

Operations Management
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Lecture - 49
Statistical Quality Control (SQC)

[FL] Friends welcome to session 49 in our course on Operations Management and currently we are in the tenth week of our discussion. And in this particular week our focus is primarily on ensuring the quality of the products that we are producing with our operations or with our production flow lines. And we have seen in the second week in the very first session the concept of quality and then we have seen in the second week the total quality management philosophy or the strategy of TQM

In third week we discussed about the TPM that is total productive maintenance and we try to understand that it is the duty of the organization or it is a responsibility of the organization to have a proper maintenance policy in order to keep all the machines up and running. So, that we are able to perform continuous production and are able to satisfy the demand as per the delivery scheduled which we have contracted with our customers or with our vendors.

Now, today our focus is on statistical quality control; we have seen the concept of quality we have understood the philosophy of total quality management. Now how we can calculate actually execute our plans and try to find out that whatever we are producing is well within the control or whatever we are producing is out of control or is beyond the variations which are acceptable by our customer.

Now, we have to see that actually measurement of certain criteria, measurement of certain variable, measurement of certain attributes which will help us to find out that whatever we are doing whether it is correct or we need certain rectification, we need certain changes which can help us to improvise our process in such a way that we are able to produce the product or to produce whatever is being made or is being fabricated or is being processed as per the requirement of the customer as per the design specifications.

Now, in statistical quality control there are 3 words the statistical, the quality and control. Now quality as we know is may be certain set of specifications that have to be met fitness for use from the customers point of view. So, whatever are the specifications or the design specifications that have to be ensured; so, we have to ensure that quality

Now, statistical means that we have to use statistics to ensure that specifications are met and control means that we have already decided the specifications we have set the limits; now we have to ensure that our process is within those limits only. So, therefore, we say that statistical quality control is we can say that the statistical method of controlling the quality of the product that we are producing. And there are number of methods or number of tools and techniques that we can use to exercise this concept of statistical quality control; that we will list today and try to understand one or two methods which we can employ to control our quality statistically.

Now, one important parameter that we must keep in mind is that always there is going to be certain variation in our output. Now whatever we are producing if the average value is 10 because of certain variations there will be some deviation in that value of 10; it can be 9.999 or it can be 10.10 or 10.09; there will be some spread across the average value that is 10 as per the specifications; why this will be there? There can be assignable causes or there can be random causes.

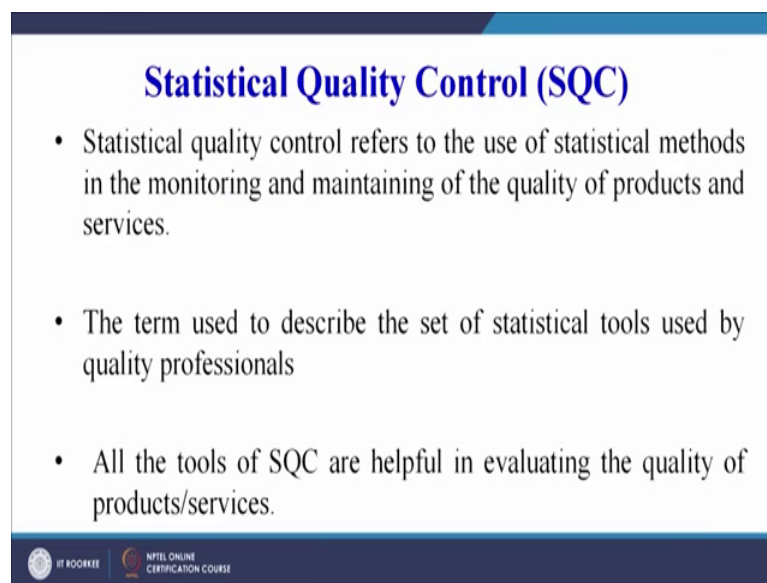
So, because of the combination of assignable and random causes; we will definitely get certain variation and this variation has to be accounted for we have to find out that whatever is the variation coming in the output characteristic, whether that is within the control it is within the acceptable limits or it is beyond the acceptable limit. Suppose this variation goes beyond an acceptable limit, we have to stop the process, we have to check, we have to look for the assignable causes we have to look that why the variation is going beyond a particular limit and how we can control the process how we can bring the process within the acceptable limits of quality or whatever is the accepted variability.

So, that is basically the concept with which we try to control the quality of our product. Now the quality of the product is dependent upon the process therefore, sometimes we call may be another technique that is statistical process control. Also in which we try to control the process in such a way that the quality that we are producing is within the

limits. So, when we exercise control on the quality part we always say that statistical quality control methodology

So, today our topic is SQC; so, we were going to see what do we mean by SQC? What are the various tools and techniques which can help us to perform statistical quality control and try to understand maybe one or two techniques with the help of certain numerical problems. So, let us quickly jump to our presentation and have a look at the concept of statistical quality control.

