasing your carrying cost will increase, because if you are ordering more number, suppose in single order only you get all your 10,000 parts which is an example for annual usage. So, I have assumed that annual usage is 10,000 components or part. So, you are ordering only once. So, in the beginning of the year in January you order all these 10,000 parts. So, all these 10,000 components have arrived at your premises. Now this 10,000 you are using them, but you are holding them maybe for the next 1 year.

So, you can see that your holding cost or carrying cost will be high. So, carrying cost is C c into Q by 2; that is the average value of the order quantity. So, we can we have seen the carrying cost also, we have seen the ordering cost also. So, both of them will add up to the total cost. Now we will try to see that the total cost has to be minimum and our total cost is minimum, where the carrying cost and the ordering cost are equal.

(Refer Slide Time: 25:36)



So, based on this ah, maybe equality we can calculate the Q optimum value. So, at our total cost minimum point our ordering cost and our carrying cost was same. So, if we ah, if we equate them very easily we can do a simple mathematics and calculate that Q optimum is equal to twice the ordering cost multiplied by the annual usage divided by the carrying cost square root of that. So, this formula is given in almost all good books of inventory management and this is used for calculating the economic order quantity or the other way of deriving the Q optimum can be.

We can write a total cost equation by adding the ordering cost and the carrying cost and then differentiating it to find out the Q optimum value and again the Q optimum by this method also, is twice the ordering cost into the annual usage divided by the carrying cost square root of that. So, the formula is same, we can get it by equating at that point where the total cost is minimum, the ordering cost and the carrying cost and then we can very easily find out Q optimum.

So, with this, I close the today's session. We have tried to see that what is the economic order quantity model and in the next session we will try to calculate the value of the ordering of the value of the optimum quantity or the economic order quantity and see how we can further calculate the number of orders, how many times we must order after how many days we must order. We will try to understand that with the help of simple numerical problems.

Thank you.