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Statistical Quality Control (SQC)

- Statistical quality control refers to the use of statistical methods in the monitoring and maintaining of the quality of products and services.
- The term used to describe the set of statistical tools used by quality professionals
- All the tools of SQC are helpful in evaluating the quality of products/services.

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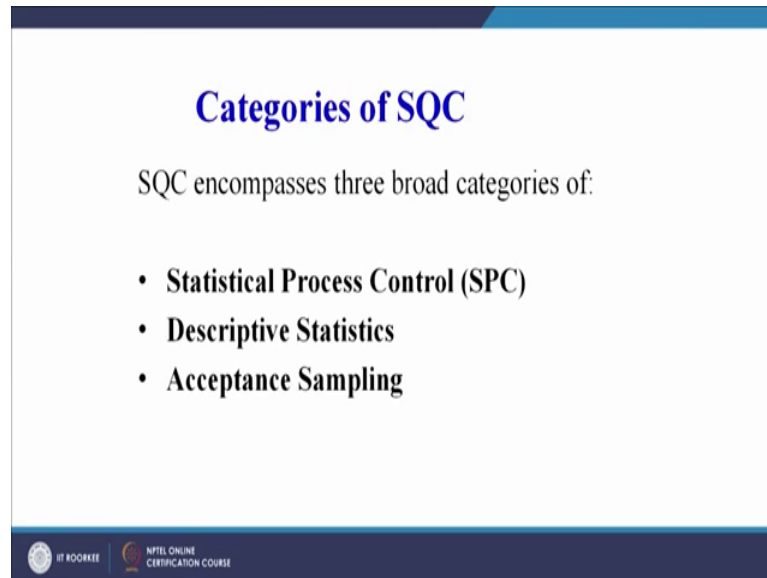
Now, statistical quality control refers to the use of statistical methods in monitoring and maintaining of the quality of products and services. So, you can see that I have explained earlier that three words statistical, quality and control; So, it as we are going to use statistical methods here for controlling the quality.

So, statistical quality control refers to the use of statistical methods in the monitoring and maintaining of the quality of products and services. The term used to describe the set of statistical tool tools used by the quality professionals. All the tools of SQC are helpful in evaluating the quality of the products and services. So, whatever I have just discussed as the introduction to today's discussion everything has been put here in the form of points.

Now, what are the categories of statistical quality control SQC and compasses 3 broad categories of statistical process control, descriptive statistics and acceptance sampling.

So, we are not going to discuss all these topics today; we are going to focus primarily on statistical quality control only.

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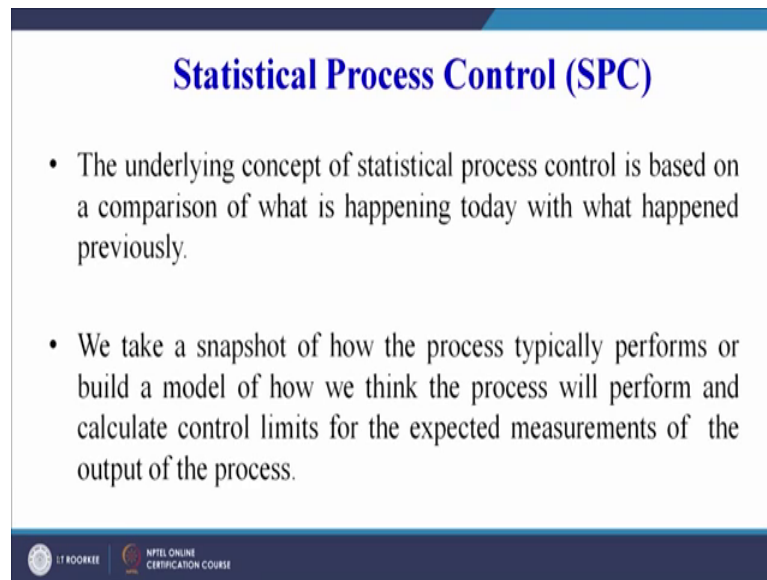


Whereas, acceptance sampling and descriptive statistics are also important tools that learners must focus on their own and try to figure out the concept of acceptance sampling the operating characteristic curve and try to figure out that what are the different inspection strategies. And how they can be used to control the quality of the product that we are producing, because our subject is an operations management and controlling the quality of the product that we are churning out that we are producing that we are processing using the operations is the most important point.

If we are managing the operations we are optimizing the use of our man-material equipment as well as resources, but we are producing something which is not of appropriate quality than the whole may be sequence or whole objective of the operations management is lost. We may have optimized all our resources, but if the product that we are producing is not of good quality; then whatever work we have put in managing the operations becomes useless.

So, therefore, checking the quality of the product that we are producing is very very important. And it can and this aspect cannot be neglected in the overall view of the operations management.

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Statistical Process Control (SPC)

- The underlying concept of statistical process control is based on a comparison of what is happening today with what happened previously.
- We take a snapshot of how the process typically performs or build a model of how we think the process will perform and calculate control limits for the expected measurements of the output of the process.

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And here we can see that statistical process control as I have already told is one important part of the statistical quality control. So, the underlying concept of the statistical process control is based on a comparison of what is happening today with what happened previously. So, that is a concept that whatever we are doing today what has happened, what we have planned earlier, how the process was performing maybe 10 weeks before how the process outcome is compared to the process outcome of the previous readings. So, all that can be is an important concept of statistical process control.

The underlying concept of statistical process control is based on a comparison of what is happening today with what happened previously. We take a snapshot of how the process typically performs or build a model of how we think the process will perform and calculate the control limits for the expected measurements of the output of the process.

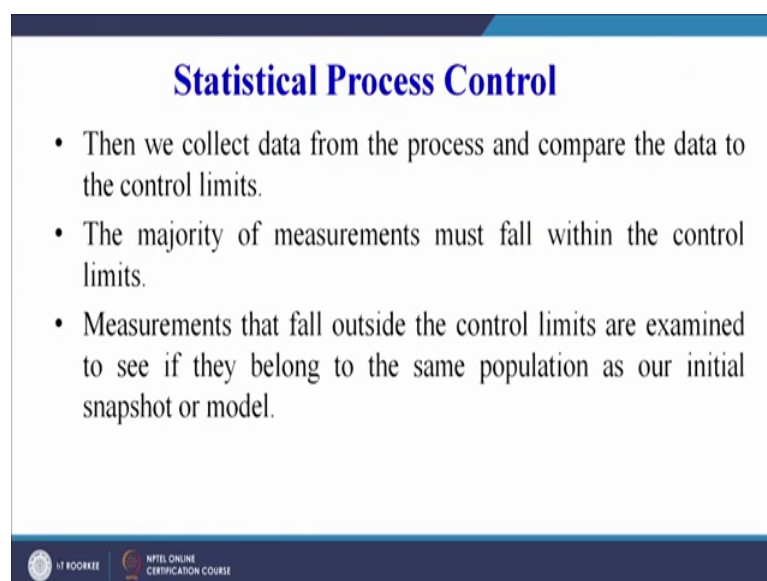
So, we can see we can we as we see a word future is mentioned here performs or build a model of how we think the process will perform. Will perform means that whatever is expected from the process that is the mean value whatever is the outcome or the expected outcome from the process; that is something we fix first and then try to see whether the process is within a certain degree of variability with that expected outcome or it is beyond a particular variation limit.

So, if it goes beyond whatever is the mean value as given here; if it goes beyond then goes far away from the mean value outside certain limits then we have to stop the process, we will say that the process is not under statistical control and the process needs to be stopped and checked. And we have to look for the assignable causes controllable causes and if we are not able to find any as and assignable causes; we may we attribute that variation to the random errors or the random causes beyond the control of the machinist or beyond the control of the supervisor.

So, we can say again I am reading this sentence that what statistical process control is all about. We take a snapshot of how the process typically performs; so, we know that how what is the expected outcome, what is the mean value of the measurement that we want to take or build a model of how we think the process will perform and calculate the control limit. So, we know the mean value we can see that what can with the control limits for this process and for the expected measurements of the output of the process.

So, we have to first fix our gauging criteria we know that this process is expected to give this mean value with this much limit of variation; which is acceptable. If the variation goes beyond the limit, then we have to see that what is going wrong and how it can be rectified or controlled.

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Statistical Process Control

- Then we collect data from the process and compare the data to the control limits.
- The majority of measurements must fall within the control limits.
- Measurements that fall outside the control limits are examined to see if they belong to the same population as our initial snapshot or model.

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So, then once we know we are we have started to exercise control, we know what is expected from this process; what is the expected degree of variation that is acceptable;

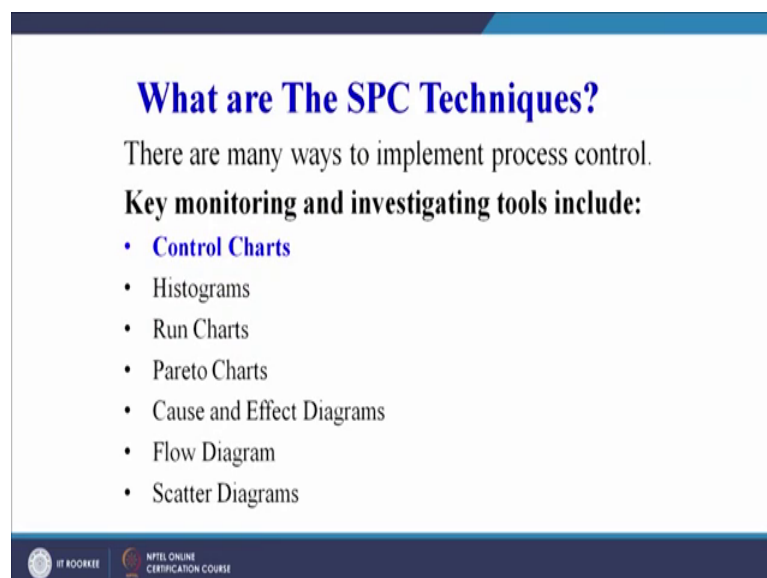
then what we do? We run the process, we collect the data from the process and compare the data to the control limits; that is control limits is the degree of variability that we are accepting as being there because of certain random variations which are cannot be controlled.

So, that variability due to random variations that we are we are accepting a double C e P t i n g; that we accept that this much variation is acceptable. So, we will we will compare the data that we have observed from the process with those control limits and see whether the data is within control limits or beyond the control limits.

The majority of the measurements must fall within the control limits; measurements that fall outside the control limit are examined to see if they belong to the same population as our initial snapshots or model. So, if the variations are beyond the control limit; we have to see that why this is happening and try to look for the causes leading to that variability in the process or the variability in the process outcome.

Now, what are the SPC techniques there are many ways to improve the implement the process control.

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What are The SPC Techniques?

There are many ways to implement process control.

Key monitoring and investigating tools include:

- **Control Charts**
- Histograms
- Run Charts
- Pareto Charts
- Cause and Effect Diagrams
- Flow Diagram
- Scatter Diagrams

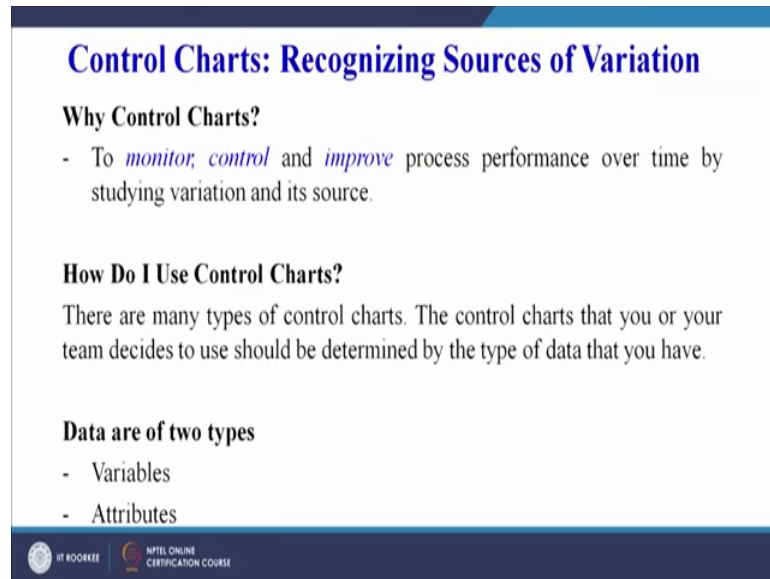
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Now we can see the key monitoring and investigating tools include control charts, histograms, run charts, pareto charts, cause and effect diagrams, flow diagrams. scatter diagrams. So, we cannot discuss each one of these, but one important point that we can

use here is the concept of control charts which is widely used in industry. So, we can see that we can use the histograms, we can use the pareto charts, we can use cause and effect diagrams in order to exercise control over our process.

Now, what are the control charts? On your screen you can see why control charts?

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Control Charts: Recognizing Sources of Variation

Why Control Charts?

- To *monitor*, *control* and *improve* process performance over time by studying variation and its source.

How Do I Use Control Charts?

There are many types of control charts. The control charts that you or your team decides to use should be determined by the type of data that you have.

Data are of two types

- Variables
- Attributes

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So, control charts are used to monitor because we will be checking taking the data plotting that data, control then we know that there are certain control limits the those are maybe the on the positive and the negative side of the mean value. So, we have mean value plus the variability that is acceptable on both ends. So, we first monitor the process that is we collect the data then we put that data on the chart and try to see whether everything is within the accepted degree of variation. And then if the sum of the values are beyond our control; then we try to improve the process performance over time by studying variation and its source.

So, as soon as the variation goes beyond the control limits; we may have to stop the process and check for the causes and then at the end try to improve the process in order to bring the process under control within the upper and the lower control limits. How do I use the control charts? Now what are the control charts is I think absolutely clear.

Now, how to use the control charts? There are many types of control charts; so, if you study the topic in detail you will see that there are control charts by variables, there are

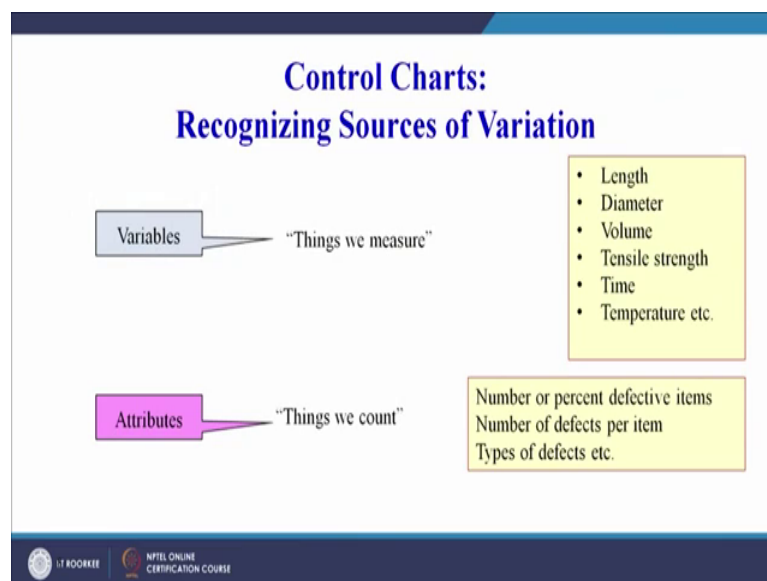
control charts by attributes. So, there are mean chart, range charts, percentage defective chart different types of charts are there; depending upon the situation, depending upon the requirement, depending upon the type of data that is available with us; we can construct these charts and use them as a controlling tool for managing the quality of the products or services that we are offering to the customers.

So, there are many types of control charts; the control charts that you or your team decides to use must be determined by the type of the data that you have; as I have already highlighted as per presentation only one criteria is given based on which we can select the type of chart to be used. But there are different types of criteria which will help us to identify the type of chart that we must use for bringing a process under control.

Now data are of two types basically; data can be variables data can be attributes. As I have already told we have two types of control charts; control charts by variables and control charts by attributes and we will try to see what are variables and what are attributes?

Now, the we can see the variables are things that we can measure for example, we can measure the length, we can measure the weight, we can measure the height, we can measure geometrical dimensions.

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So, there can be number of thing that we can measure and examples are given on the screen; you can see here length, diameter, volume, tensile strength, time, temperature etcetera all these things are variables which we can very easily measure.

So, the control charts that are based on statistical data of length diameter or things that we can measure; so, those types of control charts we will call as control charts by variables. And mean and range charts are usually you will call them as X and R charts are usually the types of charts that fall under the control charts by variables category. And then we have attributes things we count.

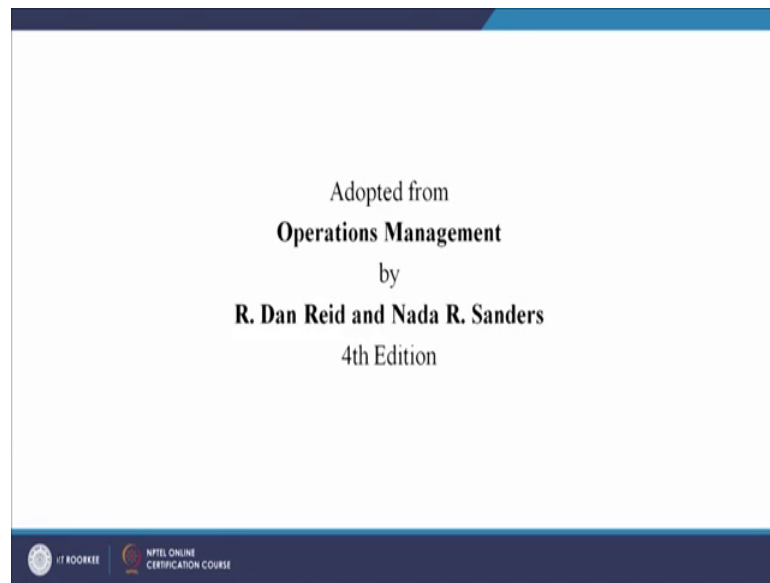
So, count means that sometimes we can count that if we switch on the light 100 times how many times the light will get switched off? Or if we check the number of bulbs for defectives for example, we a company is producing the bulbs; electric bulbs and out of 100 bulbs that have been checked we find out that 10 are defective and 90 are acceptable.

So, we can see we have counted the 10 are defective 90 are acceptable; So then number of percent defective items as I have given an example of the electric bulb number of percent defective item. So, 10 out of 100 are defective number of defects per item for example, we are using a led television at our house. So, led TV we can say that maybe in a year 3 types of 3 times there has been defect in the item or in the led, there are 3 different types of defects per item.

So, number of defects per item can be another measure which can be and falling under the control charts by attributes then types of defects also can fall under the control charts by attributes. So, usually P and C type of charts fall under the control charts by attributes category X bar and R charts usually fall under control charts by variables P and C charts are control charts by attributes.

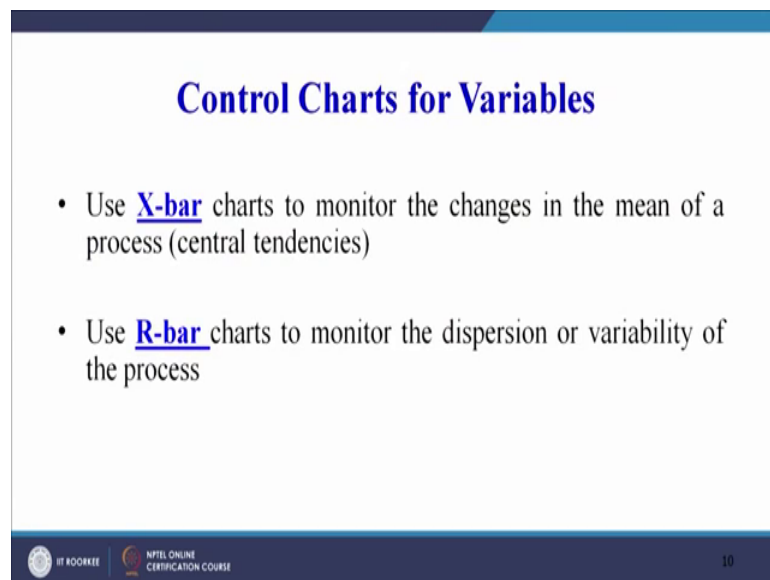
So, we will try to see this presentation further we have taken one or two example. So, these have been adopted from operations management.

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By Dan Reid and R Sanders nada R sanders 4th edition; so, we can see control charts for a variables. So, we have seen X bar chart.

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As I have already told and R bar charts are the two types of charts which fall under the control charts for variables category P and C chart fall under the control charts by attributes category

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Constructing an X-bar Chart: A quality control inspector at the XYZ company has taken **three samples with four observations** each of the volume of bottles filled. If the **standard deviation** of the bottling operation is **.2 ounces**, use the below data to develop control charts with limits of **3 standard deviations** for the 16 oz. bottling operation.

	Time 1	Time 2	Time 3
Observation 1	15.8	16.1	16.0
Observation 2	16.0	16.0	15.9
Observation 3	15.8	15.8	15.9
Observation 4	15.9	15.9	15.8
Sample means (X-bar)	15.875	15.975	15.9
Sample ranges (R)	0.2	0.3	0.2

Center line and control limit formulas

$$\bar{X} = \frac{x_1 + x_2 + \dots + x_n}{k}, \quad \sigma_x = \frac{\sigma}{\sqrt{n}}$$

where (**k**) is the no. of sample means
and (n) is the no. of observations in each sample

$$UCL_{\bar{x}} = \bar{x} + z\sigma_{\bar{x}}$$

$$LCL_{\bar{x}} = \bar{x} - z\sigma_{\bar{x}}$$

Standard deviation of process = Standard deviation of process mean / Square root of number of observations

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Now, this is an example how to construct an X bar chart this is a problem statement on your screen a quality control inspector at the XYZ company has taken 3 samples with 4 observations each. So, this is highlighted there are 3 samples taken with 4 observations each sample. Each of the volume of bottles filled now there are there is a bottling plant where the bottles are getting filled. So, the quality control inspector has taken 3 different readings of 4 samples each.

So, we can see 3 samples sorry 3 different samples at 3 different times 3 samples with 4 readings for each sample; 3 samples with 4 observations each of the volume of bottle filled what he has measured he has measured the volume of the bottles filled and this is a control chart by variable. So, we can measure the quantity for which we want to set the control limits.

If the standard deviation of the bottling operation is 0.2 ounce use the below data is given in the table to develop control charts with a limits of 3 standard deviations for the 16 ounce bottling operation. So, the mean value is given that 16 ounce is the expected volume of the bottles that have to be filled.

So, we can see sample 1 is at time 1. So, sample 1; 4 observations we have been taken sample 2 again 4 observation, sample 3 again 4 observations have been taken. So, the way the sorry the volume of the bottles filled has been measured using the equipment. So, the sample means first we have to calculate.

So, that is \bar{X} sample means and then the mean of these mean values we have to there are 3 mean values here for 3 samples each sample having 4 observation. So, we have calculated first for every sample that is sample mean, then we will calculate the mean of means that is $\bar{\bar{X}}$ that we will calculate; So, of these 3 values.

Similarly, we have calculated the range also if you can see the data here the maximum value is 16.0 and the minimum value is 15.8. So, the difference between the maximum and the minimum value is 0.2 and similarly for sample 2 and sample 3 we can calculate the range. Now the center line for our or the mean value, for our control chart that is \bar{X} bar chart will be the $\bar{\bar{X}}$ that is our central line.

So, $\bar{\bar{X}}$ is our sample means \bar{x}_1 plus \bar{x}_2 divided by the number of sample. So, we can see k is the number of sample samples sample means. So, sample means here are 3; so, we get the value of k as 3 and \bar{X} 3 values 15.875, 15.975 and 15.9

So, we have means for 3 samples here. So, these 3 values will come here divided by three. So, we will get $\bar{\bar{X}}$ which will represent the center line for our control chart that is \bar{X} bar chart. So, n is the number of observations in each sample. So, the upper control limit will be the $\bar{\bar{X}}$ plus z value which is we are already having this value 3 standard deviations.

So, z here will be 3 because it is already given in the problem that we are going to exercise control up to 3 standard deviations only. So, this is $\bar{\bar{X}}$ plus 3 standard deviations and then multiplied by the standard deviation of the process we have to calculate. And this is upper control limit, the lower control limit is the mean value center mean value minus the number of standard deviation that is 3 standard deviations multiplied by the standard deviation of the process.

So, we will calculate this upper control limit and the lower control limit for this data that is available with us. So, first for that standard deviation of the sample mean the sample mean here is 16 ounce. So, overall mean of the process we know the standard deviation of the process mean is given 0.2 divided by the square root of the number of observations. So, this $\sigma_{\bar{X}}$ which is given here $\sigma_{\bar{X}}$ how we will calculate this? $\sigma_{\bar{X}}$ represent the standard deviation of the process.

So, sigma X bar will be calculated as standard deviation of the process mean. So, standard deviation of the process mean is given as 0.2 ounce. So, this is 0.2 divided by square root of the number of observations; the square root of the number of observations is the number of observations here are 4 observations. So, we will get 0.2 divided by the square root of 4 that represents our standard deviation of the process, z is 3 standard deviations.

So, z is equal to 3 and x double bar is the mean of means that we have calculated here; these are the 3 means for the sample. So, sample 1 sample 2 and sample 3 3 mean value mean of sample 1, mean of sample 2 and mean of sample 3 So, this means 3 samples means divided by 3; so, this is a center line.

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

Solution and Control Chart (x-bar)

- **Center line (x-double bar):**

$$\bar{\bar{x}} = \frac{15.875 + 15.975 + 15.9}{3} = 15.92$$
- **Control limits for $\pm 3\sigma$ limits:**

$$UCL_{\bar{x}} = \bar{\bar{x}} + z\sigma_{\bar{x}} = 15.92 + 3\left(\frac{.2}{\sqrt{4}}\right) = 16.22$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - z\sigma_{\bar{x}} = 15.92 - 3\left(\frac{.2}{\sqrt{4}}\right) = 15.62$$

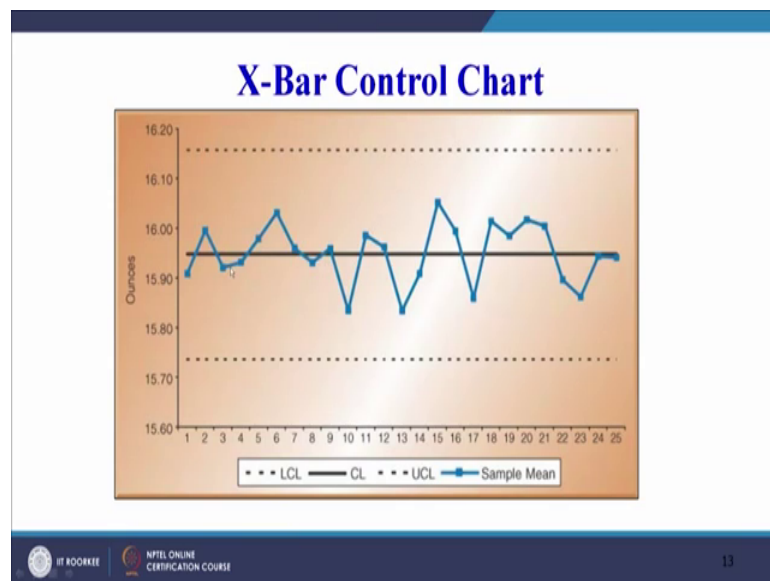


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So, I think it is clear let us see how the center line is 3 values 15.875, 15.975 plus 15.9 these are 3 sample means for 3 samples divided by 3. So, this is a center line and control limits can be calculated x double bar plus 3 times of standard deviation of the process. So, 15.92 is the grand mean we can say mean of sample means plus 3 times the standard deviation of the samp of the mean as we have seen in the previous slide; you can see standard deviation of the process mean standard deviation of the process mean divided by square root of the number of observations per sample.

So, with this we calculate 16.22 as the upper control limit and the lower control limit is again the centerline, minus 3 standard deviations. So, standard deviation of the we have

to take the standard deviation of the process which will be calculated as the standard deviation of the process mean divided by square root of, but the number of observations which is 4 per sample. So, we calculate the value as 15.62; so, this is upper and the lower control limit what is our average value? Centerline is 15.92; so, we calculate our we calculate using the formula upper and lower control limits and our control chart X bar control chart looks like this.

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This is a black line represents the center line which was fifteen point you can see 15.92. So, this is corresponding to 15.92 value and then these are the value sample mean how the sample means are changing so, so, many samples we have taken.

And upper control this is a center line this is a lower control limit and this is a upper control limit. So, we can see the upper control limit is 16.22 and lower is 15.62. So, this is representing 16.22 value and 15.22. So, this may be a upper control limit lower control limit and our all sample means are within the desired control limits.

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Control Chart for Range (R)

- Center Line and Control Limit formulas:
$$\bar{R} = \frac{0.2 + 0.3 + 0.2}{3} = 0.233$$
$$UCL_R = D_4 \bar{R} = 2.28(0.233) = 0.53$$
$$LCL_R = D_3 \bar{R} = 0.0(0.233) = 0.0$$
- Factors for three sigma control limits

Sample Size (n)	Factor for x-Chart	Factors for R-Chart	
	A2	D3	D4
2	1.88	0.00	3.27
3	1.02	0.00	2.57
4	0.73	0.00	2.28
5	0.58	0.00	2.11
6	0.48	0.00	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65

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Similarly, we can calculate the R chart also first we have to as we have seen we have calculate calculated the range for each sample. So, 3 3 samples were there the range for each sample we have calculated we will add it when divided by 3. So, we get the R bar value that is a range mean; mean of the range value for each sample.

So, there were 3 samples; so, this is the center line the mean value for all the ranges for all the samples. Then upper control limit is D 4 into mean range and lower control limit is D 3 in to the means of the ranges for the different samples. So, for getting these values D 4 and D 3; we can refer to this chart factors for 3 sigma control limit. So, our sample size n is equal to 4 4; here corresponding to this D 3 is 0 point 0 0 and D 4 is 2.28. So, the lower control limit is since D 3 is 0 so, D 3 0 in 2.223 which is the mean of the range values of the sample.

So, it is 0 only 0 multiplied by 0.233 and the upper control limit is corresponding to sample size of 4 the D 4 is 2.28. So, D 4 is 2.2 8 multiplied by 0.233 which is the mean of the range of 3 samples. So, the upper control limit comes out to be 0.53.

So, both we have seen that we can calculate the upper and lower control limits for the X bar chart, we can calculate the upper and lower control limits for the range chart also. So, we can control the sample means within the upper and lower control limit, we can also control the range or the variability within the upper and lower control limits for the range values for each sample also.

And if we get if we everything is within the control limits; we will say yes the process is producing the products within the acceptable limits. As soon as these mean values or the range values goes beyond the control limits, we will stop the process and try to check for the causes that are leading to this out of control variation in the process thus leading to the faulty output or a poor quality product. So, that is basically the concept that we exercise in order to produce good quality products.

So, with this I conclude today's session on statistical quality control; in next session we will discuss the concept of six sigma.

Thank you.